INDEPENDENT ORBIT DETERMINATION FOR COLLISION AVOIDANCE

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ABSTRACT

At ESOC, ESA's Space Operations Control Centre, the CRASS software (Collision Risk Assessment) is used to forecast close conjunctions of ESA operational satellites with any objects of the USSPACECOM catalog population. In most cases, the risk level can be considerably reduced, if the uncertainty of the chaser orbit is narrowed down by an orbit determination, which leads to an improved knowledge of the state estimate co-variances (within CRASS, these are estimated from TLE data of limited accuracy). The ODIN software (Orbit Determination via Improved Normal Equations) performs orbit determination with tracking data from radar and telescope sensors independent from the US SSN (United States Space Surveillance Network).

Key words: risk assessment; orbit determination.

1. INTRODUCTION

Long-term statistics indicate that at their operational altitudes near 780 km, and Sun-synchronous inclinations, the ERS and Envisat satellites are likely to have 2 to 3 conjunction events per year with individual collision risks exceeding 1:10,000. The Collision Risk Assessment tool (CRASS) generates conjunction forecasts and collision risk estimates of ERS-2 and ENVISAT with the objects of the USSPACECOM catalog population.

For ESA satellites, near real-time orbit determination data from the Flight Dynamics subsystem of these missions are employed. The initial covariances of ESA satellites are a by-product of the orbit determination process and they are provided together with the operational orbit files.

On the other hand, TLE (Two-Line Element) sets of limited accuracy are used to predict the orbits of the catalog objects. Metrics of the expected accuracy of the TLE sets are not available for the general public. For CRASS, the covariance of the TLE had to be independently assessed (see Alarcón (2002)). Since the actual accuracy of the TLE sets depends, among several other factors, on the type of orbit, the covariance was stored in the form of look-up tables sorted by eccentricity, perigee height and inclination. In general, the results of this analysis showed that the expected accuracy of TLE data is two orders of magnitude worse than the real-time orbit data used for operational spacecraft.

The collision risk assessments can be refined, if the accuracy of the chaser orbit is improved by an orbit determination, which also provides a better knowledge of the state estimate co-variance. The ODIN software (Orbit Determination via Improved Normal Equations) performs orbit determination which is independent from the US Space Surveillance Network (SSN). Several European sensors with known performances support this program: the FGAN radar (L-band) and the Monge radar (C-band) for LEO orbits, and the ESA telescope for GEO and GTO orbits. The state vector estimate, together with a full co-variance matrix of the solve-for parameters are used to obtain more accurate collision risk assessments from CRASS.

2. DESCRIPTION OF THE SENSORS

2.1. Characteristics of the TIRA Radar

The research institute FGAN (Forschungsgesellschaft fur angewandte Naturwissenschaften) is located at Wachtberg near Bonn/Germany. They are operating a Tracking and Imaging Radar (TIRA), which tracks non-cooperative targets in L-band (at 1.333 GHz). It is able to track objects as small as 2 cm in diameter at 1000 km distance. The range measurement resolution is typically at the meter level, while the angular resolution is around 0.0002 deg.

L-band tracking data of the TIRA radar are transmitted in ASCII format. Each tracking record contains information gained from the combination of successive L-band radar echoes within a predefined time interval that could be specified by ESOC. The minimum length of this time

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interval is given by the radar pulse period, which usually is about 30 Hz. In this case each data line would result from a single echo.

Radar measurements are azimuth (North: 0 deg, East: 90 deg), elevation (horizon: 0 deg, zenith: 90 deg), range in km, range rate in km/s, and echo amplitude (i.e. the signal-to-noise ratio) in decibel. By default, the data are not corrected for tropospheric refraction. Some tropospheric correction algorithms require local air temperature, pressure, and relative humidity. Therefore the header contains the necessary information.

2.2. Characteristics of the ARMOR Radar

Apart from FGAN's TIRA radar, the French Defence Ministry DGA operates the only other European installation of similar capabilities outside the Space Surveillance Network of the US Strategic Command. The most powerful radars of DGA are located on the vessel "Monge" of the French Navy. The "Monge" is primarily engaged in support operations for the French ballistic missile program. With its extensive equipment, however, it is a valuable asset in tracking operations of cooperative and non-cooperative targets. The "Monge" is for most of its time located at its naval home base in Brest, Bretany. It will be mainly during such periods that its tracking services could be available.

The instruments of interest for ESA applications are the two ARMOR radars. They can perform tracking of non-cooperative targets in C-band (at 5.5 GHz). The ranging resolution is typically below 1m, and the pointing resolution is normally within 0.0005 deg. The main clock of ARMOR is synchronised with the 1-sec pulses from a GPS receiver (resolution of about ± 2 micro-seconds). The local horizon and the North direction are established by means of gyro platforms (compensating for the attitude variations, particularly while on sea).

In contrast to the TIRA radar, the ARMOR radars are located on a moving platform. Hence, instantaneous position fixes are transmitted with every observation vector.

2.3. Characteristics of the ESA Telescope

ESA operates a Zeiss telescope of 1 m aperture and 0.7^o field of view (FoV), which is located on Tenerife. A liquid nitrogen cooled CCD array cumulates the received energy of the photons during exposure times on the order of 1 to 4 seconds, followed by 29-second gap times. The detection threshold is +19 to +21 mag for a signal-to-noise of S/N$\simeq$5. This allows to detect and follow objects of d$\simeq$15 cm at GEO altitudes (assuming an object albedo of 0.1). The ESA Space Debris Telescope (ESA SDT) covers a sector of $\simeq \pm 120^o$ of the GEO ring.

Pointing errors of the telescope (up to a few arc minutes) due not play a role, since object positions are defined relative to reference stars. Declination errors are approximately equal to right ascension errors. For bright objects (d$>$1m near GEO), the standard deviation is less than 0.5 arcsec.

Another error contribution is due to observation timing, resulting from

- a mechnical shutter with a (dominant) 20 ms timing error (1 sigma)
- an electronic timing error of less than 1 ms.

3. THE ODIN SOFTWARE

ODIN has been conceived as a suite of tools supporting the orbit determination process for catalogue objects. The tools that make up the ODIN suite are:

- TIRA2OTDF: Pre-processing of TIRA tracking data files
- ARMOR2OTDF: Pre-processing of ARMOR tracking data files
- ESASDT2OTDF: Pre-processing of ESA Space Debris Telescope tracking data files
- NTDF2OTDF: Conversion of Napeos tracking data files to the ODIN tracking data file format
- TLE2OTDF: Generates pseudo-tracking from a TLE
- Tracksim: Tracking simulator including simulation of moving stations
- Propag: Configurable numerical propagator including the following perturbing forces: non-spherical Earth gravity, aerodynamic drag, solar radiation pressure, and third body (Sun and Moon).
- TLEorbit: Propagates TLE using the SGP4/SDP4 model.
- PreOD Fixes: Deterministic preliminary orbit determination from radar fixes
- PreOD Angles: Deterministic preliminary orbit determination from optical angular measurements
- ODIN: Orbit determination program (includes Standard, Rank Reduction, and Levenberg-Marquardt batch least-squares techniques)
- TLEfit: Fits a TLE to a numerical orbit.
- Additional tools for simple Flight Dynamics computations

3.1. Preliminary Orbit Determination

Typically, the orbit determination process shall be initialised with an orbit based on an available TLE set. However, situations in which a-priori solutions based on TLEs lack the sufficient accuracy to start off the batch least-squares computation may eventually occur, especially during tracking of re-entering spacecraft. In this situation, a preliminary satellite orbit must be determined from a small set of available measurements without any additional information.

The type of measurements available in our application are position fixes from radar sensors (time, azimuth, elevation, and range) and angles for optical measurements, where range is not available (only time, right ascension and declination).

It is possible to obtain a preliminary state vector based on two radar fixes. Let $\vec{r}_a$ and $\vec{r}_b$ denote the satellite's geocentric position at times t_a and t_b, respectively. The area S of the sector that is bounded by $\vec{r}_a$ and $\vec{r}_b$ and the arc of the orbit between them, is

$$\Delta S = \eta A_t \tag{1}$$

Where A_t is the area of the triangle defined by the vectors. η is obtained by solving Eq. 2 from Montenbruck & Gill (2000)

$$\eta = 1 + \frac{m}{\eta^2} \cdot W\left(\frac{m}{\eta^2} - l\right) \tag{2}$$

with

$$m = \frac{GM_{\oplus} \cdot (t_b - t_a)^2}{\sqrt{2(r_a r_b + \vec{r}_a \cdot \vec{r_b})}^3} \tag{3}$$

$$l = \frac{r_a + r_b}{2\sqrt{2(r_a r_b + \vec{r}_a \cdot \vec{r_b})}} - \frac{1}{2} \tag{4}$$

$$W(w) = \frac{4}{3} + \frac{4 \cdot 6}{3 \cdot 5} w + \frac{4 \cdot 6 \cdot 8}{3 \cdot 5 \cdot 7} w^2 + \ldots \tag{5}$$

Taking into account the definition of the areal velocity:

$$\frac{dS}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} \approx \frac{\Delta S}{t_b - t_a} \tag{6}$$

The massless angular momentum vector may be defined as

$$\vec{h} = \vec{r} \times \vec{v} = r^2 \frac{d\theta}{dt} \vec{i_z} \approx 2 \frac{\Delta S}{t_b - t_a} \vec{i_z} \tag{7}$$

The unit vector perpendicular to the orbital plane, i_z, is given by

$$\vec{i_z} = \frac{\vec{r}_a \times \vec{r}_b}{|\vec{r}_a \times \vec{r}_b|} \tag{8}$$

Finally, the velocity vector may be expressed in terms of the radius vector and the momentum and eccentricity vectors as shown in Battin (1999):

$$\vec{v_a} = \frac{\mu}{h^2} \vec{h} \times (\vec{e} + \frac{\vec{r}_a}{|\vec{r}_a|}) \tag{9}$$

Where the eccentricity vector may be obtained from

$$\vec{e} = \frac{(\frac{h^2}{\mu} - |\vec{r}_a|)\vec{r}_b \times \vec{i_z} - (\frac{h^2}{\mu} - |\vec{r}_b|)\vec{r}_a \times \vec{i_z}}{|\vec{r}_a \times \vec{r}_b|} \tag{10}$$

The problem of finding an orbit from angle observations can be reduced to that of finding an orbit from two position fixes. Each set of angle measurements (for example right ascension and declination) defines a unit vector, which describes the direction from the station to the satellite at the observation epoch. Since range is not available, the distance has to be derived during the process of determining the orbit. In order to obtain the state vector in an unambiguous manner, three sets of observations must be available. From these values, and the known station location, the satellite position at the observation epoch can be derived in an iterative way. The method is described in Montenbruck & Gill (2000). Knowing the position vectors, the velocity vector may finally be computed from Eq. 9.

The methods presented above determine the orbit from a minimum number of measurements. Nevertheless, one expects a series of measurements distributed over one or more passes. The tracking data may be split into subsets of the minimum number of measurements (two for radar and three for optical measurements). For each subset of consecutive observations the state vector is computed and propagated up to a common orbit determination epoch, where the mean value and the standard deviation for each state vector component may be computed. Next, in order to remove any outliers, the observations with residuals larger than a pre-defined number of standard deviations from the average are discarded and the final average is computed with just the remaining measurements.

As an example, we will use the very low orbit specified in the first column of Tab.1. From the nine passes over TIRA with a minimum elevation of $5°$ detected with the tracking simulator of ODIN for the period 2003/05/04 to 2003/05/07, we selected the three passes shown in Tab. 2.

First of all, simulated measurements were generated over the visibility periods at a rate of one per second. The sim-

Table 1. Orbit comparison on 2003/05/07-00:00:00.0.

Element	Reference	Prelim.	Determ.
S/M Ax., km	6595.085	6603.571	6594.963
Eccentr.	0.00348	0.00357	0.00350
Inclin.	$98.524°$	$98.515°$	$98.524°$
Asc.Node	$60.671°$	$60.674°$	$60.673°$
Arg.Per.	$148.504°$	$157.425°$	$149.225°$
Tr.Anom.	$99.653°$	$84.107°$	$98.982°$
Drag	2.0	1.8	2.001

Table 2. Selected TIRA passes from 2003/05/04 to 2003/05/07.

Pass	AOS	LOS	Culmination
1	04-01:07:45	04-01:13:47	$73.2°$
2	05-01:09:43	05-01:15:34	$75.2°$
3	06-12:43:55	06-12:48:25	$12.3°$

ulated noise level of 0.011 km for range, $0.010°$ for azimuthn and $0.012°$ for elevation was taken from a calibration of the TIRA with SPOT4 precise DORIS ephemeris (see Ameline (2001)).

A preliminary orbit was obtained from the tracking data of the first pass, using just one observation every ten seconds. The orbital parameters at the orbit determination epoch (2003/05/07-00:00:00.0) are shown in the second column of Tab. 1. The drag coefficient was not estimated, but fixed at 1.8. The measurement residuals for this orbit determination are shown in Fig. 1.

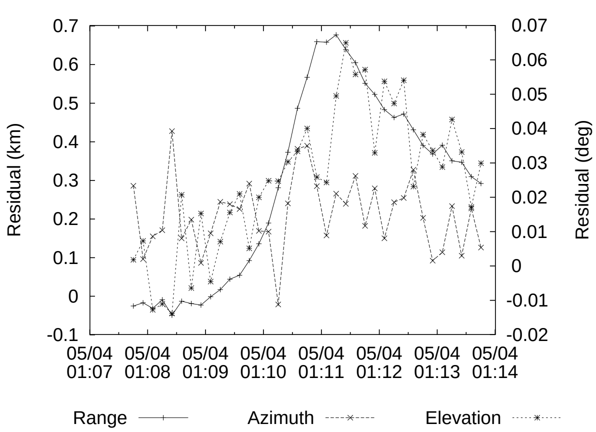

Figure 1. Residuals of the preliminary orbit determination.

3.2. Singular Value Decomposition

In the basic least-squares estimation method, the solution is the extended initial state vector that minimizes the loss function:

$$J(\hat{X}_0) = \frac{1}{2}\varepsilon^T\varepsilon \quad (11)$$

Where the quantities ε_i are the residuals that account for the difference between actual and modeled observations due to measurement and model errors.

The iterative solution using the Gauss-Newton method is:

$$X_{k+1} = X_k + \left(F^T F\right)^{-1} F^T \varepsilon_k \quad (12)$$

Where F is the matrix of observation equation coefficients, which contains the partial derivatives of the computed observations with respect to the estimated parameters.

The common procedure to account for the a-priori knowledge is through the insertion of the a-priori covariance of the estimated parameters, P_0. In this case, one tries to minimize the following loss function

$$J(\hat{X}_0) = \varepsilon^T \cdot \varepsilon + (\Delta\hat{X}_0 - \Delta X_0)^T \cdot P_0^{-1} \cdot (\Delta\hat{X}_0 - \Delta X_0) \quad (13)$$

The iterative solution in this case is

$$X_{k+1} = X_k + (F^T F + P_0^{-1})^{-1} \cdot (F^T \varepsilon_k + P_0^{-1} \cdot \Delta X_0) \quad (14)$$

Assuming that the a-priori covariance matrix may be defined as

$$P_0^{-1} = {S^{-1}}^T S^{-1} \quad (15)$$

One may use matrix S^{-1} to change the definition of the estimated parameters:

$$X' = S^{-1} X \quad (16)$$

The matrix of partial derivatives corresponding to the new non-dimensional parameters is:

$$F' = F \cdot S \quad (17)$$

Taking into account the above definitions, The non-dimensional form of Eq. 14 yields

$$X'_{k+1} = X'_k + (F'^T F' + I)^{-1} \cdot (F'^T \varepsilon_k + \Delta X'_0) \quad (18)$$

ODIN uses the singular value decomposition method to solve the normal equations. This method is well suited for ill-conditioned problems. It allows the detection of singularities or near singularities in the normal matrix.

Furthermore, the formulation of the method is convenient for the Levenberg-Marquardt algorithm. On the other hand, the method is computationally more involved than other techniques, like the QR factorization. However, due to the limited amount of tracking data that will be handled by the current application this is not considered a major draw-back.

The singular value decomposition of the partial derivative matrix is denoted by

$$F_{m\times n} = U_{m\times n} D_{n\times n} V^T_{n\times n} \tag{19}$$

where U and V are orthonormal matrices, which means that both $U^T U$ and $V^T V$ are equal to the n-dimensional identity matrix and D is a diagonal matrix of elements $d_1 \geq d_2 \geq \ldots \geq d_n \geq 0$ known as singular values. There are exactly r positive singular values for a matrix of rank $r \leq n$.

Introducing the following definitions:

$$\vec{s} = V^T \Delta \hat{X}_0 \tag{20}$$

$$\vec{t} = U^T \varepsilon_{ref} \tag{21}$$

Substituting the new variables in Eq. 18 yields:

$$s_{k+1} = s_k + (D^2 + I)^{-1} \cdot (D t_k + \Delta s_0) \tag{22}$$

3.3. Rank Reduction and Levenberg-Marquardt

Due to the limited observability of a given orbit from Europe, and due to possible applications for re-entry orbit estimations, the orbit determination algorithm has to be robust, and provide convergence also for poorly conditioned systems. Apart from the standard least-squares technique, ODIN implements the Rank Reduction and Levenberg-Marquardt methods.

The form of Eq. 22 in a case without a priori covariance information is:

$$s_{k+1} = s_k + (D^2)^{-1} D t_k \tag{23}$$

If F is not a full-rank matrix, only the first r components of $\vec{s}$ can be determined, while the remaining components are arbitrary. Setting $s_i = 0$ for all $i > r$ yields the smallest norm solution.

The same principle may also be applied in the case that F has full rank but is nevertheless near-singular as indicated by a high ratio d_1/d_m of the largest and smallest singular value. This ratio, which is also known as the condition number of the normal matrix, gives a general indication of the quality with which the solution is defined by the given measurements. In order to avoid a deterioration of the solution it may be preferable to neglect contributions from small singular values. This method is known as rank reduction. The loss function obtained in this manner is slightly higher than the exact minimum, but it may be preferable to a solution that is far off the correct value due to the strong influence of measurement errors.

The Levenberg-Marquardt method applied to the least-squares problem searches the minimum of the loss function given by Eq. 11, subject to

$$|dx| = \sqrt{\left(X_{k+1} - X_k\right)^T \left(X_{k+1} - X_k\right)} \leq \delta \tag{24}$$

It can be shown that the solution to this problem has the form (see Dennis & Schnabel (1996)):

$$X_{k+1} = X_k + dx = X_k + \left(F^T F + \mu I\right)^{-1} F^T \varepsilon_k \tag{25}$$

for the unique $\mu \geq 0$ such that $|dx(\mu)| = \delta$, unless $|dx(0)| \leq \delta$, in which case $\mu = 0$ defines the solution.

In order to obtain μ, let us define:

$$\eta(\mu) = |dx(\mu)| - \delta = |(F^T F + \mu I)^{-1} F^T \varepsilon| - \delta \tag{26}$$

The root of $\eta(\mu)$ may be obtained iteratively by Newton's method, with

$$\eta'(\mu) = \frac{d|dx(\mu)|}{d\mu} = -\frac{dx^T (F^T F + \mu I)^{-1} dx}{|dx(\mu)|} \tag{27}$$

The Singular Value Decomposition of the system is very convenient for the Levenberg-Marquardt algorithm. Indeed, the solution to the problem in the variable s has the form

$$s_{k+1} = s_k + (D^2 + \mu I)^{-1} D t_k \tag{28}$$

In this case:

$$\eta(\mu) = |s(\mu)| = |\left(D^2 + \mu I\right)^{-1} D t_k| \tag{29}$$

$$\eta'(\mu) = -\frac{s^T (D^2 + \mu I)^{-1} s}{|s(\mu)|} \tag{30}$$

The iterative process to obtain μ is very efficient, since the modified Hessian matrix $(D^2 + \mu I)$ is diagonal and its factorization is straightforward.

The orbit determination problem from the three passes of Tab. 2 and the preliminary solution in the second column of Tab. 1, could be solved using the Levenberg-Marquardt method The solution is shown in the third column of Tab. 1.

Table 3. Numerical orbit fit of a TLE.

Element	Value
Epoch	2003/05/01-00:00:00.000000
S/M Axis	7168.490 km
Eccentr.	0.00122
Inclin.	$98.528°$
Asc.Node	$315.702°$
Arg.Per.	$70.058°$
Tr.Anom.	$289.931°$
Drag Coeff.	2.170
Solar Rad. Coeff.	1.300

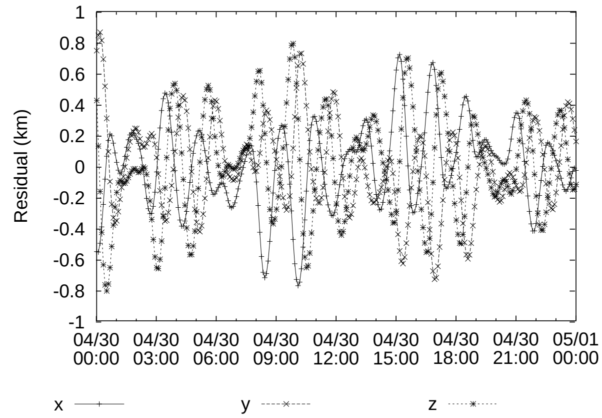

Figure 2. Residuals of the TLE fit.

3.4. Pseudo-t racking from TLE

ODIN may generate pseudo-tracking from a given TLE in the form of inertial positions propagated with the SGP4/SDP4 theory. In this manner, the software may incorporate TLE state vector information to complement the independent tracking. This information may also be used in the orbit determination process to fit a numerical orbit to the TLE, thus improving the long-term stability of the orbit in case of infrequent TLE updates.

As an example of this capability, the Levenberg-Marquardt method was used to fit a numerical orbit to the following TLE:

1 23560U 95021A 3121.00000000 -.00000062 00000-0 -68250-5 0 8212 2 23560 98.5482 315.7474 0001243 94.2397 265.8923 14.32249494489401

The selected orbit determination arc goes from 2003/05/01-00:00:00.0 to 2003/05/02-00:00:00.0. We use the Levenberg-Marquardt method with a maximum weighted step size of 1.0. The increment of the parameters are non-dimensionalised with 1 km for positions, 1m/s for velocities, and 0.01 for the drag and solar radiation pressure coefficients. The iteration stops when the change in RMS of the weighted residuals is less than 0.1%. The initial state vector is taken from the TLE prediction, the initial drag coefficient is 2.2, and the initial solar radiation pressure coefficient is 1.3. The fitted orbit is shown in Tab. 3 and the residuals of the TLE fit are shown in Fig. 2.

We may see here how the Levenberg-Marquardt method works. The effects of the drag coefficient and solar radiation coefficients are almost unobservable, as indicated by the large condition number 0.19634E+08. Therefore, they remain almost unchanged in the process.

REFERENCES

Alarcón J.R., Martinez F.M., Klinkrad H., Development of a Collision Risk Assessment Tool., *Advances in Space Research*, vol. 34, pages 1120-1124, 2004.

Alarcón J.R., Development of a Collision Risk Assessment Tool. Technical Report (final report) ESA contract 14801/00/D/HK, GMV S.A., Mar.2002.

Ameline P., Calibration of the TIRA and ARMOR Radar Systems. Technical Report (final report) ESA contract 14475/00/D/HK, GMV S.A., Dec.2001.

Montenbruck O., Gill E., *Satellite Orbits*, Springer-Verlag, 2000

Battin R.H., *An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition*, AIAA, 1999

Dennis J.E., Schnabel, R.B., *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, SIAM, 1996