# OPTIMIZATION OF PROTECTION STRUCTURES


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## ABSTRACT

The cost of every kilogram of a load placed in orbit is high and it makes us to treat carefully the problem of spacecraft protection against space debris and meteoroid impacts. This problem is multifold one and it gets under way from a rough estimation of the damage probability for the spacecraft and finding out a way for enhancing spacecraft reliability by improved assembling. On the other hand you need to develop a protection system for the spacecraft: to create protection shella, to find out the ballistic limit equations, and, if it is necessary, to verify them in experiments. Constructions using metal-mesh screens are currently underway. In the last stage the assembling process that optimizes mass distribution between protection structure elements is worked out again.

The optimization process can't be algorithmizable sometimes, but convenient software can be useful to solve the problems, especially when complex objects are addressed. Three finding algorithms are developed in the program COLLO2000: the graphical visualization of the optimization process, the Monte-Carlo method and the finite elements method. The results on the model simulations are provided.


## 1. INTRODUCTION

Designing of the optimal protection for a spacecraft includes several stages as following:

-obtaining initial data on the configuration of the spacecraft, its orbital parameters and life time,

-preliminary evaluation of reliability of the spacecraft as a whole and each of its elements,

-preliminary evaluation of the ballistic limit parameters of the outer surface elements apart,

-choice of spacedebris and meteoroid environment models,

-calculating the collisional risk in the mission,

-development of suggestions on the protection system and enhancing spacecraft's elements,

-optimization of protective constructions.

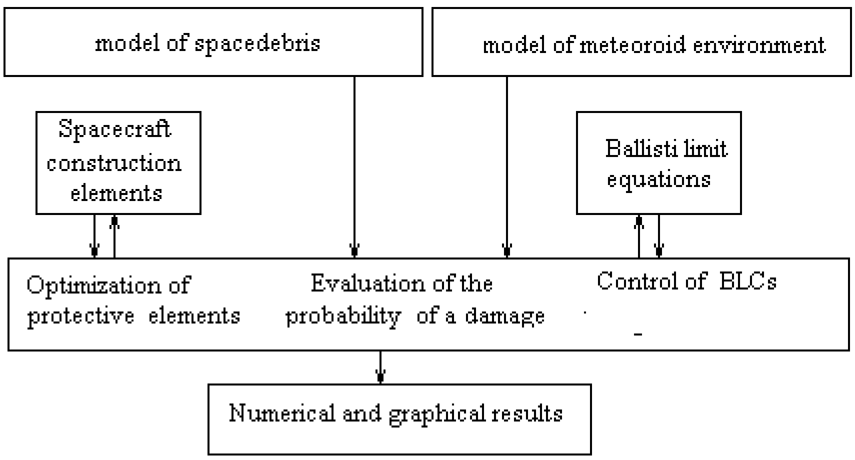


*Fig.1. A scheme of optimization problem for a spacecraft protection.*

This problem is a complex one and can be solved .in the iterative way. The software to provide probability calculations considering data on space pollution and ballistic data of differential protective structures is developed and some results are presented here.

By now the optimization problem is examined in (Reimerdes Hans-G., 2001, 2003.), (Stokes H, 1999, 2000).There is supposed a new method of hyperbolic elements instead of usually used the Monte Carlo simulations.


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## 2. TARGET SETTING

A basic function to evaluate probability of a damage of the spacecraft as a consequence of an impact of orbital debris or meteoroids is the damaging stream:

$$F_d = \sum_k \sum_i \int_{T_1}^{T_2} \iint_{S_k} \int_v \iint_\Omega \oint_L F(d_{ki}) \cdot v_n \cdot di \cdot dS \cdot dv \cdot \cdot d\Omega \cdot dt \ , (1)$$

where $L$ is the orbit of a spacecraft,

$S_k$ – $k$-th element of the outer surface area of the spacecraft,

$N_i$ – space density of distribution (concentration) of the $i$-th component of space debris or meteoroid environment,

$T_1$ and $T_2$ – time of beginning and ending of the spacecraft orbital mission,

$\Omega$ - a solid angle opened to impact for the considered part of the spacecraft outer surface,

$v$ – a speed of the particle,

$v_n$ – the normal component of a particle velocity,

$F(d_{ki})$ – cumulative flux of space particles with size greater than $d_{ki}$,

$d_{ki}$– critical size of a damaging particle of the $i$-th component of environment for $k$-th element of the surface.

It is supposed that that the collisional damage is an occasional event governable by the Poisson's statistics. So the probability of a damage is determined by the following expression: $PNP = e^{-F_d} \cong 1 - F_d$. Accoding to the dasic equation (1) the protective system consists of separate elements protecting different parts of the spacecraft, and when the value of integral damaging stream by expression (1) is given it is needed to calculate an optimal distribution of the additional mass between the elements:

$$M = \sum_k S_k m_k \ ,$$

Here $m$ is a surface density of the $k$-th element.

The solution in a great part is determined by the chosen models of spacedebris and meteoroid environment. Calculations have shown that the distribution of trajectories of spacedebris and meteoroids plays a large role in the choice of the protective constructions. In this work the model ORDEM2000 (Liou Jer-Chyi, 2000) is used.

## 3. ON THE OPTIMAL VIEW OF THE INFORMATION ON THE ORBITAL ENVIRONMENT RELEVANT TO THE PROTECTION

It is impossible to overestimate the role of models of space debris and meteoroid environment in estimation of damage risks and designing an optimal protection construction. The laboriousness of development of reliable models is several orders comparatively to development of codes to calculate and optimize a spacecraft protection (But without building and calibration of ballistic limit equations) – the protection problem brings only the calculations multidimensional collisional integrals. But nevertheless it is desirable to separate the protection problem which begins with recalculations of the orbital distributions of spacedebris and meteoroids into its velocity speed and trajectoties distributions in the concomitant coordinate system. So there should be a note of this trasformation. How it was noticed by (Reimerdes H-G., 2001), the model ORDEM-96 (Kessler D.J.,.1996) was ideal in this regard. But used multidimensional approximations are rather laborious and rough. In the model (Liou Jer-Chyi, 2000) the table form of information presenting in the concomitant and geocentric coordinates is used. The shortage of such approach is a forced roughness of the net in sizes of particles that leads to a large dependence of results on an used interpolation methods. In this case it is impossible to meet requests of industry on the accuracy of calculations of the protection constructions ( to be exact, in concordance of the results). Maybe the contradiction would be a table form of distributions in the Keplerian coordinates which differ by smoothness.

## 4. AN OPTIMIZATION PROBLEM FOR A SPACECRAFT PROTECTION MASS

Reliable spacecraft protection against meteoroid and space debris needs substantial additional mass outlay which can be compared with the payload mass. But some directions are more dangerous than others, so the mass redistribution essentially affects the effectiveness of the protection system. It is possible to distribute the additional mass between protective elements by hand, but if there are too many protective elements such approach would be rather difficult. This problem was analyzed

In some works (Reimerdes Hans-G., 2001, 2003.), (Stokes H, 1999, 2000) the genetic approach to the optimization was suggested. It is a variant of the Monte Carlo method (MCM) and it allows substantially reduce time of the designing work. Such choice was determined by the characteristics of target functions which have local extremums, so standard "fast" algorithms of the kind of the steepest descent method doesn't work without additional analysis. But the results can be displaced in certain respects. Like with some other problems the method of finite elements (MFE) is an alternative to the MCM, and provide a fast and accurate solution of the problem. . Its disadvantage however lies in comparative complexity. This method is rather perceptible to the alteration of the problem's details. Nevertherless MFE allows to develop fast and exact codes.

## 5. A MATHEMATICAL PROBLEM. A METHOD OF HYPERBOLIC ELEMENTS IN A PROBLEM OF CONVEX ANALYSIS

Let a nonlinear operator *M(x):* $X \to Y$ be differetiable in the sense by Frechet. *X* and *Y* – Banach spaces (Trenogin V.A., 1980.). Let determine the norm of the operator *M(x)* as $\|M(x)\| = \sup_{x \in D} \|M(x)\|$, where *D* – some convex subset in *X*. Fixed points of this operator are determined by the equality of its Frechet differential to zero: $dM(x_0, h) = 0$, where *h* is a differential of the variable *x*.

The basic conditions of the target functional that provide existence and uniqueness of the solution for the mentioned above optimization problem are the following:
1) a fixed point exists if the operator *M(x)* is continuous and is mapping some convex set $\Omega$ from n-dimentional Banach space into itself (by Brauer's theorem);
2) the fixed point exists if M*(x)* is maooing a convex subset *D* (not necessarily finite one) of the Banach set *X* into a compact subset $R \subset D$ (by Shauder's theorem).
Let introduce the target function as a sum of piece-wise 1-D hyperbolic functions of the instrumental variables $M(x_1,...,x_n) = \sum_{i=1}^{n} f_i(x_i)$ subject to $g = x_1 + ... + x_n$ and $x_1 \geq 0$, …, $x_n \geq 0$. The above conditions of existance and uniqueness of the solution are implemented. The algorithm is stable. Approximation is obvious. So obtained converge to an exact solution. The real rate of convergence is determined by properies of the target function rather than the approximation order. In the practice the convergence is high. .

## 6. OF THE PROTECTION AGAINST SPACE DEBRIS FOR A CUBE MODEL

Let's examine a cube model (Fig. 2) in a standard orbit (orbit altitude of 400 km and inclination of 51.6 degrees).

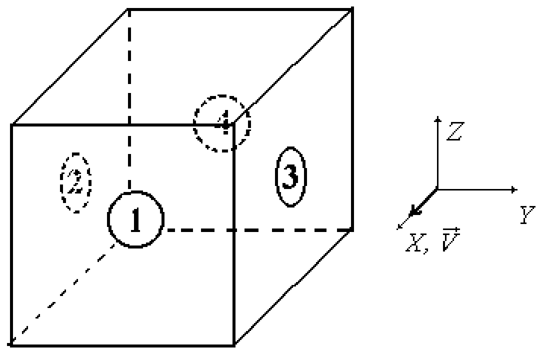


*Fig. 2. A cube model. 1 – front, 2 – right, 3 – left, 4 – back faces.*

According to the model ORDEM2000, streams at upper and lower faces are zero, and these faces do not take part in the optimization process. Due to the symmetry of the distribution functions for orbital objects in the concomitant coordinates the left and right faces are in equal conditions. But following the traditions these faces are analyzed separately. The symmetry of the solution is an additional criterion of its accuracy.

**1-st case.** Every surface of the cube is considered a single wall strcture with a given thickness. Let the total limit for damaging stream to be $10^{-3}$ particles per year. Calculated values for the shell plate thicknesses are given in Tab. 1, the sections of the target function in coordinares $x_1$ and $x_2$ with the minimum point are given in Fig. 3 and 4.

*Table 1. Optimization of thickness of the single wall structure the cube model when the total damaging stream is $10^{-3}$ particles per year.*

| Faces of the cube | Damaging streams (1/year) | Thickness (mm) | MDPANTO Thickness (mm) |
|---|---|---|---|
| Front | $0{,}345 \cdot 10^{-3}$ | 7,11 | 15.0 |
| Starboard | $0{,}299 \cdot 10^{-3}$ | 7,48 | 7.8 |
| Port | $0{,}289 \cdot 10^{-3}$ | 7,23 | 7.4 |
| Back | $0{,}068 \cdot 10^{-3}$ | 2,11 | 1.0 |
| Average | | 6,00 | 7,8 |

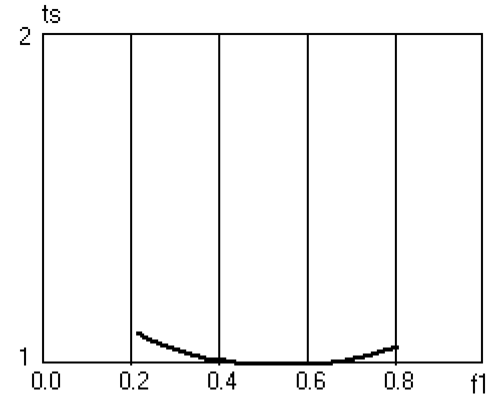


*Figure 3. The section $x_1 - x_2$ of the target function for the cube model with the single wall structure.*

The Fig. 3 depicts the dependence of the summary thickness $t_s$ for two surfaces of the cube – front and right depending on the density part of the stream that penetrates the front surface is given. It is the section $x_1 - x_2$ of the target function. In the global minimum the sum of the damaging streams on the front and right faces is 0,000568 particles/year. In the fig. 4 the section $x_1 - x_4$ of the target function is given. The corresponding sum of the damaging streams for these surfaces is 0,000419 particles per year.

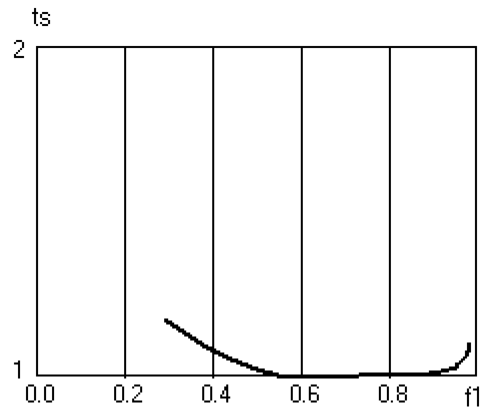


*Figure 4. A section $x_1 - x_4$ of the goal functions for a cube model with single walls.*

In Fig. 4 shown the expected tendency of the curve when the thickness of the back surface goes to zero.
In Fig. 5 the level lines of the target function on the plane $X_1X_2$ (a section x1 – $x2$ – $x3$ )are given. The total damaging stream on the 1-st, 2-nd and 3-rd surfaces is 0.000858 particles per year. The line level of (1 + 1/80),…, (1 + 1/20) related to the minimum are shown.

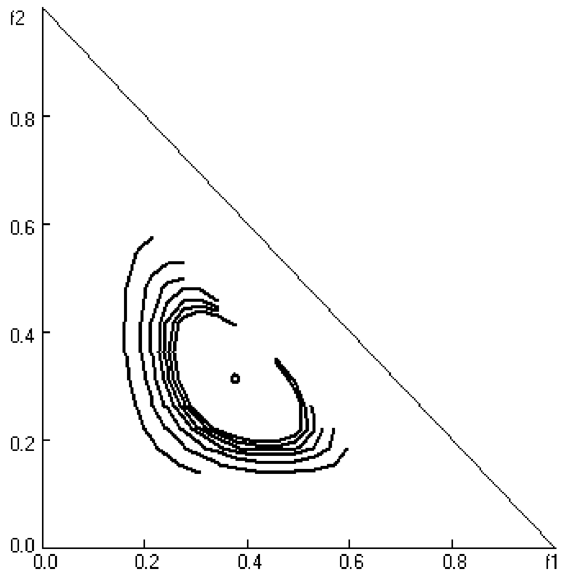


*Figure 5. Section x1 – $x_2 - x_3$ of the target function for the cube model with single wall structures.*

If the cube is protected by single wall structures of equal thickness for the four sides mentioned above then their total thickness must be no less than 7.09 mm to endue the stream of PNP=$10^{-3}$ particles per year. The theoretical economy from the optimized construction is about 11.8 kg.

**2-nd case**. Each surface in the cube model has a Whipple shield (0.1mm+10cm+1mm): the back wall thickness of 1 mm; the gap between the bumper and the back wall of 10 cm. The thickness of the back wall can be varied. Let the total damaging stream to be $10^{-4}$ particles per year. The ballistic limit equation by Christiansen (Christiansen E. L., 2001.) is used. The calculated back wall thickness is given in the Tab. 2.

*Table 2. Optimization of the back wall thickness in the Whipple protection for a cube model (total damaging stream is of $10^{-4}$ particles per year).*

| Face | Damaging stream through the face, (1/year) | Back wall thickness (mm) |
|---|---|---|
| Front | $0{,}277 \cdot 10^{-4}$ | 0,224 |
| Left | $0{,}291 \cdot 10^{-4}$ | 0,284 |
| Right | $0{,}290 \cdot 10^{-4}$ | 0,282 |
| Back | $0{,}142 \cdot 10^{-4}$ | 0,146 |
| Average | | 0,234 |

If the Whipple shield will be the same for all the considered sides of the cube then the thickness of the back wall must be about 2.62 mm to endure the level of PNP=$10^{-4}$ particles per year. The theoretical mass economy from the optimized construction is about 3.0 kg.

In Fig. 6 the section $x_1$ - $x_2$ for the target function is shown. This section includes the point of the minimum of the target function. (the total damaging stream on these walls is $0{,}568\cdot10^{-4}$ particles per year). And in Fig. 7 the section $x_1$ - $x_4$ is given. In Fig. 8 the level lines for the target function over the plane $X_1X_2$ are given: these lines correspond to the levels of (1 + 1/50), …(1 + 1/10) related to the minimum of the target function. The total damaging stream in the last case is $0{,}858\cdot10^{-4}$ particles per year.

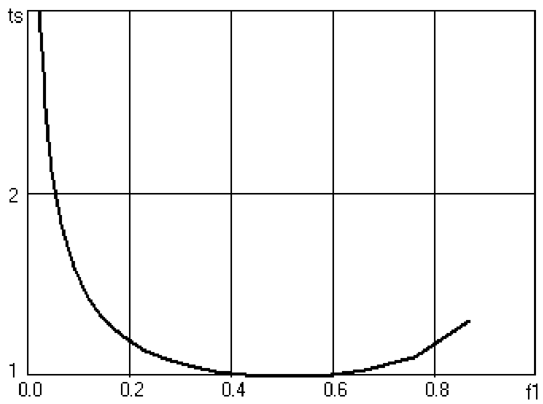


*Fig. 6. The section $x_1$ –$x_2$ of the target function for the cube model with the Whipple shield.*

It should be noted that in both case with the single wall and the Whipple shoeld the target function is sufficiently smooth. In the first case the minimum is faded, the second case the minimum is expressed fairly. The reason is that the thickness of the back wall has a little influence on the optimal distribution.

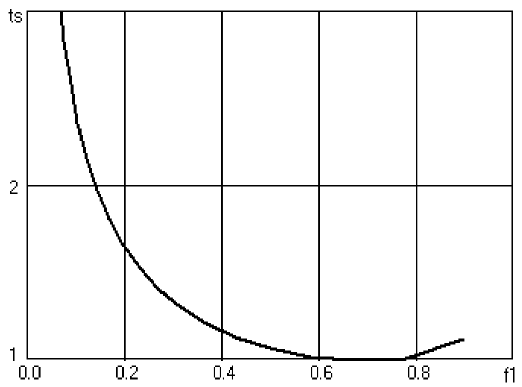


*Fig. 7. The section $x_1$ –$x_4$ of the target function for the model cube with the Whipple shield.*

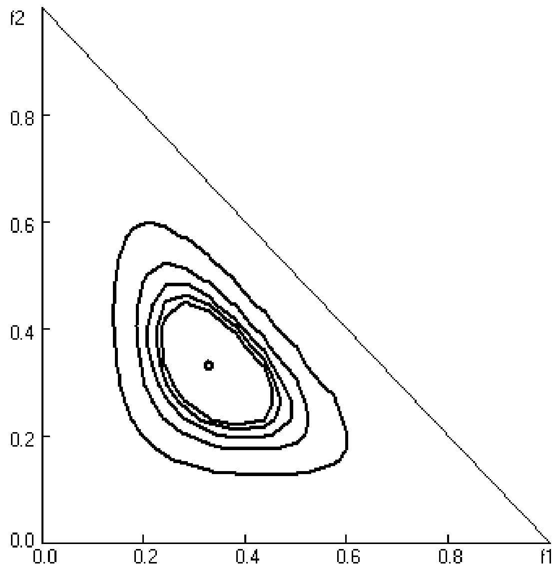


*Fig. 8. The section x1 – x2 – x3 of the target function for the cube model with the Whipple shield.*

## 7. CONCLUSIONS

Thus the analysis of the protection optimization problem reduces to the investigation of the additive properties of a goal operator. If the operator is a sum of smooth piece-wise monotonically descending functions belonging to the same class, then there exists the unique solution of the optimization problem. And the particular unique solution exists if the target function can be approximated by hyperbolic finite elements. The minimization with the ORDEM2000 and BLCs for a single wall structures and the Whipple shield gives a smooth solution. In the case of the single wall the minimum is faded, in the case of the Whipple shield the minimum is fair, but at the nearest adjacency of the optimum the solution is very flat. The comparison between calculations using the direct Monte Carlo and the hyperbolic elements methods was made. The DMCM allows to calculate the optimum when there are about 5-6 protective elements, otherwise the convergence is unsufficient. At the same time the MHE allows to optimize any number of elements

It is shown that an optimal mass distribution between the elements gives a considerable economy in the weight of the spacecraft. It should be noted that the distribution of trajectories of space debris and meteoroids plays a decisive role and is more important than the distribution of velocities.

Optimization problem is included in the COLLO2000 program and in general can be extended for the case when there are mutual shading and the varying spacecraft orientaton.

## 10. REFERENCES


Christiansen E. L. and Kerr J., Ballistic limit equations for spacecraft shielding. *International Journal of Impact Engineering,* 26, 93-104. 2001.

Kessler D.J., Zhang J., Matney M.J., Eichler P., Reynolds R.C., Anz-Meador P.D., and Stansbery E.G. A computer based orbital debris environment model for spacecraft design and observations in low Earth orbit. NASA Technical Memorandum. NASA Johnson Space Center, Houston, Texas.1996.

Liou Jer-Chyi, Matney Mark J., . Anz-Meador Phillip D, Kessler Donald, Jansen Mark, Theall Jeffery R., The New NASA Orbital Debris Engineering Model ORDEM2000, NASA/TP—2002-210780, Johnson Space Center, Houston, May 2000.

Reimerdes Hans-G.. Optimization of Micrometeoroid and Space Debris Protection Systems. IADC 21 Meeting, Bangalore, India,2003.

Reimerdes H-G., Wohlers W. Optimization of Micrometeoroid and Space Debris Protection Systems. ESA SP-473, Proc. Third European Conference on Space Debris, Darmstadt, Germany, 2001.

Stokes H, Swinerd G., Walker R., and Wilkinson J. SHIELD: a New Model to Identify Optimum Debris Protection for Unmanned Spacecraft. IAA-99-IAA.6.5.02, presented at the 50th International Astronautical Congress, Amsterdam, The Netherlands, 4-8 October 1999

Stokes H, Swinerd G., Walker R., Wilkinson J., and Martin C. Achieving Cost-Effective Debris Protection of Unmanned Spacecraft using SHIELD. IAA-00-IAA.6.5.06, presented at the 51st International Astronautical Congress, Rio de Janeiro, Brazil, 2-6 October 2000.

Trenogin V.A. Functional analysis. Moscow, “Nauka”, 1980.