OPTIMIZATION OF THE INTERFACE BETWEEN SPACE DEBRIS ENVIRONMENT MODELS AND DAMAGE PREDICTION TOOLS

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ABSTRACT

The problem of determining the composition and discreteness of the initial data for damage prediction tools, ensuring computation time minimization under limitation on errors of penetration probability (PP) estimates, is considered in this report. The investigations are carried out on the basis of application of the software, in which the construction of space debris environment characteristics is combined with PP calculation.

The results of PP estimation are presented depending on the composition and discreteness of the initial data representation. As a result, the recommendations for the optimal form of the initial data representation are worked out.

1. INTRODUCTION

Fig. 1 shows schematically the link between the SDPA environment model and damage prediction tool (SDPA-PP).

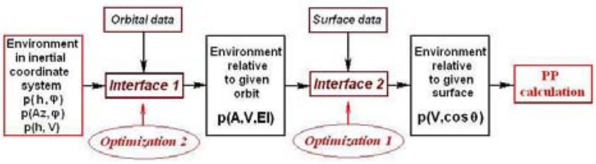

Figure 1. Scheme of the link between environment model and damage prediction tool.

Three types of space debris environment models are used in the SDPA-PP software for calculating the probability of space debris penetration through SC walls:

1. In the inertial coordinate system;
2. Relative to the given orbit;
3. Relative to the given surface.

The SD environment in the inertial coordinate system, used in our model, includes the following characteristics:

1. The subdivision of SD sizes into ranges (8 for LEO and 9 for GEO). Fragment of this subdivision is presented in the table below.

2. The spatial density $\rho(h,\varphi)_j$ depending on the altitude and latitude of a point as well as on the SD size (d_j, d_{j+1}).

3. Two-dimensional statistical distributions of the tangential/radial velocity vector component as a function of altitude of a point ($p(h,V_\tau)_j \,/\, p(h,V_r)_j$) for each range of SD size.

4. Statistical distributions of velocity directions ($p(Az,\varphi)_j$) of approaching (i. e. flying up to SC) SD particles for various latitudes.

5. Statistical distribution of particle's density ($p(\rho_p)_j$) for each range of SD size.

№ of range	1	2	3	4	5	6	7
d_j, d_{j+1}, cm	0.1-0.25	0.25-0.5	0.5-1.0	1.0-2.5	2.5-5.0	5.0-10	10-25

The blocks "*Interface 1*" and "*Interface 2*" carry out transformation of one type of SD environment into another. In the final analysis, their content depends on the technique of penetration probability calculation.

2. THE TECHNIQUE OF A PP EVALUATION

The principles of our technique are stated in a series of our publications (Nazarenko, 2000, 2001, 2003) rather adequately. The known ballistic limit equations for the critical particle size (d_c) as a function of wall design parameters, SD size, collision velocity direction, as well as particles density, are used here (Cour-Palais, 1982, Christiansen, 1998). These equations (the so-called ballistic curves) are as follows:

$$d_c = f\left(V, cos(\theta), \rho_p, t_b, S, t_w, \rho_w\right). \qquad (1)$$

The following designations are applied here: V is the relative velocity of a particle, θ (teta) is the angle between the relative velocity and perpendicular to the surface, ρ_p is the density of a particle, t_b is the thickness of an external wall layer (bumper or shield), S is the distance between a bumper and a wall, t_w is the thickness of a wall as such, ρ_w is the wall material density.

Consider some elementary platform dS on the SC surface. Its orientation, characterized by some angles α and β, is assumed to be known. This elementary platform collides with particles of various size (d) and various densities (ρ_p) approaching with various velocities (V) and under various angles ($cos(\theta)$). The statistical approach is applied to estimate the penetration

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probability. The required probability of penetration of particles from the given size range (d_j, d_{j+1}) through the wall is convenient to be presented as a product of this collision probability by the conventional penetration probability at collision:

$$PP(d_j, d_{j+1}) = P_{col}(d_j, d_{j+1}) \cdot P(d_c < d | collision)_j \quad . \quad (2)$$

We introduce into consideration the normalized distribution $p(d, V, cos(\theta), \rho_p)$ of probable values of four characteristics of SD particles listed above at the instant of their collision with an elementary platform. This distribution satisfies the condition

$$\int_d \int_V \int_{cos(\theta)} \int_{\rho_p} p(d, V, cos(\theta), \rho_p) \cdot dd \cdot dV \cdot dcos(\theta) \cdot d\rho_p = 1 \quad . \quad (3)$$

Designate the region of space of considered four arguments, in which the condition

$$d > d_c = f(V, cos(\theta), \rho_p), \quad (4)$$

is satisfied, as Ξ. Then the integral of function $p(d, V, cos(\theta), \rho_p)$ over the region Ξ will be equal to the probability of penetration of the given wall under its collision with particles

$$P(d_c < d | collision) = \int_{\Xi} p(d, \cos(\theta), V, \rho_p) \cdot d\Xi \quad . \quad (5)$$

The application of the natural and sufficiently correct assumption, that the distribution $p(V, cos(\theta))$ is identical for all particles from some size range (d_j, d_{j+1}), allows to essentially simplify the calculation of integral (4). In this case the considered four-dimensional distribution can be presented as

$$p(d, \mathrm{V}, \cos(\theta), \rho_\mathrm{p}) = p(d)_j \cdot p(\rho_\mathrm{p})_j \cdot p(\mathrm{V}, \cos(\theta))_j \; . \quad (6)$$

The algorithm of conventional probability calculation is reduced to fulfillment of the following operations:
1. The cycle on discrete values of arguments V_m, $cos(\theta)_i$, $\rho_{p,k}$ is organized.
2. The critical size of particles d_c is calculated by formula(1) for each of combinations of these arguments.
3. The conventional penetration probability is calculated for the given values of arguments

$$P(d_c < d | collision)_{j,m,i,k} = \begin{cases} 0, by \quad d_c > d_{j+1} \\ \dfrac{Q(> d_c) - Q(> d_{j+1})}{Q(> d_j) - Q(> d_{j+1})}, by \quad d_j < d_c < d_{j+1} \\ 1, by \quad d_c < d_j \end{cases} . \quad (7)$$

Here the value of a flux for particles with size larger than d_c ($Q(> d_c)$) is determined by interpolation of known values $Q(> d_j)$ and $Q(> d_{j+1})$.
4. The estimates (7) are summed up with regard to distributions $p(\mathrm{V}, \cos(\theta))_j$ and $p(\rho_p)_j$

$$P(d_c < d | collision)_j = \sum_m \sum_i \sum_k P(d_c < d | collision)_{j,m,i,k} \cdot p(V_m, \cos(\theta))_j \cdot p(\rho_{p,k}) \cdot \quad (8)$$

As a result, the required conventional penetration probability for elementary platform dS under its collision with particles from the considered size range is determined. The calculation of these estimates is the most labor-consuming operation.

The collision probability $P_{col}(d_j, d_{j+1})$ is determined simply enough:

$$P_{col}(d_j, d_{j+1}) = dS \cdot [Q(> d_j) - Q(> d_{j+1})] \cdot \sum_m \sum_i \cos(\theta)_i \cdot p(V_m \cdot \cos(\theta)_i) \cdot \quad (9)$$

Thus, the substitution of estimates (8) and (9) into (2) leads to determination of the penetration probability for the given elementary platform dS under considered conditions.

3. OPTIMIZATION OF PARAMETERS OF ENVIRONMENT RELATIVE TO THE GIVEN SURFACE

Consider the influence of discreteness of the $p(V, cos(\theta))$ distribution representation on the accuracy of penetration probability estimates. This influence was estimated experimentally as a result of some calculations of the conventional penetration probability for various limits of bins. The parameters of the circular orbit are as follows: the altitude is 450 km and the inclination is 51.6°. Three versions of wall configuration were chosen: two panels distinguished by orientation, and a cylinder. In all versions the wall characteristics were accepted identical: t_b =0.05 cm, S=2 cm, t_w =0.16 cm, ρ_w =2.7 g/cm^3. 5 versions of discreteness of V and $cos(\theta)$ arguments, characterized by the number of their subdivision into boxes (*nV* and *nCos*), were considered. The results of determination of the conventional penetration probability for particles of size 0.1-0.25 cm are presented in Fig. 2.

One can see from these data that for all versions of wall configuration the estimates are steady enough: the corresponding deviations from the data with maximal discreteness do not exceed 10 % for panels and 3 % for a cylinder. For panels, the tendency of increasing deviations with decreasing discreteness is revealed. The higher stability of estimates for a cylinder may be explained by a greater number of realizations, used in the $p(V,cos(\theta))$ distribution construction, since the additional cycle on its surface was applied.

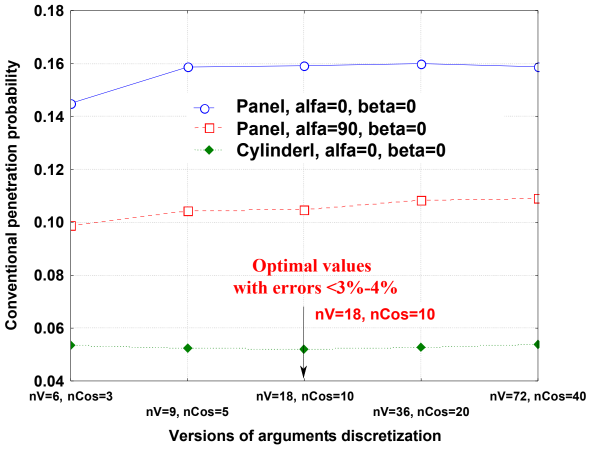

Figure 2. Estimates of $P(d_c < d|collision)$ for various limits of bins

On the basis of these results the conclusion is made, that the application of discreteness parameters nV=18 and $nCos$=10 provides determination of conventional penetration probability with errors not higher than 3 % - 4 %. So, this discreteness is applied further in our analysis.

The total number of probability estimates, contained in the histogram $p(V,cos(\theta))$, is equal to $nV\cdot nCos$=18·10=180. They relate to one of eight ranges of sizes.

4. CONSTRUCTION OF CHARACTERISTICS OF SD ENVIRONMENT RELATIVE TO THE GIVEN SURFACE

This file is initial one for calculating the penetration probability. The initial data for its construction are contained in characteristics of SD environment relative to the given orbit. They have a form of the normalized three-dimensional distribution of SD cross-sectional area flux $p(A,V,El)_j$ as a function of azimuthal deviations of the relative velocity (A), of relative velocity values (V) and velocity vector deviations from the horizontal plane (El). These distributions are constructed for each of considered SD size ranges (d_j,d_{j+1}). The example of such a distribution for particles of size 0.1-0.25 cm is presented in Tab. 1.

The following values of bins ("boxes") are applied here: $dA=360/nA$=5°, $dV=18/nV$=1 km/s, $dEl=180/nEl$=5°. Only those values are saved in the considered file, which have $Probability$>0. Generally, this file consists of $nA\cdot nV\cdot nEl$ lines. However, in the majority of cases it is much smaller, since for many values of arguments $Probability$=0. In particular, the file given in Tab. 1 above consists of 318 lines, which equals only 0.68 % of the maximum number of lines ($nA\cdot nV\cdot nEl$ = 72·18·36=46658).

Table 1. The file of the SD environment relative to the given orbit $p(A,V,El)$

A°	V, km/s	El°	$p(A,V,El)$
2.5	15.5	-2.5	2.773E-0004
2.5	15.5	2.5	1.851E-0004
7.5	14.5	-2.5	7.156E-0007
7.5	14.5	2.5	1.389E-0006
7.5	15.5	-2.5	5.027E-0004
7.5	15.5	2.5	5.087E-0004
12.5	14.5	-2.5	5.205E-0003
12.5	14.5	2.5	6.206E-0004
12.5	15.5	-2.5	9.812E-0004
12.5	15.5	2.5	3.863E-0004
...	...	...	...

The distribution $p(V,cos(\theta))$ is constructed on the basis of the considered file as follows.

1. The cycle on lines of $p(A,V,El)$ file is organized.
2. The values $\theta = f(A,El,\alpha,\beta)$ and $\cos(\theta)$ are calculated.
3. The values $p(A,V,El)$ are added to corresponding "boxes" of the $p(V,\cos(\theta))$ histogram.

5. OPTIMIZATION OF PARAMETERS OF ENVIRONMENT RELATIVE TO GIVEN ORBIT

Consider the influence of discreteness of the $p(A,V,El)_j$ distribution representation on the accuracy of penetration probability estimates. 5 versions of discreteness of A, V and El arguments, characterized by the number of their subdivision into boxes (nA, nV and nEl), are considered:

1	2	**3**	4	5
nA=18 nV=6, nEl=9	nA=36 nV=9, nEl=18	**nA=72 nV=18, nEl=36**	nA=120 nV=36, nEl=60	nA=180 nV=72, nEl=90

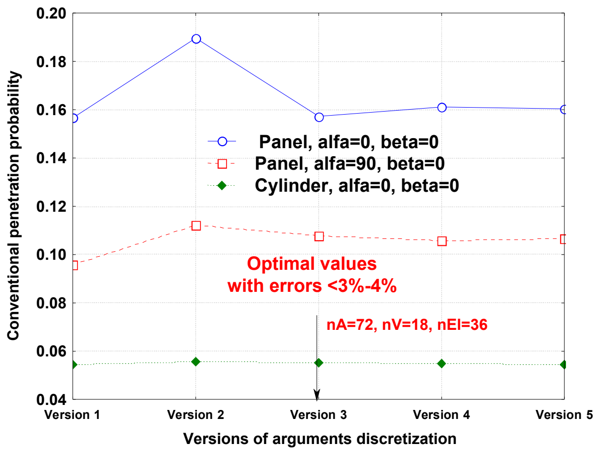

Figure 3. Estimates of $P(d_c < d|collision)$ for various limits of bins

The results of determination of the conventional penetration probability for particles of size 0.1-0.25 cm are presented in Fig. 3. One can see from these data that for all versions of wall configuration the estimates are steady enough: the corresponding deviations from the data of the right column do not exceed 18 % for panels and 2 % for a cylinder. For panels, the tendency of increasing deviations with decreasing discreteness is revealed.

On the basis of these results the conclusion is drawn, that the application of parameters of discreteness nA=72, nV=18 and nEl=36 provides determination of conventional penetration probability with errors not higher than 3% - 4%. So, it is this discreteness, which is applied further in our analysis.

The *further optimization step* consists in refusal from using the three-dimensional distribution $p(A,V,El)$ and in applying the statistical distribution of flux directions $pQ(A)$ and the dependence of the average relative velocity on its direction $V(A)$. In this case, only nA lines are required for file representation at subdivision of probable directions of approaching particles on nA sectors (nA=180 is applied in the model). It is obvious that disadvantages of this simplification can be revealed for elliptical orbits.

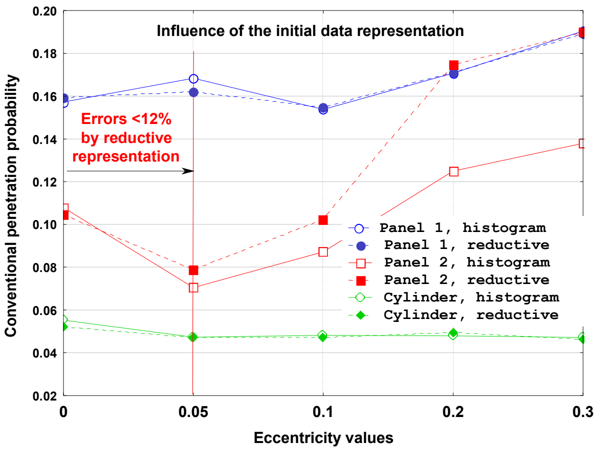

Figure 4. The estimates of conventional penetration probability for various eccentricities (j=1)

The influence of the orbit eccentricity on the accuracy of determination of the conventional penetration probability depending on a more complete (the $p(A,V,El)$ histogram) or simplified representation of environment relative to the given orbit was estimated. In so doing, for fixed perigee altitude (450 km), inclination (51.6°) and argument of perigee (20°) the eccentricity has been varied. The calculations were carried out at more complete (the $p(A,V,El)$ histogram) and simplified ($pQ(A)$, $V(A)$) representation of the initial data. The calculation results are presented in Fig. 4.

One can see from these data that for the second panel the transition to the simplified initial data representation results in considerable changing of corresponding estimates. The distinctions of the data reach 40 %.

On the basis of these results the conclusion was drawn that the simplified (and more economical) structure of environment relative to the given orbit is expedient to be applied for eccentricity values lower than 0.05. For such a simplification the error in estimates of conventional penetration probability does not exceed 12 %. For higher eccentricity values it is expedient to apply the three-dimensional histogram of initial data distribution (see Tab. 1). So, it is this recommendation, which is used further in our analysis.

6. CONSTRUCTION OF THE SD ENVIRONMENT RELATIVE TO THE GIVEN ORBIT

The technique, used in the SDPA model for constructing the SD environment relative to the given orbit, was outlined in sufficient detail in a series of our publications (Nazarenko, 2002, 2003). Its distinctive feature is that the characteristics *averaged* over one revolution are determined in this case. Consider briefly the technique for constructing the azimuthal distribution of the SD cross-sectional area flux.

The instantaneous value of the cross-sectional area flux $Q(t)$ is equal to the product of particle's spatial density ρ by the relative velocity at the given point:

$$Q(t) = \rho(t) \cdot V_{rel}(t). \tag{10}$$

This dependence is used as a basis for further calculations. On the interval of one orbit the trajectory is subdivided into nU=180 sections with a step of 2° in the argument of latitude. The feature of this algorithm is the method of determining the dependence of relative velocity on its direction, which is characterized by angle A. The calculation is based on the use of statistical distribution of directions of SD velocity $p(t,Az)$ at the given point of the inertial space. This distribution is constructed for various latitudes with a step of 2° in the angle Az. At each of considered trajectory points the cycle on possible values of angle Az_j is organized, and the angle A_j, and the horizontal component of relative velocity are calculated from a triangle of velocity vectors. The total value of relative velocity is calculated with regard to the radial component of SC velocity.
The corresponding value of SD flux through the considered azimuthal sector ($(A_j, A_j + \Delta A)$) is equal to

$$P(t_i, A_j) = \rho(t_i) \cdot V_{rel}(t_i, A_j) \cdot p(t_i, Az)_j \cdot \Delta t_i = Q(t_i, A_j) \cdot \Delta t_i. \tag{11}$$

Here Δt_i is the duration of SC stay on the i-th section of a trajectory ($u_i, u_i + \Delta u$).
The deviation of relative velocity from the horizontal plane (the angle of elevation El) is determined as well. Thus, for each of the time instants (t_i) and for each of

azimuthal directions of approaching of particles (Az_j), the following quantities are determined: a) the relative velocity, b) the azimuthal deviation of relative velocity (A_j), c) the relative velocity deviation from the horizontal plane (*El*) and d) the SD flux through the considered sector (11). The number of such data is equal to the product of a number of considered time instants by a number of intervals of angle *Az* subdivision into sectors. This number equals $nU \cdot nA$=32400 for one SC revolution and for some SD size range. This file is a basis for further calculations of penetration probability. On the basis of this file, the three-dimensional distribution $p(A,V,El)$ is constructed, and the other summary characteristics of SD flux are determined.

The SD flux through the azimuthal sector $(A_j, A_j + \Delta A)$ is determined by summation of all estimates of type (11). In this case the number of events (*k*), when the direction of a relative velocity vector *A* falls into the sector ($(A_j, A_j + \Delta A)$), is determined for each of time instants. As a result, one obtains

$$P(A_j) = \sum_i \sum_k P(t_i, A_j)_k \,. \qquad (12)$$

The summation of estimates (12) over all possible values of angle A_j (nA=180) results in estimation of the total flux P_Σ

The calculation of ratios of estimates (12) and P_Σ for various values of A_j angle results in constructing the azimuthal distribution of a SD flux relative to given orbit:

$$pQ(A_j) = P(A_j) / P_\Sigma \,. \qquad (13)$$

7. OPTIMIZATION OF PARAMETERS OF ENVIRONMENT RELATIVE TO THE GIVEN ORBIT (CONTINUATION)

It was mentioned above, that the three-dimensional distribution $p(A,V,El)$ is constructed on the basis of a file containing the values of arguments at each of considered trajectory points (*nA* values at one point). Such an output data file of NASA ("ORDEM") and ESA ("MASTER") models is suggested to be used for damage prediction tools (Reimerdes, 2004). In this connection, the natural question arises: whether the essential worsening of accuracy does not occur at replacement of the initial file by a smaller three-dimensional histogram $p(A,V,El)$. In the example considered above this histogram consisted of 318 lines. If the worsening of accuracy does not occur, then the transition to the three-dimensional histogram is an efficient measure for optimizing the output data of models.

As a first approximation, the answer to this question is contained in the materials of our paper (Nazarenko, 2003). Below, this question is solved on the basis of additional test calculations. The calculations were carried out for two orbits considered above: circular and elliptical ones (for *e*=0.2, ω=200°). The complete form of data representation (for each range of SD size) consisted of *nA* sets of estimates (*A, V, El, Flux*) at *nU* points of the trajectory. The file included those sets, in which *Flux* > 0. The other form of data representation was the histogram $p(A,V,El)$ (see Tab. 1). The results of calculations are presented in Tab. 2.

Table 2. Influence of the form of initial data representation on the estimates of conventional penetration probability

Versions of design and orientation data	Orbit type	Form of interface		Ratio
		Full	Histogram	
Panel, α=0°, β=0°	Circular	0.1642	0.1581	**0.962**
	Elliptical	0.1785	0.1742	0.976
Panel, α=90°, β=0°	Circular	0.1065	0.1077	1.011
	Elliptical	0.0655	0.0662	1.010
Cylinder, α=0°, β=0°	Circular	0.0542	0.0550	1.015
	Elliptical	0.0468	0.0477	1.019

One can see from these data, that for all versions of wall configuration the application of the three-dimensional histogram $p(A,V,El)$, rather than the complete set of initial data, results in worsening the accuracy of penetration probability estimates not greater, than by 4 %. At the same time, the volume of data file is reduced tens or hundreds times, respectively, for elliptical or circular orbits. On the basis of these investigations the conclusion was drawn on expediency of applying the three-dimensional histogram $p(A,V,El)$: this allows to essentially save computation time in penetration probability calculation

An important parameter of the considered algorithm is the number of trajectory subdivisions into bins. The subdivision into *nU*=180 bins was applied above. It was determined that the transition to a larger step in the argument of latitude results in essential distortion of azimuthal distributions of SD flux. By this reason, as well as with account taken of the aforementioned weak influence of this measure on computation time expenses (with using the $p(A,V,El)$) distribution), the step Δu=2° is applied in our model.

It should be noted, that at present there is no consistent approach to constructing azimuthal distributions of SD flux (the relative velocity). This is testified, in particular, by the materials of the report (Beltrami, 2004). Our investigations demonstrate the essential influence of the type of azimuthal flux (the relative velocity) distribution on penetration probability estimates. The distinction between corresponding estimates reaches 50 %.

In summary to this chapter, we shall consider the influence of SD size range discreteness. Our investigations demonstrate that the 2- and 3-fold increase of SD size "boxes" results in increasing wall penetration probability. The maximum deviations of estimates from those corresponding to initial discreteness reach 18 % and 22 %, respectively. The conclusion is drawn from the results of investigations, that it is inexpedient to increase the SD size "boxes".

8. BRIEF INFORMATION ABOUT THE SDPA-PP MODEL.

The measures on optimization of the interface between space debris environment models and damage predicti-on tools, considered above, are implemented in the SDPA-PP model. This allowed one to reduce, two orders of magnitude at least, the volume of information used in the "PP calculation" block (Fig. 1). Accordingly, the computation time expenses also decreased. The applied simplifications result in errors of PP calculation not higher than 10%, which is appreciably lower, than possible errors of modern space debris models (Beltrami, 2004).

The model is implemented in the operational system of the win32 platform (Microsoft Windows 95, 98). The time of calculations for one version of initial data (one component of elementary shape) does not exceed several seconds. When addressing to the program, the first panel of the program is opened, which is designed for choosing one of three versions of initial data ("GEO", "LEO" and "Trajectory"). The "Trajectory" version differs from previous ones only by the fact, that the SC orbital parameters are specified as a sequence of state vector values in the Earth-Centered Inertial coordinate system, rather than in the form of orbital elements.

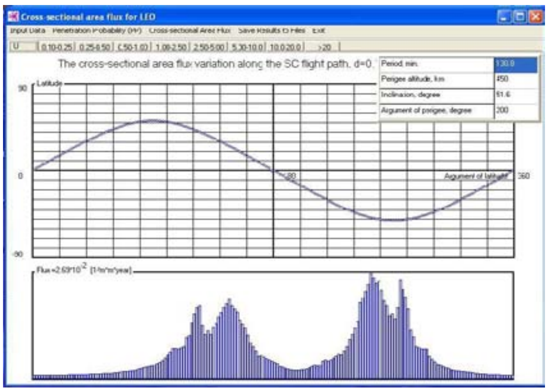

Figure 5. Panel "The cross-sectional area flux along the SC flight path"
(click to magnify)

Fig. 5 presents the panel "The cross-sectional area flux along the SC flight path" for the "LEO" point of menu.

Fig. 6 presents the panel "Penetration Probability for SC in … ". The content of the panel consists of four pictures and a text. All these data, except the right bottom picture, relate to one of considered SC components. More complete data are written in files. The text, displayed on the screen, contains the estimates of conventional and full penetration probability for each of components and for particles of various sizes.

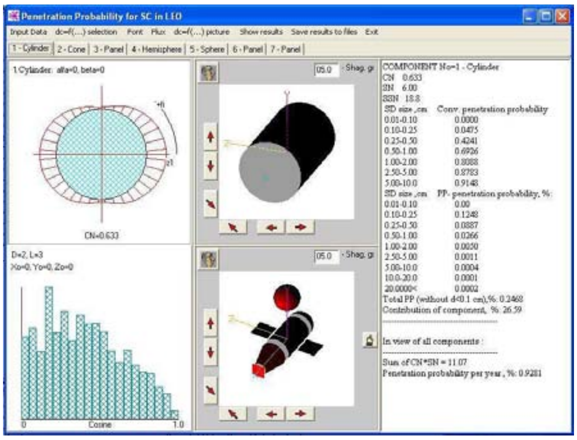

Figure 6. The panel of results of PP calculation
(click to magnify)

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