EVALUATION OF THE PROBABILITY OF CORRECT SATELLITE IDENTIFICATION BASED ON ORBITAL DATA

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ABSTRACT

The paper considers the situation when newly detected satellite is to be either identified with one of the previously cataloged satellites or correlated with a group of satellites characterized by certain set of parameters. Such identification problems must be solved for satellite break-ups, separations or when the satellites are lost for significant period of time.

The work describes the technique for determination of the probability of the fact that the newly detected satellite correlates with some of the satellites already present in the catalog. The technique is based on comparing the orbital parameters of the new satellite with the orbital parameters of the candidates for the parent. We must take into account the accuracy of the orbits of the analyzed satellite and parent-candidates and also the probability of presence of other satellites with close orbital parameters («background satellite density»). Using the suggested technique we can determine the probability of the fact that the analyzed satellite was generated by certain event.

1. USED ORBITAL PARAMETERS

Comparing of the parameters of motion of two satellites performed to determine the probability of generation of the object by certain event and for construction of the parametrical model of background satellite density (bsd) is based on criteria of "proximity" in the space of selected set of orbital parameters. For this set we use the vector of orbital momentum $\overline{c}\{c_x, c_y, c_z\}$ and the semimajor axis a, which together constitute the vector $\overline{q}\{q_1, q_2, q_3, q_4\}$.

2. MODEL OF THE BACKGROUND

The model of the background is the probability density function for detecting a satellite in the given point $\overline{q}$ of the space of selected orbital parameters. This model is constructed by clustering the cataloged satellites using closeness in the space of parameters $\overline{q}$. The number m of these clusters is chosen by the analyst depending on the required details of the model and the capacity of available computers. This number is assumed to vary within 100 - 800. Section 2.3 describes in general the technique for clustering the set of orbital elements.

2.1. Forming of the average values of the parameters of groups of satellites and the respective covariance matrices

We assume that the set of orbital elements $\boldsymbol{Q}\{\overline{q}_1, \overline{q}_2, ..., \overline{q}_n\}$ is already divided into m groups (clusters):

$$\boldsymbol{Q} = \boldsymbol{Q}_1 \cup \boldsymbol{Q}_2 ... \cup \boldsymbol{Q}_m ,$$

where $\boldsymbol{Q}_k\{\overline{q}_k^1, ..., \overline{q}_k^{m_k}\}$ – is the set of orbital elements in k-th group. For each group we determine the vector of average orbital parameters $\hat{\overline{q}}_k$

$$\hat{\overline{q}}_k = \frac{1}{m_k}\sum_{i=1}^{m_k}\overline{q}_k^i ,$$

where $\overline{q}_k^i$ – orbital elements of the i-th satellite of the group $\boldsymbol{Q}_k$

If the clustering was efficient then the density of each group $\boldsymbol{Q}_k$ is the highest near the vector of average orbital parameters $\hat{\overline{q}}_k$. The number of the satellites of the group $\boldsymbol{Q}_k$ becomes smaller with the distance from the center, for which we take the point $\hat{\overline{q}}_k$. We assume that the background density for each group $\boldsymbol{Q}_k$, $(k\ \ 1,...,m)$ in the space of the selected orbital parameters $\overline{q}$ has normal distribution. The a posteriori estimate of the covariance matrix C_k of the scattering of the orbital parameters $\overline{q}$ in the group $\boldsymbol{Q}_k$ is determined by the formula

$$C_k = \mathbf{M}\left[\Delta\overline{q}_k, \Delta\overline{q}_k^T\right] = \frac{1}{m_k}\sum_{i=1}^{m_k}\left[\left(\overline{q}_k^i - \hat{\overline{q}}_k\right)\left(\overline{q}_k^i - \hat{\overline{q}}_k\right)^T\right],$$

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where $\mathbf{M}[\;]$ denotes the mean of the stochastic value in brackets. These calculations will determine m vectors of the centers $\hat{\bar{q}}_1, \hat{\bar{q}}_2, ..., \hat{\bar{q}}_m$ and m respective covariance matrices $C_1, C_2, ..., C_m$. In the total the number of the groups m, the set of vectors $C_1, C_2, ..., C_m$ and the set of covariance matrices $C_1, C_2, ..., C_m$ constitute the set of the parameters of the background. Using these parameters of the clusters we can determine the probability density for detecting a satellite in the certain point $\bar{q}$ of the space of selected orbital parameters.

2.2. Calculation of the background density for the given state vector

According to the made assumptions the probability density of detecting a background satellite from the group Q_k in the point $\bar{q}$ of the space of orbital parameters is determined by the formula of multi-dimensional normal distribution

$$p_k(\bar{q}) = \frac{1}{\sqrt{(2\pi)^4 |C_k|}} \exp\left[-\frac{1}{2}\Delta\bar{q}_k (C_k)^{-1} \Delta\bar{q}_k^T\right]$$

where $\Delta\bar{q}_k = \bar{q} - \hat{\bar{q}}_k$.

In the vicinity of this point, within the volume of the space of the parameters $dQ = dq_1 \cdot dq_2 \cdot dq_3 \cdot dq_4$ the mean of the number n_{back}^k of satellites from the group $\boldsymbol{Q}_k$ is given by the formula

$$\mathbf{M}\left[n_{back}^k\right] = p_k(\bar{q})\, dQ$$

This number does not depend on the number of the objects of other groups within the vicinity dQ of the point $\bar{q}$, thus the probability density for detecting in this point a satellite from one of the groups of the set $\boldsymbol{Q}$ is determined by the formula

$$p_{back}(\bar{q}) = \sum_{k=1}^{M} p_k(\Delta\bar{q}_k)$$

2.3. Clustering procedure

The input data for the procedure is the set of orbital parameters $\bar{q}_1, \bar{q}_2, ..., \bar{q}_n$, which should be clustered into groups using some criteria of closeness. The procedure clusters the satellites into disjoint groups using closeness to the centers. As a result the procedure generates the set of m centers $\hat{\bar{q}}_1, \hat{\bar{q}}_2, ..., \hat{\bar{q}}_m$ and the respective set of covariance matrices $C_1, C_2, ..., C_m$. In general the procedure can be described as follows:

1. We perform the initial screening of all the orbital elements that result in initial clustering of satellites into $M >> m$ groups which constitute the extended set.
2. From the extended set we select the subset of m groups containing the majority of the satellites and determine for them the means and covariance matrices
3. Then we perform cycle of iterations. Each iteration re-correlate the satellites to the groups using the criteria of maximum closeness to the average orbital elements, generate new mean elements and covariance matrices.

2.4. Using the model of the background for evaluation of the probability of identification

The constructed model of the background provides the possibility to determine the probability of detecting a background satellite in the given area of the space of orbital parameters. Parameters of the model are the orbital elements of the centers of the clusters and the respective covariance matrices. The centers generate probability density independently from each other. Thus the total probability density is the sum of the densities generated by individual centers. This provides a helpful possibility to manipulate with parameters of the background when we are solving different tasks. In particular, the clustering procedure can be applied not to the whole catalog, but to certain its part. For example, we can exclude from the set of satellites used for generating the background density the fragments of break-ups. These fragments should be clustered into groups (one parent satellite – one group of fragments). Each group will have its center and covariance matrix.
Doing so we can use the group composed of the fragments of certain break-up in two aspects. On one hand it can be considered a part of the background when we solve the task of identification of two sets of orbital parameters, for example the new satellite and the lost one (in this case the presence of fragments will make identification more difficult). On the other hand the center and the covariance matrix of the group can be used for identification of new object with this break-up, as we describe in section 3. We can operate similarly with other groups of satellites, for example with satellites generated by launches or separations.

3. CALCULATION OF THE STATE VECTOR AND COVARIANCE MATRIX OF A SATELLITE FOR THE TIME OF CONSIDERED EVENT

Correlation of newly detected object with certain event basically depends on how close are the orbital elements of this object and the orbital elements of the objects potentially generated by this event. If the epoch of the orbital parameters of the analyzed satellite differs from the time of the considered event, its orbital elements will differ from those referred to this time. Thus the state vector and the covariance matrix of the satellite

should be propagated to the time of the event. This is valid equally for break-ups, launches and separations.
The calculation of the state vector of the analyzed satellite and the derivatives of the current state vector with respect to initial one for the given time is performed by numerical integration of the equations of motion and the variation equations.
Model of motion includes the influence of the following perturbing factors

- Earth gravitational field represented by expansion in series of spherical functions including harmonics up to 16x16.
- Gravitational influence of the Moon and the Sun
- Solar radiation pressure
- Atmosphere

The covariance matrix C_{X_s} of the errors of the state vector for the time t_s with the known covariance matrix of $\overline{q}_k$, referred to the time t_0 is determined using the formula

$$C_{X_s} = \frac{\partial \overline{X}_s}{\partial \overline{X}_0} C_{X_0} \left(\frac{\partial \overline{X}_s}{\partial \overline{X}_0} \right)^T$$

where $\frac{\partial \overline{X}_s}{\partial \overline{X}_0}$ is the matrix of derivatives obtained by solving the variation equations, $\left(\frac{\partial \overline{X}_s}{\partial \overline{X}_0} \right)^T$ – the matrix transposed with respect to matrix $\frac{\partial \overline{X}_s}{\partial \overline{X}_0}$

4. CORRELATION WITH BREAK-UPS

When we get new orbital parameters $\overline{q}$ based on the certain set of measurements we should determine whether the orbital elements correspond to the fragment of known break-up. The probability of this fact depends on the average number of the fragments of the break-up

$$\mathbf{M}\left(n^*\right) = p^*\left(\overline{q}\right) dQ$$

and the average number of background satellites

$$\mathbf{M}\left(n_{back}\right) = p_b\left(\overline{q}\right) dQ,$$

located in the vicinity dQ of these orbital parameters and is determined by the formula

$$P_{\overline{q}} = \frac{n^*}{n^* + n_{back}} = \frac{p^*\left(\overline{q}\right)}{p^*\left(\overline{q}\right) + p_{back}\left(\overline{q}\right)}$$

If we have to identify the new orbital elements with the fragments of one of several break-ups $B_1,\dots,B_n$, then the probability of correlation to the k-th break-up is determined by the formula

$$P_{\overline{q}}^k = \frac{p_k^*\left(\overline{q}\right)}{\sum_{k=1}^{n} p_k^*\left(\overline{q}\right) + p_{back}\left(\overline{q}\right)},$$

where $p_1^*\left(\overline{q}\right),\dots,p_n^*\left(\overline{q}\right)$ - probability densities for the fragments of break-ups $B_1,\dots,B_n$ respectively

The procedure for calculation of the probability density for the background $p_{back}\left(\overline{q}\right)$ is presented in section 2. Performing this calculations we should exclude (see section 2.4) from the total set of satellites the satellites affiliated to the considered break-ups.
The model of the probability density for the fragments of break-up is constructed with assumption of normal distribution of their orbital elements around the mean value of orbital elements of this break-up. We use the same set of orbital elements (four parameters) as we have used constructing the model density for the background. The mean of the distribution is calculated as the average of the orbital elements of all known fragments of this break-up:

$$\hat{\overline{q}} = \frac{1}{m} \sum_{k=1}^{m} \overline{q}_k$$

where $\overline{q}_k$ – orbital elements of the k-th fragment, m – the number of known fragments.
Covariance matrix of scattering is calculated using formula

$$C^q = \frac{1}{m} \sum_{k=1}^{m} \left(\hat{\overline{q}} - \overline{q}_k\right)\left(\hat{\overline{q}} - \overline{q}_k\right)^T$$

The probability density for detecting the fragment in the vicinity of the given orbital parameters $\overline{q}$ is determined by the formula

$$p\left(\overline{q}\right) = \frac{1}{\sqrt{\left(2\pi\right)^4 \left|C_q\right|}} \exp\left[-\frac{1}{2} \Delta\overline{q} \left(C_q\right)^{-1} \Delta\overline{q}^T \right]$$

where $\Delta\overline{q} = \overline{q} - \hat{\overline{q}}$

5. CORRELATION WITH LOST SATELLITES

Let us assume that processing of certain set of measurements resulted in the state vector $\overline{X}\{\overline{r},\overline{v}\}$ of a satellite and its covariance matrix $C_X(t)$ for the time t.
We are going to determine the probability of the fact that this satellite correspond to the previously lost satellite S-1, which has the state vector referred to the time t_s, denoted $\overline{X}'_s\{\overline{r}'_s,\overline{v}'_s\}$, and the respective covariance matrix C'_{X_s}.
Knowing the vector $\overline{X}$ for the time t and using the adequate model of motion we can calculate its state vector $\overline{X}_s$ and covariance matrix $C_X\left(t_s\right)$ for the time t_s. We assume that the errors of orbital parameters $\overline{q}_s$ and $\overline{q}'_s$ have normal distribution and are characterized by covariance matrices C_{q_s} and C'_{q_s} respectively. Thus the difference $\Delta\overline{q}_s = \overline{q}_s - \overline{q}'_s$ has normal distribution with covariance matrix $C_{q_s}^{\Delta} = C_{q_s} + C'_{q_s}$.

The probability of presence of a background object in the area dQ is characterized by the probability density of the background $p_{back}\left(\overline{q}'_s\right)$. The mean of the number of satellites within the volume dQ is

$n_{back} = p_{back}\left(\overline{q}'_s\right) dq_1 \cdot dq_2 \cdot \ldots \cdot dq_n$.

The mean of the number of detections of the considered object in the area dQ is

$n_{object} = p_{object}\left(\Delta\overline{q}_s\right) dq_1 \cdot dq_2 \cdot \ldots \cdot dq_n$

where $p_{object}\left(\Delta\overline{q}_s\right)$ is the probability density for detection of the considered satellite.

The probability of correlation of the satellite to the given event is calculated using the formula

$$p = \frac{n_{object}}{n_{object} + n_{back}} = \frac{p_{object}}{p_{object} + p_{back}}$$

The probability of detecting the considered satellite in the area k of the four-dimensional space of orbital parameters has normal distribution. The probability density is determined by the formula

$$p_{object}\left(\Delta\overline{q}_s\right) = \frac{1}{\sqrt{(2\pi)^4 \left|C^{\Delta}_{q_s}\right|}} \exp\left[-\frac{1}{2}\Delta\overline{q}_s \left(C^{\Delta}_{q_s}\right)^{-1} \Delta\overline{q}_s^{\,T}\right]$$

where $\left|C^{\Delta}_s\right|$ is determinant of matrix C^{Δ}_s, $\left(C^{\Delta}_s\right)^{-1}$ is its inverse, $\Delta\overline{q}^{\,T}_s$ – the vector transposed with respect to vector $\Delta\overline{q}_s$.

Calculation of the covariance matrix C_{q_s} of the errors of the orbital parameters use the formula

$$C_{q_s} = \frac{\partial\overline{q}_s}{\partial\overline{X}_s} C_{X_s} \left(\frac{\partial\overline{q}_s}{\partial\overline{X}_s}\right)^T$$

the general technique of calculation of covariance matrix C_{X_s} is presented in section 3.

6. EXAMPLE

For the example of efficiency of the procedure we selected two geostationary satellites with close orbital elements (Table 1)

Table 1

	Sat-1	Sat-2
Sat. Number	1990-112D	1990-016D
Epoch, UTC	20/12/1990 11:35	15/02/1990 07:52
Period, min	1439.40	1439.80
Inclination, ◦	7.8	8.4
Eccentricity	0.002688	0.004451

Fig. 2 demonstrates the evolution of the semi-major axes of the satellites. One can see that visual analysis is rather difficult.

We performed the updating of the orbits for each satellite for the beginning and the end of the interval of the measurements and then – the cross-correlation of the obtained orbital parameters. The scheme of identification is presented in Fig. 1.

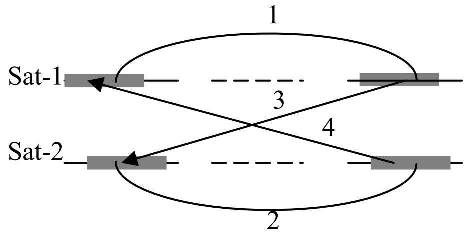

Figure 1. Scheme of identification.

Two parallel lines depict the complete interval of the measurements for Sat-1 and Sat-2. The thicker (grey) sections of the lines present the intervals of the measurements in the beginning and the end of the intervals that were selected for orbit determination. The arrows point to the area with which the identification was performed. The end of the trajectory of Sat-1 (1) was identified with the beginning of the trajectory of Sat-2 (2). The end of the trajectory of Sat-1 was identified with the beginning of the trajectory of Sat-2(3), the end of the trajectory of Sat-2 was identified with the beginning of the trajectory of Sat-1 (4).

Results of determination of the probability of identification for different variants are presented in Table 2. The header of the table contains the satellite number and the year of processing. The cells of the table present the probabilities of correct identification. We can see that for this case we have almost certain identification.

Table 2

	1990-112D	1990-016D.
1990-112D, 1991	0.99999	0.00000
1990-016D, 1990	0.00000	0.99999

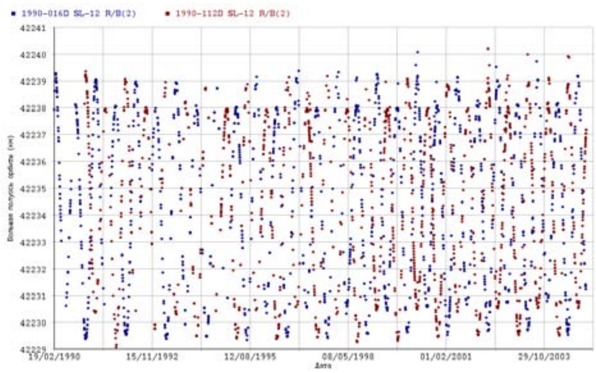

Figure 2. Evolution of the semi-major axis of two satellites – ID 1990-112D (red) and ID 1990-016D (blue)

7. REFERENCES

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