STATISTICAL ANALYSIS OF THE ORBITAL CHARACTERISTICS OF GEO DEBRIS ENVIRONMENT

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ABSTRACT

This present paper brings out the statistical nature of the characteristics of the tracked non-functional GEO (Geo stationary Earth Orbit) debris objects and proposes a distribution model of the GEO environment by utilizing the approach and philosophy as laid out for the ***SIMPLE*** (Stochastic IMPressionistic Low Earth) Model proposed recently for LEO. It is noted that the catalogued objects, as of now of around 800, by USSPACECOM across the years 1998 to 2004 have the same semi major axis mode (highest number density) around 35750 km above the earth. Just by properly excluding the objects in the small bin of operational region, say (35700, 35800) km containing about 40 percent of objects, the rest of the objects have a number density distribution of Single Laplace distribution with two parameters, namely location and scale. It is further noticed that the percentage of objects in the bin around mode is nearly same across the years. The Laplace parameters observed across the years can be modeled mathematically, in fact the location parameters do not vary and the scale parameters shows a definite trend. These observations are successfully utilized in providing a statistical model for the GEO debris environment.

1. INTRODUCTION

The increase of space debris and threats to commercial space activities in the Geosynchronous Earth Orbit (GEO) predictably cause concerns regarding the environment over the long term. A variety of activities regarding space debris such as detection, modeling, protection and mitigation measures, has been going on during the past couple of decades. Due to the absence of atmospheric drag to remove debris in GEO and the increasing number of utility satellites in GEO, the number of objects in GEO will continue to increase. The GEO debris measurements have been limited to objects with size more than about 60 cm, and hence the scenario has to be expanded for smaller objects, which are untracked. Hence modeling aspects of GEO debris environment has to take momentum and the characterization of the GEO environment is critical for risk assessment and protection of future satellites and also to design effective debris mitigation measures.

Any object placed in an orbit around the earth has a risk of colliding with other orbiting objects, and especially the debris objects. For mission planning and safety or mitigation reasons, it is necessary to estimate the level of risk by determining the probability that a given target object will collide with currently orbiting objects. The database available for such an assessment is the orbital parameters obtained through NORAD catalogued data. In an earlier paper the authors proposed a Stochastic IMPressionistic Low Earth (SIMPLE) model for the space debris environment in LEO up to about 2000 km altitude and for eccentricity less than 0.2 (Ananthasayanam et. al, 2003). This paper proposes a GEO debris environment model in line with the SIMPLE model for LEO objects. This paper brings out the details of the analyses carried out with respect to catalogued Geo stationary objects as provided by the TLE sets. The study is basically aimed at obtaining the distribution nature of various orbital characteristics semi major axis, eccentricity and inclination of the GEO objects and hence to arrive at a proper model for GEO. The functional objects are well maintained and have the station keeping and the objects, which are non-functional, pose threat to the GEO missions. Hence we mainly deal with the second category of objects for the GEO environment modeling. In order to recall the SIMPLE model proposed for the LEO, next section provide a brief summary of the model.

2. SIMPLE MODEL

SIMPLE model (Anilkumar 2004) for the space debris was based on the database provided by the Two Line Elements of these objects from 1999 to 2002 catalogued in USSPACECOM. It was shown that in LEO (altitude less than 2000 km), the characteristics of these debris objects with eccentricity less than 0.2 can be specified in terms of suitable probability distributions. The number of objects in the altitude

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and perigee height bins in the model can be either specified or represented in terms of a mixture of Laplace distributions (McLaughlin, 1999), the eccentricity and ballistic coefficient based on suitable lognormal distributions. Further the model described the statistical nature of the debris objects with respect to five inclination bands also.

SIMPLE Model provide the number density with respect to altitude for objects below 2000 km and eccentricity less than 0.2, through a tertiary mixture of Laplace distributions which is characterised by 8 parameters, namely, three location, three scale and two weight. The SIMPLE model parameters[1] for the whole inclination band as well as five separate inclination bands, namely, (0, 36], (36, 61], (61, 73], (73, 91] and (91, 180] are given in the Tab. 1.

While other debris environment models utilize deterministic (semi-empirical) functions to describe the distributions, SIMPLE model uses probability distributions and all the functional forms of the altitude as well as the perigee height distribution follow a tertiary mixture of Laplace distributions. The number of parameter values in SIMPLE is, in general, just 8 for each description of altitude or perigee distributions and in other models it is observed that the parameter values in functional forms are more. SIMPLE model captures closely all the peak densities without losing the accuracy at other altitudes.

Table 1 SIMPLE model parameters for number density in altitude
μ – Location parameter, λ - Scale parameter ,
p – Weight parameter

Inclination Band	Parameters of the Tertiary mixture of Laplace Distributions							
	μ_1	λ_1	μ_2	λ_2	μ_3	λ_3	p_1	p_2
(0, 180]	792	158	974	087	1467	093	0.7	0 .7
(0, 36]	550	120	710	120	1450	250	0.4	0.8
(36, 61]	575	060	815	087	1440	090	0.2	0 .5
(61, 73]	900	160	815	087	1440	090	1.0	1.0
(73, 91]	740	100	950	060	1450	070	0 .4	0 .7
(91, 180]	815	165	930	130	1485	120	0.8	0 .7

3. SIMPGE (Stochastic IMPressionistic GEO Environment) Model -Extension of the SIMPLE model to GEO

3.1. Number Density Distributions with respect to Semi Major Axis

Fig. 1 provides the histograms of the semi major axis of the catalogued GEO objects for one set in the 2004. It may be noted that the first plot shows a very high peak at its modal value, in which the objects can be considered as those of the functional satellites. Second plot in Fig. 1 is the frequency distribution of the non-functional objects after removing about 40 % of the functional objects. Studies indicated that the semi major axis of these non-functional objects can be best fitted with the Tertiary mixture of Laplace distributions as used for the SIMPLE model for LEO.

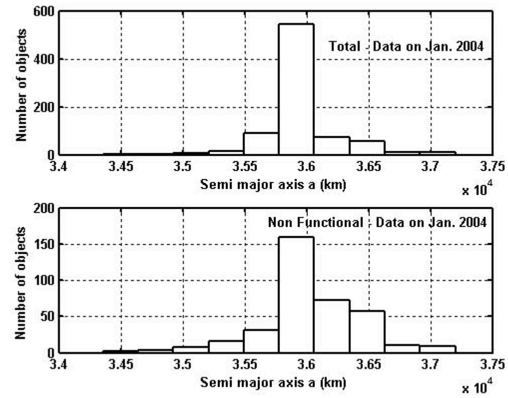

Figure 1 Histograms of the semi major axis of the catalogued GEO

Fig. 2 and Fig. 3 provide the distribution fit for the semimajor axis of the non-functional GEO objects using the tertiary mixture of the Laplace distributions for the year 2004 and 2002. Tab. 2 provides the estimated distribution parameters for the semi major axis for the years 2000 to 2004, quoted each for one set of TLEs in January. It may be noted that these parameters does not vary significantly across the years, except for the first scale parameter. The weight parameters are selected as 0.3 and 0.5 after some sensitivity studies as they do not vary much across the TLE sets considered. It is seen that this parameter is not sensitive in the distribution fit.

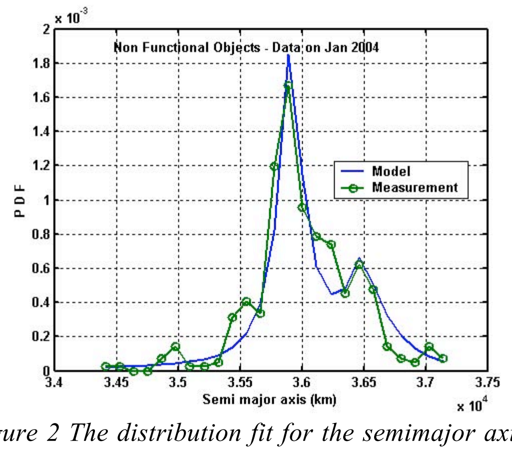

Figure 2 The distribution fit for the semimajor axis of the non functional GEO objects using the tertiary mixture of the Laplace distributions for the year 2004

We propose a model by estimating the parameters across the five years based on a least square technique, which is close to the average of the parameters across the years. Hence the model values can be taken as

$$(\mu_1\ \lambda_1\ \mu_2\ \lambda_2\ \mu_3\ \lambda_3\ p_1\ p_2) = (35500, 800, 36500, 250, 35900, 146, 0.3, 0.5)$$

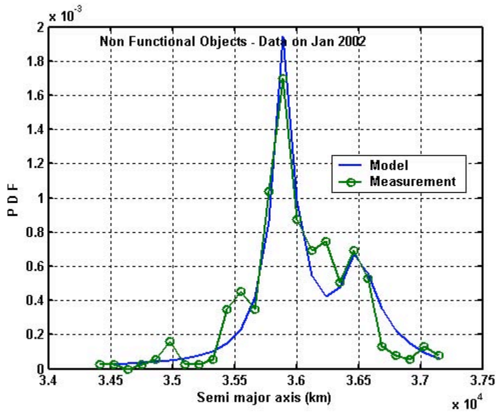

Figure 3 The distribution fit for the semimajor axis of the non functional GEO objects using the tertiary mixture of the Laplace distributions for the year 2002

Table 2 Model parameters for number density of GEO objects in semi major axis

μ – Location parameter, λ - Scale parameter ,

p – Weight parameter

Year	Parameters of the Tertiary mixture of Laplace Distributions							
	μ_1	λ_1	μ_2	λ_2	μ_3	λ_3	p_1	p_2
2000	35490	770	36450	244	35875	150	0.3	0 .5
2001	35490	900	36400	242	35865	142	0.3	0.5
2002	35500	710	36420	253	35890	154	0.3	0 .5
2003	35510	760	36510	247	35870	145	0.3	0.5
2004	35500	825	36500	255	35900	140	0.3	0.5

Fig. 4 provides a comparison between the model prediction and the measurement frequencies for all the five years from 2000 to 2004. It may be noted that the matching is quite well.

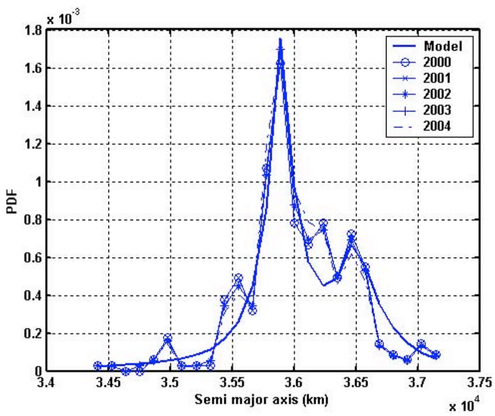

Figure 4 Comparison of SIMPGE model with observations for the year 2000 to 2004

3.2. Number Density Distributions with respect to Eccentricity

Fig. 5 shows the eccentricity distributions of the total objects. First of the two plots shows the eccentricity histogram and the second shows the histogram of the logarithm of eccentricities. The second plot clearly tells that the log (eccentricity)can be best modeled with binary mixture of normal distributions. The logarithm of eccentricity can be modeled with binary mixture of normal distributions and the same is shown in Fig. 6.

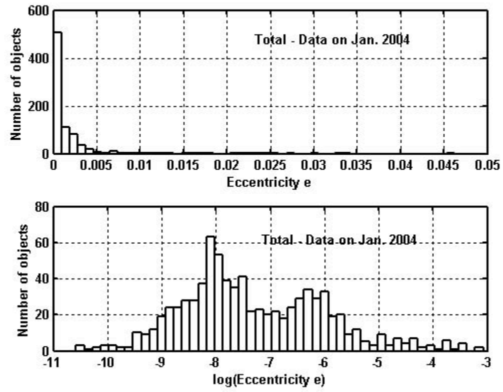

Figure 5 Histograms of the Eccentricity of the catalogued GEO

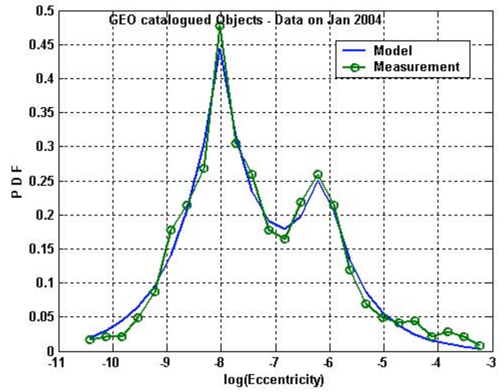

Figure 6 The binary mixture of normal distribution fit for Eccentricity of the non functional objects

The distribution parameters for the eccentricity are provided in Table 3.

Table 3 Model parameters for number density of GEO objects in Eccentricity

μ – Location parameter, λ - Scale parameter ,

p – Weight parameter

Year	Parameters of the Binary mixture of log Normal Distributions				
	μ_1	λ_1	μ_2	λ_2	p_1
2000	-7.89	0.93	–6.18	0.71	0.68
2001	-7.86	0.96	–6.23	0.72	0.66
2002	-7.95	0.95	–6.33	0.81	0.60
2003	-7.94	0.92	–6.20	0.69	0.67
2004	-8.01	0.89	–6.13	0.69	0.67

Here also it can be seen that the parameters are not varying much across the years. Hence we suggest the model parameters as

$$(\mu_1 \quad \lambda_1 \quad \mu_2 \quad \lambda_2 \quad p_1) = (-7.95 \quad 0.95 \quad -6.23 \quad 0.75 \quad 0.65)$$

3.3. Number Density Distributions with respect to Inclination

It may be noted that the inclination of the functional satellites are nearly 0 deg and the distribution of the inclination of the non-functional objects is shown in the Fig. 7. This shows the pattern of inclination distribution for a typical set of GEO debris objects for the year 2004. Similar pattern is observed in all the years. Here we propose to use uniform distribution for inclination as generalized assumption and further studies are required to arrive at some deterministic / Statistical model for Inclination.

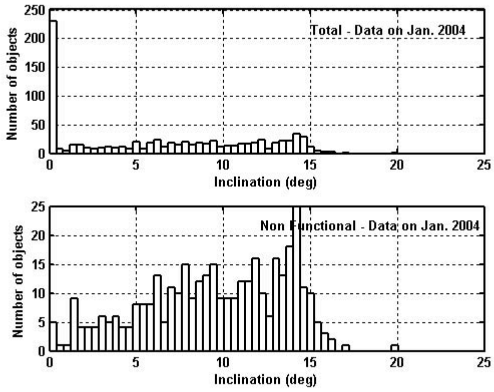

Figure 7 Histograms of the Inclination of the catalogued GEO

3.4 Correlation between Inclination and the longitude of ascending node

Fig. 8 shows the relation between the inclination and the longitude of ascending node _ of the GEO objects. From the second plot it can be seen that inclination and _ of the non-functional objects are highly negatively correlated.

4. CONCLUSIONS

This paper analyses the statistical nature of the characteristics of the tracked GEO debris objects and proposes a distribution model of the GEO environment by utilizing the approach and philosophy as laid out for the SIMPLE model proposed recently for LEO by the authors. It is noted that the catalogued objects by USSPACECOM across the years 1998 to 2004 have the same semi major axis mode (highest number density) around 35750 km above the earth. Just by properly excluding the objects in the small bin of operational region, say (35700, 35800) km containing about 40 percent of objects, the rest of the objects have a number density distribution of Single Laplace distribution with two parameters, namely location and scale. It is further noticed that the percentage of objects in the bin around mode is nearly same across the years. The Laplace parameters observed across the years are modeled mathematically, in fact the location parameters do not vary and the scale parameters shows a definite trend. These observations are successfully utilized in providing a statistical model for the GEO debris environment.

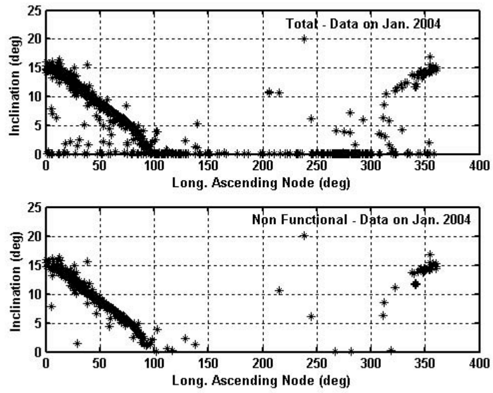

Figure 8 Correlations between the Long. Ascending Node and Inclination of the catalogued GEO

5. REFERENCES

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