DISPOSAL ORBIT CHARACTERISTICS FOR GALILEO INCLUDING ORBIT PROPAGATION TECHNIQUES

C.J. Saunders[(1)], C.E. Martin[(1)], H.G. Lewis[(2)]

[(1)]Space Division, QinetiQ Ltd, Cody Technology Park, Farnborough, Hampshire, GU14 0LX, United Kingdom. cjsaunders@space.qinetiq.com, cemartin@space.qinetiq.com
[(2)]School of Engineering Sciences, University of Southampton, Highfield, Southampton, SO17 1BJ, United Kingdom. hglewis@soton.ac.uk

ABSTRACT

A study has been undertaken to investigate the long-term (200-year) stability of disposal orbits for the Galileo constellation of navigation spacecraft. If not initially set-up correctly, the orbits exhibit exponential growth in eccentricity which would cause them to re-enter the operational MEO region, and if left unchecked eventually the LEO and GEO regions as well. This is due to a resonance in the angle $2\omega+\Omega$, which at Galileo altitudes oscillates sinusoidally with a period of ~5000 years. This is driven by secular drift in both ω and Ω caused by perturbations from the Earth's J_2 harmonic. This growth is also sensitive to the inclination of the orbit in question. A study has also been performed to look at the suitability of various orbit propagation techniques for MEO orbits. A semi-analytical propagator was found to perform best in terms of accuracy and computer run-time.

1. INTRODUCTION

The nominal operational Galileo orbit will have a semi major axis of 29993.7km, an eccentricity of 0.001 and an inclination of 56°. The orbit altitude means that the satellites follow a 5/3-day repeat orbit. An End-of-Life (EOL) disposal orbit has been proposed at 300km above the operational altitude (Chabot et. al. 2004).

However, simply placing the satellite into an approximate circular orbit at this altitude is not sufficient to ensure long-term stability. Perturbations to the orbit can drive large unexpected changes in the orbital elements, which can cause the satellite to depart from the idealised circular disposal orbit.

Previous studies have shown that at Galileo orbit altitudes the orbit should be perturbed through secular variations due to the J_2 zonal harmonic in the Earth's gravitational field, resonance with certain tesseral harmonics in the Earth's gravitational field (notably C_{55} and S_{55}), due to the ~5/3 day repeat orbit, and, luni-solar gravity.

For this study a time period of 200 years has been adopted as the period over which the orbit must remain stable. The stability is required so that the satellite does not start to re-enter the region of space occupied by operational navigation satellites, or starts to penetrate the Geostationary Orbit (GEO) protected region (see Fig.1).

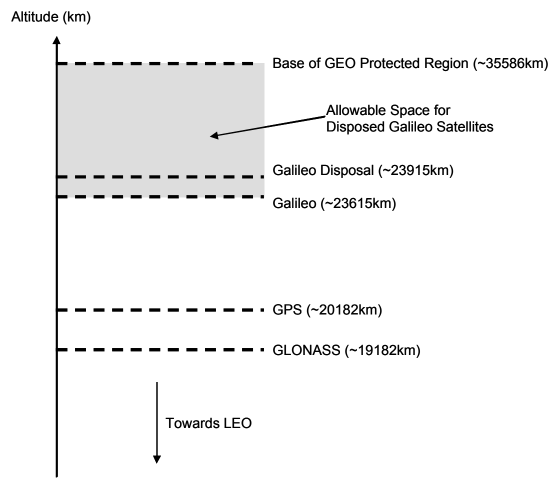

Figure 1. Allowable region of orbit space for Galileo satellites after EOL.

2. SECULAR ECCENTRICITY GROWTH

If the Galileo disposal orbit is not to interfere with operational Galileo satellites, then apogee and perigee variations must be limited to less than 300km. Assuming the disposal orbit to be circular this corresponds to a maximum allowed eccentricity growth of $\Delta e \approx 0.01$. However, previous studies have shown an eccentricity growth of $\Delta e \approx 0.7$ is possible in the Medium Earth Orbit (MEO) region for uncontrolled objects over 150 year time periods (Chao and Gick 2004).

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This large eccentricity growth is caused by a resonance in the angle 2ω+Ω (where ω is the argument of perigee and Ω is the right ascension of ascending node). Chao and Gick 2004 show that the long-term ('doubly-averaged') variation in the Galileo orbit eccentricity due to third body perturbations is:

$$\frac{de}{dt} = -(15/8)e\gamma s[-0.0072\sin 2(\omega-\Omega) \\ -0.1277\sin(2\omega-\Omega) - 0.511\sin(2\omega) \\ +0.4714\sin(2\omega+\Omega) - 0.0984\sin 2(\omega+\Omega)], \quad (1)$$

where e is the orbit eccentricity, $s=(1-e^2)^{1/2}$ and $\gamma=n_3^2 R_m/n$, where n_3 is the mean motion of the third body, n is the mean motion of the Galileo orbit, and R_m is a mass ratio (1 for solar perturbations, 182.3 for lunar perturbations).

However, ω and Ω are changing with time as they are driven by the Earth's oblateness (J_2) and luni-solar gravity effects. The time rate of change due to J_2 is modelled through the well known equations,

$$\dot{\Omega}_{J2} = -\frac{3nR_e^2 J_2}{2p^2}\cos(i), \quad (2)$$

$$\dot{\omega}_{J2} = \frac{3nR_e^2 J_2}{4p^2}\left[4-5\sin^2(i)\right], \quad (3)$$

where $p=a(1-e^2)$, a is the semi-major axis, R_e is the equatorial radius of the Earth, i is the orbit inclination and J_2 is the second order zonal harmonic (~1082.64×10^{-6}). Third body effects on ω and Ω are modelled through the following expressions (Larson and Wertz 1999),

$$\dot{\Omega}_{Sun} = -0.00154\frac{\cos(i)}{n}, \quad \dot{\Omega}_{Moon} = -0.00338\frac{\cos(i)}{n},$$
$$\dot{\omega}_{Sun} = 0.00077\frac{(4-5\sin^2 i)}{n}, \quad \dot{\omega}_{Moon} = 0.00169\frac{(4-5\sin^2 i)}{n}, \quad (4)$$

where the nomenclature is the same as for Eq. 2 and Eq. 3. Summing together the contributions from J_2, the Sun, and the Moon, the total rate of change for Ω and ω in Galileo disposal orbits is:

$$\dot{\Omega}_{Total} = -0.02553\ ^{o}/\text{day}, \quad \dot{\omega}_{Total} = 0.01286\ ^{o}/\text{day}. \quad (5)$$

Applying the rates in Eq.5 to the sinusoidal terms in Eq. 1, allows the oscillation rates of the various sinusoidal terms to be established. These are shown in Table 1.

Table 1. Angular rates and oscillation periods for Galileo disposal orbits.

Sinusoidal Term	Angular Rate (°/day)	Oscillation Period (yr)
2(ω-Ω)	0.07678	~12.8
(2ω-Ω)	0.05125	~19.8
2ω	0.02572	~38.3
(2ω+Ω)	0.00019	~5191.1
2(ω+Ω)	-0.02534	~38.9

Over long periods of time (200 years was chosen for this study), the (2ω+Ω) term acts as a constant resonance term. It is this very long period oscillation which is responsible for the large eccentricity growth predicted to affect MEO

As mentioned by Chao and Gick (2004), the large growth in eccentricity predicted in the MEO region is very sensitive to orbital inclination. Fig. 2 shows the results of applying the sinusoidal terms in Eq. 1 to a range of inclinations between 0° and 90° at the nominal Galileo disposal altitude.

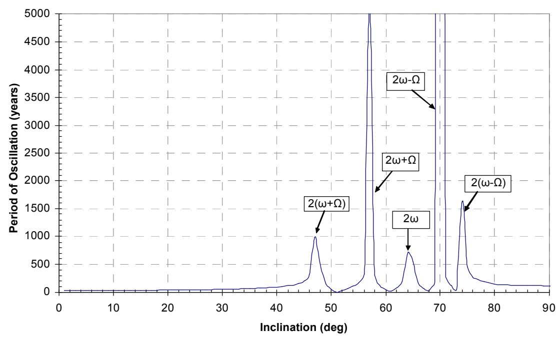

Figure 2. Oscillation periods (limited to 5000 years for clarity) for given sinusoidal terms at Galileo disposal altitude.

Fig. 2 shows that the Galileo disposal orbit lies right on the edge of the inclination range responsible for the long oscillations in 2ω+Ω. Other inclinations are more sensitive to the other combinations of ω and Ω, and these are illustrated in Fig. 2 also. Merz and Jehn (2004) show that the long term behaviour of the disposal orbit eccentricity is also dependent on the final value of ω and Ω adopted in the disposal orbit, and the initial eccentricity of the disposal orbit (smaller initial values leading to smaller future values).

Because of the very long periods considered for disposal orbits, the eccentricity growth is very weakly dependent on the disposal epoch (Balashova, et. al. 2004). The influence of ω and Ω is more complicated and is related to the inclination of the orbit.

As Fig. 2 shows, the Galileo orbit lies just outside the main 2ω+Ω resonance zone. However, the inclination of MEO objects follows a ~37.5 year oscillation caused by the Sun and Moon (similar to the 54 year cycle seen in GEO objects). This oscillation can move the orbit either directly into the 'heart' of the 2ω+Ω zone, or move it away towards the oscillation 'null' at ~50° inclination. However, the behaviour of this inclination cycle is dependent on the initial values of ω and Ω in the disposal orbit (again in an analogous fashion to GEO). Hence, the initial values of ω and Ω indirectly effect the eccentricity growth by influencing the orbit inclination, which then affects the eccentricity growth.

Fig. 3 shows the inclination history over 200 years for a satellite at Galileo disposal orbit altitude with varying values of 2ω+Ω. Fig. 3 was produced using the 'LOP' part of Satellite Tool Kit (STK). The gravity field used was 5×5, luni-solar gravity was modelled and Solar Radiation Pressure (SRP) was included with an area-to-mass ratio of 0.01 m^2/kg. The time step for the propagation was 5 days. Note that it is the combination of ω and Ω (i.e. 2ω+Ω) that is important to consider, rather than the individual elements themselves (Merz and Jehn 2004). As Fig. 3 shows for certain values of 2ω+Ω (e.g. 90°, 135°), the inclination suffers a secular decrease with a superimposed ~37.5 year oscillation. Other values (e.g. 315°) show slight increases over 200 years with a superimposed oscillation. Fig. 4 shows the eccentricity history for the same objects as in Fig. 3. As can be seen those objects with initially decreasing inclinations (e.g. 2ω+Ω = 90°, 135°) have the smallest eccentricity growth over the 200 year simulation. Here the initially decreasing inclination moves the orbit out of the resonance zone near 56° (c.f. Fig. 2).

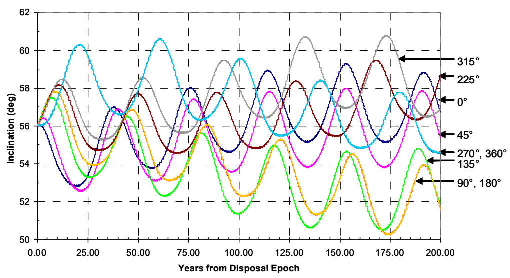

Figure 3. Inclination history for an object in a Galileo disposal orbit with different values of 2ω+Ω (shown by arrows to the right).

Conversely those orbits with increasing initial inclination suffer the biggest eccentricity growth over the 200 years (e.g. 2ω+Ω =225°). However, it should be noted that there is not a well defined trend. Certain orbits show initially decreasing inclination (e.g. 2ω+Ω =45°), but result in a high final eccentricity. It appears therefore that evidence for a general trend exists, but that a more comprehensive survey of possible disposal orbit parameters is required to fully establish whether or not such a trend truly exists.

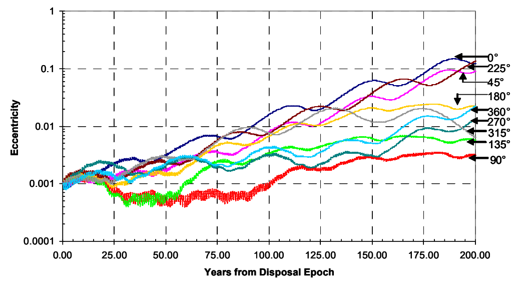

Figure 4. Eccentricity history for an object in a Galileo disposal orbit with different values of 2ω+Ω (shown by arrows to the right).

3. MINIMISING EOL INTERFERENCE

Section 2 has shown that a natural mechanism exists to drive the eccentricity of Galileo disposal orbits to values large enough to cause the orbit to penetrate operational navigation satellite orbits. To minimise this growth certain EOL manoeuvres can be adopted. These are shown below (note that these assume a 1000kg spacecraft and a 250s Isp chemical thruster system):

4.1 Reduce Initial EOL Eccentricity. In general a 100-fold increase in eccentricity can be observed over 200yr. A simplistic approach to ensuring that e<0.01, is to set e<0.0001 at EOL. This can be achieved for a ΔV and propellant cost of:

$$\Delta V \approx \frac{\Delta e \cdot V_{orb}}{4} \approx 0.82 m/s \equiv 0.33 kg \qquad (6)$$

Where V_{orb} is the orbit velocity and Δe is the eccentricity change required.

4.2 Reduce Orbit Inclination. If the orbit inclination is reduced to near 50° at EOL, then the satellite will be outside the 2ω+Ω oscillation zone. This is a reduction of Δi=6°, which would cost:

$$\Delta V = 2V_{orb} \sin\left(\Delta i / 2\right) \approx 326 m/s \equiv 121 kg \qquad (7)$$

4.3 Change Argument of Perigee. Targeting the final value of 2ω+Ω can result in lower eccentricity growth. Changes to ω are cheaper in ΔV terms than changes in Ω. The worst case Δ2ω change will be 180°, so that Δω=90°. This would cost:

$$\Delta V \approx \frac{nae}{2} \cdot |\Delta\omega| \approx 163.2 m/s \equiv 64.4 kg \, , \qquad (8)$$

where n is the mean motion, and a is the semi-major axis.

Thus, minimising the initial eccentricity is clearly the most cost-effective option in terms of ΔV and propellant.

4. ORBIT PROPAGATORS FOR MEO

Of vital importance in modelling the long-term behaviour of any disposal orbit, is to have an orbit propagator that can predict the perturbations and resonances that may occur. However, an overly complex model can add a significant overhead to the overall run-time. To determine the best type of propagator to use with MEO disposal orbits, two numerical (STK 'HPOP' and an internal QinetiQ model), one semi-analytical (STK 'LOP') and one purely analytical orbit propagator (an internal QinetiQ model) were compared against each other. When performing a 50 year propagation of a MEO object with Earth and luni-solar gravity, as well as SRP included the following run times were produced.

Table 2. Run-times for 50 year MEO propagation.

STK 'HPOP'	QinetiQ Numerical	STK 'LOP'	QinetiQ Analytical
-13.5min	~1min	~1.5min	~3sec

The QinetiQ numerical model, and the semi-analytical 'LOP' model are very closely matched in terms of run-time, but the numerical model is limited to 50 year propagations. Only STK 'LOP' and the QinetiQ Analytical model can perform 200 year propagations. When comparing these propagators, specifically to look at Galileo orbits it was found that only the semi-analytical model had sufficient model complexity to predict eccentricity growth. This was surprising as theoretically only J_2 and luni-solar gravity are needed to produce the resonance in 2ω+Ω (J_2, J_3, J_{22}, luni-solar gravity and solar radiation pressure are included in the QinetiQ Analytical model). As the QinetiQ analytical model has very fast run-times (necessary for large scale constellation collision risk simulations) we intend to study this phenomenon in future studies. Using the 'LOP' propagator the complexity of the internal Earth gravity field was also investigated. Denoting the harmonics of the gravity field as C_{lm} and S_{lm}, it was found that a sharp transition occurs between models with l,m ≤ 2 and those with l,m>2. The runs with l,m>2 provided a much better fit to the numerical models. Therefore it appears that l,m=3 as a minimum must be included in any propagator force model (see Fig. 5 which shows this comparison).

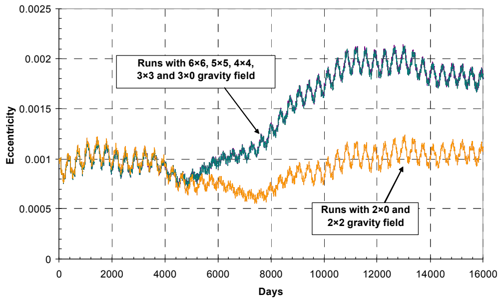

Figure 5. Influence of gravity field on propagators.

As the evolution of the inclination appears to strongly influence the eccentricity growth, the luni-solar force modelling is also very important. Therefore we also intend to study how various orbit propagators calculate luni-solar ephemeris, and how to maintain the highest possible level of accuracy over very long time periods.

ACKNOWLEDGEMENTS

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REFERENCES

Chabot, T., Legendre, P., Fraysse, H., *Galileo & GSTB Orbit Evolution*, Presentation to 22nd Inter-Agency Debris Committee, Abano Terme, Italy, April 2004.

Chao, C.C., Gick, R.A., *Long-term Evolution of Navigation Satellite Orbits: GPS/GLONASS/GALILEO*, Adv. Space. Res., Vol. 34, 1221-1226, 2004

Wertz, J.R., Larson, W.J., *Space Mission Analysis and Design 3rd Edition*, Microcosm Press and Kluwer Academic Publishers, 1999.

Merz, K., Jehn, R., *Stability of MEO Disposal Orbits*, Presentation to 22nd Inter-Agency Debris Committee, Abano Terme, Italy, April 2004.

Balashova, N.N., et. al., *Research of MEO Navigating Systems Stability*, Presentation to 22nd Inter-Agency Debris Committee, Abano Terme, Italy, April 2004.