THE CRITICAL DENSITY THEORY IN LEO AS ANALYZED BY EVOLVE 4.0

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ABSTRACT

The critical density theory is revisited with NASA's long-term debris environment model, EVOLVE 4.0. Previous studies were based on incomplete data and simplifying assumptions. Recent data of ground-test and on-orbit breakups and fragment decay have been utilized within the EVOLVE 4.0 structure to realistically model the projected low Earth orbit (LEO) debris environment. The EVOLVE 4.0 predictions over a 1000-year time period are shown to be consistent with the earlier predictions of simpler models; however, EVOLVE 4.0 is shown to be able to eliminate the simplifying assumptions in these earlier models and be a more accurate tool in understanding the most effective migration measures to limit future population growth.

1. INTRODUCTION

A debris population is said to be at the point of critical density when the rate of increase of objects due to random collisions just balances the rate of decrease due to atmospheric decay. If the population is below critical density it will tend to a lower equilibrium since the decay will be dominant over the generation process. If above, it will tend to a higher equilibrium as the fragments grind to smaller pieces, which will remain in the environment but will not be effective colliders.

This critical condition may exist independently of a phenomenon in which collision fragments dominate the collisional activity. Collision fragment dominance has not occurred in LEO (low Earth orbit) yet, but a state above critical density may already exist within certain altitude regimes in LEO, as noted in [1] through [7] and references therein. These studies made use of simple particle-in-box type models to illustrate the basic growth of the debris population over long time periods.

Our analysis refers specifically to the critical density as defined in [7]. This work expanded the definitions and quantified the phenomenon with recent data that is also incorporated in EVOLVE 4.0. The term critical density, itself, was penned in [1] and assigned to the 1989 spatial density of collisional breakup fragments within an altitude band. This assumed a constant intact population collisions in any other band. Referring to the then-current catalog which indicated that about one half the cataloged objects were fragments and the other half were intacts, the expression for the critical density for cataloged objects became,

$$S = \frac{1}{V\sigma N_o \tau} \qquad (1)$$

Here, S is the total spatial density of cataloged objects in a 100-km altitude band, N_o is the canonical number of cataloged fragments generated in a breakup cloud that are capable of causing a catastrophic collision, τ is the average lifetime of the breakup cloud within the altitude band (assuming a constant solar flux), and V and σ, respectively, are the average relative velocity and collision cross section within the altitude band.

The derivation leading to Eq. 1 was honed in [7] to acknowledge the differences in size and mass between intacts and fragments. Considering intact-intact collisions and intact-fragment collisions separately, [7] further categorized the critical density phenomenon as 'unstable' or 'runaway'. Both conditions were shown to depend on the intact population. The critical intact spatial density initiating an unstable environment was given by,

$$_U S_i = \frac{1}{V(\sigma_f + k\sigma_i) N_o \tau} \qquad (2)$$

Eq. 2 is the re-derivation of Eq. 1. In both, if the intact population exceeds the stated spatial density then the fragment population increases until it reaches a higher equilibrium. The factor k is defined as the ratio between the current intact and fragment populations, *i.e.*, S_i / S_f, The collision cross sections, σ_i and σ_f, are the average values within an altitude band for an intact-intact collision and an intact-fragment collision, respectively. A runaway environment, one in which the fragment population will increase without bound, was derived to be the critical intact spatial density,

$$_R S_i = \frac{1}{V\sigma_f N_o \tau} \qquad (3)$$

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Eqs. 2 and 3 were derived under the simplifying requirements of Eq. 1. In addition, it was assumed that intact-intact collisions resulted in twice as many cataloged fragments as those of intact-fragments.

Relaxing the simplified conditions, *i.e.*, allowing movement of fragments into lower and higher altitude bands through decay and ejection, respectively, and permitting near-circular fragment orbits, resulted in slightly more complicated forms of Eqs. 2 and 3. But the overall theory did not suffer a major revision. Additionally, when changed from critical spatial density to critical number of intacts above some height, *h*, that produce an unstable or runaway environment, the equations became, respectively,

$${}_U N_i = \frac{4\pi a^3 V_o \rho_a C_D}{(\sigma_f + k\sigma_i) V W (m/A)_a N_o} \qquad (4)$$

and

$${}_R N_i = \frac{4\pi a^3 V_o \rho_a C_D}{\sigma_f V W (m/A)_a N_o} \qquad (5)$$

The new terms are defined within altitude bands to be,

N_i	= number of intact objects above *h*
a	= semi-major axis at *h*
V_o	= circular orbit velocity at *h*
ρ_a	= average atmospheric density at *h*
C_D	= drag coefficient
V	= average relative velocity at *h*
W	= weighting factor for elliptical orbits ($W \geq 1$ if $e \geq 0$)
$(m/A)_a$	= average mass-to-area of fragments massive enough to cause a catastrophic collision

The final conclusion of [7] was that in the ‘best case’, where upper stages do not contribute to the collisional activity, the 700 km to 1500 km altitude region is currently above the unstable threshold. The altitude region of 1300 km to 1420 km is above runaway.

This paper directly extends the work of [7] by applying NASA’s long-term debris environment model, EVOLVE 4.0, to the task. This code is a high fidelity simulation model that explicitly incorporates source models and future interactions in a Monte Carlo process. With the EVOLVE 4.0 code, the simplifying assumptions of [7] need not be assumed directly.

2. EVOLVE 4.0 AND UPGRADES

The EVOLVE 4.0 code and underlying analyses are detailed in [8]. The main advantages of using this code over the techniques of past studies in the analysis of the

(1) EVOLVE 4.0 uses NASA’s latest data-derived breakup model as described in [9] and [10],
(2) the Monte Carlo processes in the breakup event assignment, as well as the subsequent breakup fragment deposition mimic the natural variability of individual breakups, and
(3) the ability to apply mitigation procedures, *i.e.*, *n*-year deorbit rules and explosion suppression, directly to the EVOLVE projected environment allows the analyst to easily test future scenarios.

Specialized upgrades to the code are necessary for this study. They include a tagging or separation of the spatial density arrays associated with intacts, explosion fragments, and collision fragments. Also, groupings of arrays to allow the long (1000-year) projection periods required consideration of objects no smaller than 10 cm in size. This does not impede the study since EVOLVE limits collisional interactions to be between objects greater than 10 cm in size.

3. THE EVOLVE 4.0 LONG-TERM LEO DEBRIS ENVIRONMENT

All EVOLVE 4.0 projection environments in this study build on the same EVOLVE 4.0-generated present-day LEO environment. This includes a NASA-generated launch history file from 1957 to present, and a list of all known breakups run through the EVOLVE 4.0 fragment deposition software. This generates an historical environment that closely matches the Space Surveillance Network (SSN) Catalog. Use of this environment in place of the Catalog serves to verify the breakup model software.

For a single projection, that historical environment is used as the basis of several iterative projections that are distinguished by their random number seed values. Ten such iterations are performed and the EVOLVE 4.0 projection is then defined as the mean of the ten.

In all cases here, explosions are forbidden within the projection period. This is an international goal and one that simplifies the understanding of the phenomenon and helps keep the 1000-year projected environment within computational constraints.

4. THE EVOLVE 4.0 CRITICAL DENSITY STUDY RESULTS

The first tests in our study are performed to determine how closely the EVOLVE 4.0 projection follows one of the results of [7]. There, maintaining the current intact population, *i.e.*, payloads and upper stages, was noted to result in an unstable environment within the altitude region of 600 km to 1500 km and a runaway between the altitudes of 800 km and 970 km and between 1300 km and 1400 km. Fig. 1 displays the EVOLVE-generated LEO growth for three cases: maintenance of 1×, ¾×, and ½× the current intact population.

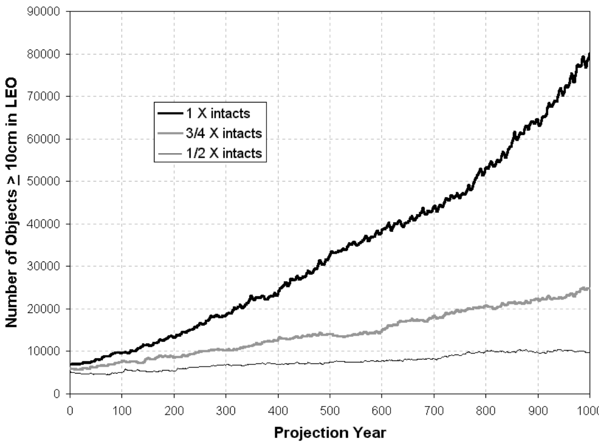

Fig. 1. Number of objects in LEO greater than 10 cm in size over a 1000-year projection. The stated intact population is maintained throughout.

Fig. 1 shows agreement in kind between the EVOLVE results and those of [7]. The overall 1× intact population in Fig. 1 appears to be in a runaway state. The break between the runaway and the unstable state is somewhere between the 1× and the ¾× cases. A breakdown of collision debris by 100-km altitude bands in Fig. 2 reveals the region of maximum runaway to be within the 900 km to 1000 km altitude band, with runaway within 700 km to 1100 km. Not shown are the other altitude bands in LEO, which also appear to be increasing, though much more slowly. This is because the fragment population is decaying through the lower altitudes and being ejected into higher altitudes at an ever-increasing rate due to the ever-increasing collisional fragmentation. The EVOLVE 4.0 results, then, suggest that the current population is in a runaway state throughout most of LEO. This is a more strident runaway state than is found by [7] and is due mainly to two sources. First the EVOLVE 4.0 orbital propagator uses the updated function for exospheric temperature in [11] that results in higher values during mid- and low-level solar flux periods. Second, the fragment area-to-mass values derived in [9] from observations of numerous on-orbit breakups are generally higher than those of the P-78 breakup used exclusively in [7]. Both result in a higher level of atmospheric decay and, therefore, a higher level of coupling between altitude bands than is permitted in [7].

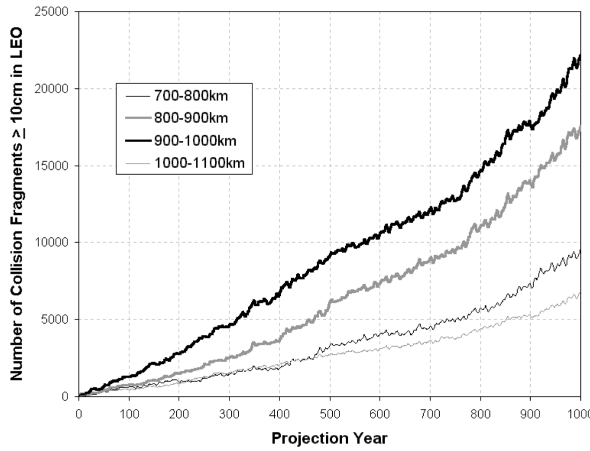

Fig. 2. Growth of collisional debris (greater than 10 cm in size) within selected 100-km LEO altitude bands for the 1× intact test case.

Though Eqs. 4 and 5 are strictly applicable to the conditions set forth by [7], they are used here with this EVOLVE 4.0 environment as a test of compliance between the two methods. The values W=1.1 and C_D=2.2 were used in [7] and are used here. The other terms are derived for each altitude band through simple calculation (h_1, a, and V_o) or via the EVOLVE 4.0 breakup model and environment (N_o, k, σ_i, σ_f, ρ_a, V, and $(m/A)_a$). The current EVOLVE 4.0 environment and purported regions of instability are estimated in Fig. 3.

Strictly speaking, the result indicates that the current EVOLVE 4.0 environment is in an unstable state within the altitude range of about 700 km to 1700 km. This is similar to the result of [7], in which the current environment was calculated to be unstable between 600 km and 1500 km. Contrary to the results of [7] and the present study displayed in Fig. 1, however, there is no evidence of a runaway state by this analysis.

Strong adherence of Eqs. 4 and 5 to the EVOLVE 4.0 result as displayed in Figs. 1 and 2 cannot be expected since, as noted above, the derivations of these equations in [7] required specific simplifying assumptions on the environment. These include the use of average values of collision cross section and other parameters over several altitude bands, the near-circular orbit assumption of all intacts and debris, and the use of the P-78 fragmentation as representative of all collisional fragmentations in ejection velocity and area-to-mass of fragments. Even so, the EVOLVE 4.0 environment is within an order of magnitude of compliance with Eqs. 4 and 5.

The test results shown in Fig. 1 require blanket maintenance of some existing intact population. These are artificial scenarios designed in [7] to approximate projected environments in which different levels of mitigation were assumed. But a detailed mitigation scenario is one for which EVOLVE 4.0 is well suited. Our next test then applies an 8-year cycled launch traffic to the projection period, excluding all operational debris, and including a strict 25-year deorbit rule. After deployment, all upper stages are required to move immediately into a decay orbit, and all payloads after 10 years of service. As in the previous EVOLVE 4.0 tests no explosions are permitted in the projection period. This scenario corresponds to the 'best case' of [7]. There, the upper stage contribution to the collisional activity was simply ignored (Here, the upper stages are moved to decay orbits.).

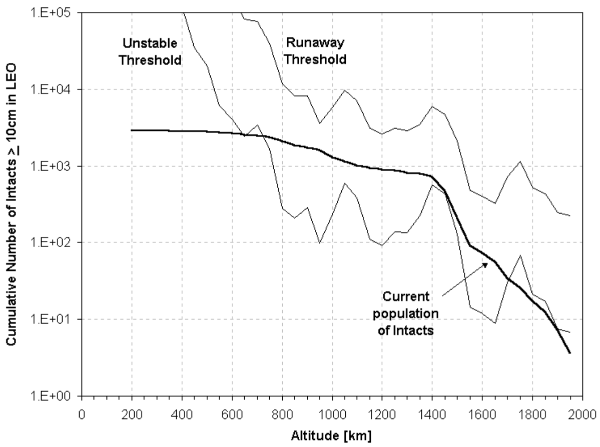

Fig. 3. Possible regions of instability using the EVOLVE 4.0 parameter values for the 1× intact test case in text Eqs. 4 and 5.

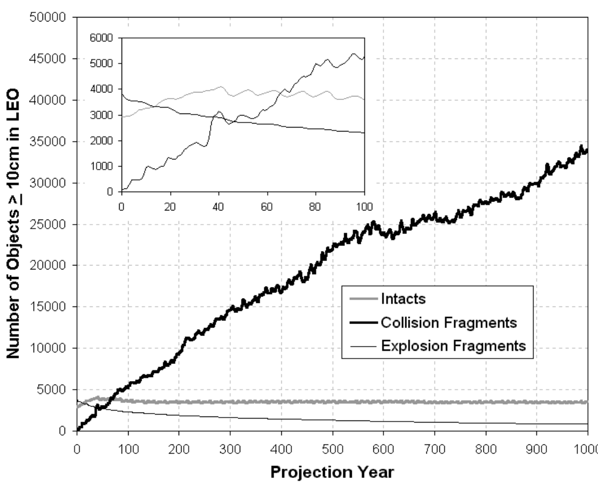

Fig. 4. EVOLVE 4.0 projected environment. Intacts, explosion fragments and collision fragments are separated.

The growth of the resulting environment is depicted in Fig. 4 and its inset. The population split shows the effect of the 25-year rule on the intacts. That population continues to increase as the deorbit rule is applied then

3500 objects after about 100 years. The explosion fragment population continually decreases as the historical period low-altitude fragments decay. The 800 fragments remaining after 1000 years are those of the high-altitude upper stage breakups around 1400 km.

Collision fragments dominate the environment within 100 years, a result that has been noted in previous EVOLVE 4.0 studies [12] and [13]. According to the critical density theory of [7] they appear to be in an unstable state moving to a higher, but finite, equilibrium until about 600 years into the projection period when they begin to exhibit the exponential growth of a runaway state.

Fig. 5 displays the collision history categorized by collider type. Intact-intact catastrophic collisions do dominate for about the first half of the 1000-year projection period, but are overtaken by the intact-collision fragment events at this point. The long-term dominance of intact-fragment events is also noted in [6]. There, as here, the explosive growth of debris is attributed to these intact-fragment collisions. In addition, EVOLVE 4.0 demonstrates a non-trivial role for the collision fragment-collision fragment events. These fragments represent an exponentially increasing population as long as the intact population is forced to remain constant.

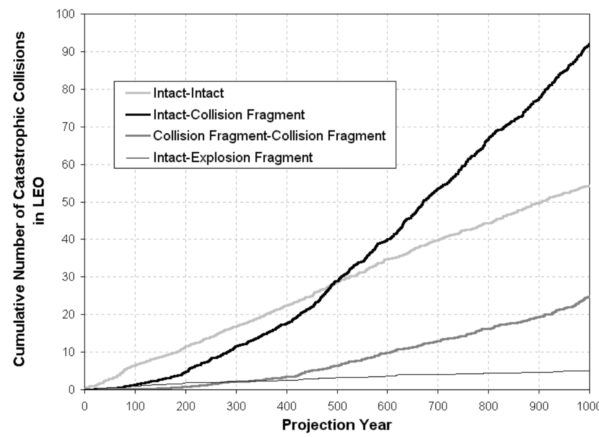

Fig. 5. Cumulative number of catastrophic collisions categorized by the four major collider combinations.

The apparent runaway state after year 600 in Fig. 4 may be further investigated through the altitude dependence of the collision fragment population over time. During the projection period the dominant region of collisional activity remains at lower altitudes (less than 1000 km) where the intacts are more numerous and larger. But the collision fragment population, itself, shifts from lower to higher (greater than 1000 km) altitudes as evidenced by Figs. 6 and 7, respectively. This is a function of the atmospheric decay rate, which decreases exponentially

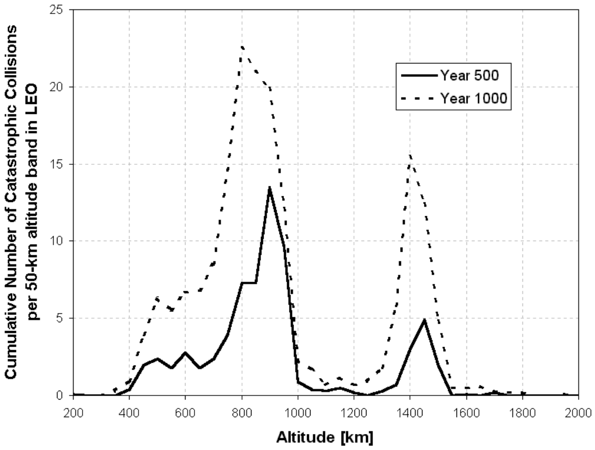

Fig. 6. Catastrophic collision activity as a function of altitude and time.

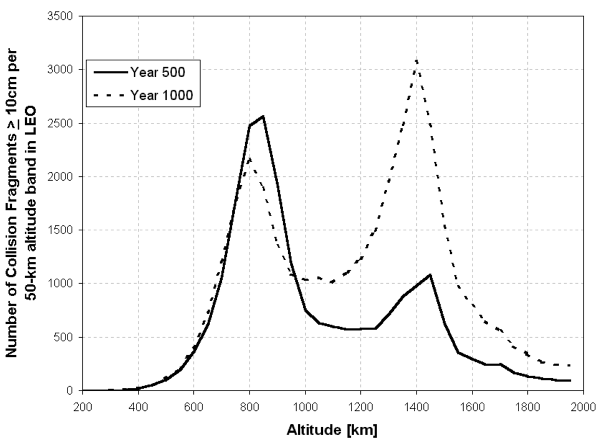

Fig. 7. Collision debris as a function of altitude and time.

Figs. 8 and 9 display the collision debris growth by altitude. In agreement with the analysis of [7] the low-altitude and high-altitude regions appear to be uncoupled, with low altitudes being unstable and high altitudes being in a runaway state.

To summarize, in the case with a mitigation standard applied, *i.e.*, the 25-year deorbit rule, a runaway state appears to exist in the high-altitude regions only. This is primarily due to the low decay rate of fragments at those altitudes. The low-altitude regions, where the majority of the collisional activity occurs, appear to be in an unstable condition moving to a finite equilibrium.

Finally, an interesting observation that displays the importance of the actual intact population and confirms the results of [7] is a comparison between the first test, that of maintaining the current 1× intact population, shown in Fig. 1, and the second test, that of applying a mitigation standard, Fig. 4. The maintained current 1× average intact population of 3500 is maintained through the 25-year rule mitigation standard test. Yet, it is the first test (2915 intacts) that displays runaway instability at the majority of altitudes. The second test (3500 intacts) displays runaway characteristics at high altitudes only and at a much lower rate than that of the first test (compare Figs. 2 and 9).

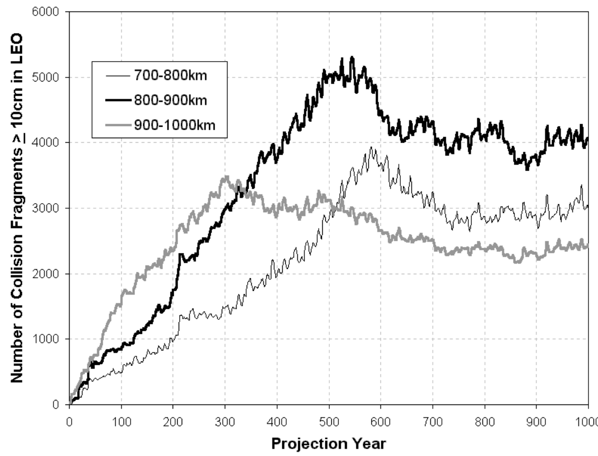

Fig. 8. Major (low) altitude regions of growth of collision debris (700km to 1000km).

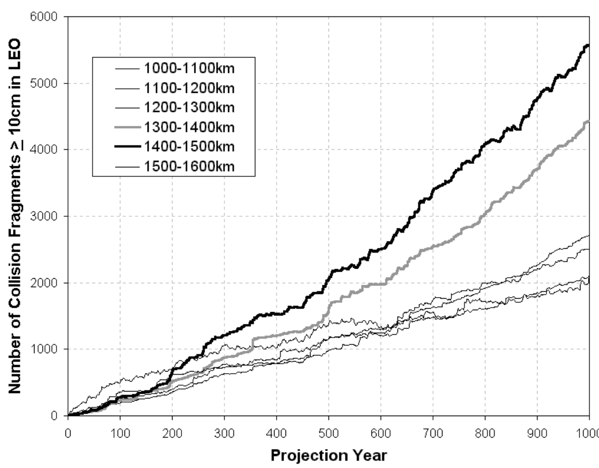

Fig. 9. Major (high) altitude regions of growth of collision debris (1000km to 1600km).

The point is that getting upper stages out of LEO increases the critical levels. In the language of Eqs. 4 and 5, the smaller σ_i (and smaller resulting σ_f) lead to higher levels of instability thresholds. Collisions become less likely and it becomes less likely that the environment will seek a higher equilibrium whether finite or infinite as the intact size decreases. This can be true even as the number of intacts increases.

5. CONCLUSIONS

The purpose of this study is to illustrate the utility of analysis of the critical density phenomenon. The theory as formulated by [7] most closely implements the EVOLVE 4.0 data sources so that work is used for direct comparison. However, other studies based on the simpler particle-in-box models also generally agree with EVOLVE 4.0 results. This study has an advantage over those previous studies in that the EVOLVE 4.0 code explicitly applies source models to debris environment projections: source models which are based on extensive data analysis.

This EVOLVE 4.0 critical density study confirms the long-term value of applying mitigation measures of deorbiting spent vehicles. In line with [7], it appears that the current environment is in a runaway state throughout most of LEO. The mitigated environment, *i.e.*, with a 25-year deorbit rule applied, is unstable, with runaway conditions occurring in the higher altitude regimes only. The intact-collision fragment collisions are shown to become the dominant interaction in the distant future if the intact population is replenished in agreement with [6].

In addition, though the intacts are the ultimate drivers of the instabilities, the EVOLVE 4.0 results show a minor but non-trivial contribution by collision fragment-collision fragment collisions. This has not been noted before.

Where the comparisons of EVOLVE 4.0 results and previous work falter is in the specific applications of [7] equations for unstable and runaway intact populations. This is not surprising given that the theory of [7] is derived with specific simplifying assumptions. Among these are that all LEO intacts and debris remain in nearly circular orbits, that the intacts are much more massive than any fragment, and that the P-78 collision is representative of all collisions. These restrictions are not assumed in EVOLVE 4.0, which allows for a distribution of fragment characteristics based on other observed breakups and on the Monte Carlo process. Even so, the EVOLVE .40 application of the equations is within an order of magnitude of the results of [7].

Finally, it must be noted that projections of this duration (1000 years) have not previously been attempted with EVOLVE 4.0 or any other long-term environment simulation model of its capacity. The code, itself, is continually under refinement as new data sources become available. However, the tools derived for this analysis provide a starting point for future more in depth studies of the critical density phenomenon using debris simulation models such as EVOLVE 4.0.

6. REFERENCES

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