POPULATIONS FOR A DIVINE-BASED SPACE DEBRIS MODEL

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1 ABSTRACT

A new analytical tool to calculate flux and spatial density resulting from space debris was developed with the aim to allow fast and accurate analysis for any target orbit and altitudes ranging from LEO to GEO. The model is based on the approach presented by N. Divine in 1993, which was developed for the meteoroid flux analysis. This approach was adapted to the Earth's debris environment, and is implemented in ESA's meteoroid and space debris reference model MASTER '99 as so called 'Standard application'. An extended version of the model called DIADEM, which is restricted to flux and spatial density analysis at high altitudes, includes a debris particle cloud model. This model may be applied to known cloud generation events, e.g. solid rocket motor (SRM) firings.
The accuracy of the analysis results of the Divine-based model is essentially depending on the implementation of the underlying debris population. This applies to both the background population and to particle clouds. This paper is intended to describe the adaptation of the MASTER reference population to the needs of the new model, and to give a survey of its cloud modelling capabilities.

2 INTRODUCTION

The main motivation to develop a new tool for the calculation of debris flux and spatial density has been the lack of a fast analytical model which covers the geostationary orbit region, and the fact that the existing debris models are working with their own populations only. The development of the new model based on the approach of N. Divine [1] offers the possibility to make use of any population which can be described by distributions of the particle size and the orbital elements of the particle orbits. Thus, not only populations which are based on mathematical models (e.g. break-up models like POEM or EVOLVE) but also populations based on in-situ measurements (e.g. GORID) may serve as input for the new debris model.
Each (sub-) population (e.g. fragments) has to be described by appropriate probability density distributions, considering cross-couplings between the particle diameter and the particle orbital element distributions on one hand, and cross-couplings between the orbital elements on the other hand. A detailed description of the population implementation philosophy is given in section 4.1. The currently ongoing optimisation and the automation of the establishment of the required probability density distributions is addressed in section 6.1.

In-situ measurements (e.g. of the GORID detector aboard a GEO satellite) prove the existence of another kind of populations, so called particle clouds. The implementation of a simple cloud model within the Divine-based debris model is described as well as the results of the cloud model application to a particular SRM firing cloud, which was detected by GORID in section 5.

3 IMPLEMENTATION

3.1 Flux/spatial density calculation

Since analysis is carried out for Earth-bound target objects, the co-ordinate system has to be changed from a heliocentric, ecliptic system to an Earth-centred, equatorial system. Thus, the orbital elements of the target as well as those of the population objects have to be described in the Earth-centred, equatorial system.
Divine makes use of the following equation describing the phase space density in the six-dimensional position-velocity space:

$$g_0 = \frac{1}{2\pi e}\left(\frac{r_1}{GM_0}\right)^{\frac{3}{2}} N_1 p_e p_i \qquad (1)$$

This equation is independent of the particle mass and independent of time, and it is based on the assumption of ecliptical (equatorial) and rotational symmetry of the population.
The re-formulation of the original Divine phase-space-density equation (1) by Kessler/Matney [2] with respect to a description of the population by probability density functions of the orbital parameter distributions results in the conversion from these so called 'textbook' distributions D_1, D_e, D_i to the distributions N_1, p_e, p_i used in the original Divine equation (3):

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$$N_1 = \frac{D_1}{r_1^2} \qquad p_i = \frac{D_i}{2p^2 \sin i} \qquad p_e = (1-e)^{\frac{3}{2}} D_e \quad (2)$$

where D_1 is the distribution of the perigee radius r_1, D_e is the distribution of the eccentricity e, and D_i is the distribution of the inclination i of the particle orbits.

The number of objects (in a certain particle mass interval $[m_l, m_u]$) per unit volume (or spatial density) N is calculated by

$$N = \frac{H_M}{p} \cdot \int_{c_{\min}}^{p/2} N_1(\sin c)\, dc \cdot \int_{e_c}^{1} \frac{p_e}{\sqrt{e - e_c}} de \cdot \int_{|d|}^{p-|d|} \frac{(\sin i) p_i}{\sqrt{\cos^2 d - \cos^2 i}} di \quad (3)$$

with the number of particles derived from the mass distribution

$$H_M = \int_{m_l}^{m_u} H_m dm \quad (4)$$

Equation (4) may also be established for the diameter distribution, which was used in the new model.

The auxiliary variable c, and the minimum eccentricity e_c (equation (3)), which is necessary to reach the target position, are defined as

$$c = \arcsin(r_1 / r)$$
$$e_c = \frac{r - r_1}{r + r_1} = \frac{1 - \sin c}{1 + \sin c} \quad (5)$$

The current position, for which spatial density or flux shall be calculated, is given by the spherical co-ordinates r (distance from the Earth's centre) and d (declination). Due to the a.m. symmetry assumption the third co-ordinate (right ascension) is not needed here. The lower and upper bounds of the integrals in (3) ensure that particles are contributing only, if their orbits are intersecting the position specified by r and d

Some minor changes (removal of all detector related factors) have to be implied to derive the adapted flux equation:

$$F = \frac{1}{4} \sum_{dir=1}^{4} \left[N \cdot (v_{imp})_{dir} \right] \quad (6)$$

where the impact velocity is derived by the difference of the particle velocity and the target velocity [1]

$$v_{imp} = \left| \vec{v}_{part} - \vec{v}_{tar} \right| \quad (7)$$

The summation in equation (6) takes into account the assumption of rotational and equatorial symmetry of the debris population, which results in four possible particle directions with the same probability.

The described theory has been coded in two versions:

- MASTER Standard application ([4],[7]),
- DIADEM (Divine-based analytical debris environment model) for high altitudes ([6]).

The software has been extensively tested, and the results have been validated against those of the MASTER '99 Analyst application.

3.2 Model Capabilities

As already mentioned, two different sets of requirements were defined for the debris model to be developed. Some requirements are valid for both, while others are specific, as for example the requirement to implement a cloud model in the high altitude model DIADEM. The user requirements defined for the new analytical debris model are reflected by the model capabilities.

A list of the capabilities of the two versions of the model is given in Table 1.

Table 1: Capabilities of the Divine-based debris model

	MASTER Standard Application	DIADEM
General	Flux and spatial density calculation for all target orbits and altitudes from LEO to GEO	Flux and spatial density calculation for high altitudes (target perigee radius > 35000km)
Population	MASTER reference population (the population sources and the particle size range may be selected by the user)	
		additionally: any (user defined) population; any (user defined) particle cloud; function defined populations/clouds possible

	MASTER Standard Application	DIADEM
Output	Spatial density as a function of altitude and declination (2D, 3D) for fixed altitude and declination ranges	Spatial density as a function of altitude and declination (2D, 3D) for user defined altitude and declination ranges
	2D flux spectra as a function of ➢ impact velocity, azimuth angle, elevation angle ➢ impact location (target orbit true anomaly, altitude, right ascension, declination) ➢ particle orbital elements (semi-major axis, perigee radius, eccentricity, inclination) ➢ particle size (diameter, mass) and any 3D combination of these	

The remarkably low storage needs for the population data of about 750 Kbytes, and the fast computation make the new model a highly efficient tool for debris flux/spatial density analysis.

4 BACKGROUND POPULATION

4.1 Implementation

In principle, any existing or newly created population may serve as input for the new debris model, if it is possible to describe the population by means of the diameter or mass distribution, and the orbital element distributions D_I, D_e, D_i.

For the applications described in this paper, the orbital element distributions D_I, D_e, D_i are derived from the MASTER '99 reference population, which consists of seven source-based sub-populations: catalogued objects, fragments, NaK droplets, SRM slag particles, SRM Al2O3 dust particles, paint flakes, and ejecta. The cumulative diameter distribution of the population is given in Fig. 1:

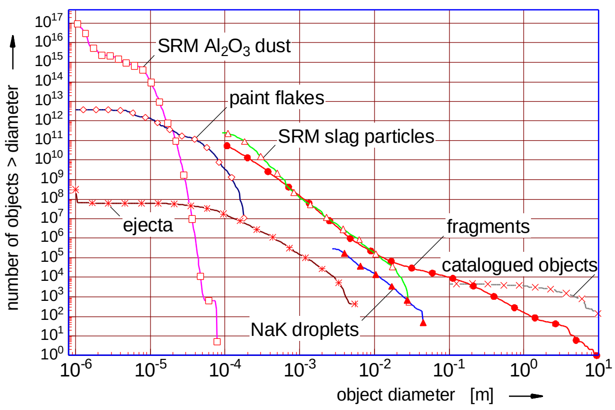

Fig. 1: Cumulative diameter distribution of the MASTER '99 reference population

In order to obtain a reproduction of the population as detailed as possible, high emphasis has to be put on an appropriate establishment of the orbital element distributions (so called probability density tables).

To consider the different orbital element distributions of particles of different size, the complete population ranging from 1micron to 100m in size is sub-divided into eight diameter (as well as mass) classes, each diameter class covering one decade. For each size class, the distributions of the orbital elements perigee radius, eccentricity, and inclination of each sub-population are derived as follows:

Extraction of the perigee radius distributions. Fig. 2 gives the perigee radius distributions for different size classes of the fragment population.

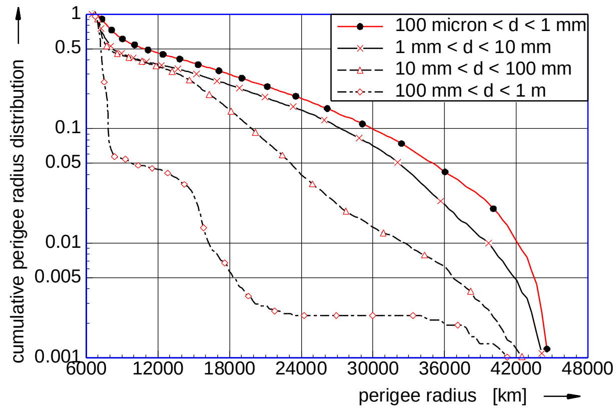

Fig. 2: Perigee radius distributions of the fragment population, four diameter classes

The different perigee radius distribution characteristics show that a consideration of different orbital parameter distributions is required.

Sub-division of the perigee radius range to up to five sub-ranges (e.g. low, medium, high perigee). This additional sub-division considers the different eccentricity and inclination distributions for particles with for example low, medium, and high perigees.
Extraction of eccentricity and inclination distributions for each selected perigee radius range. Fig. 3 gives an example of the cumulative eccentricity distributions related to four different perigee radius ranges, while Fig. 4 shows the corresponding cumulative inclination distributions.

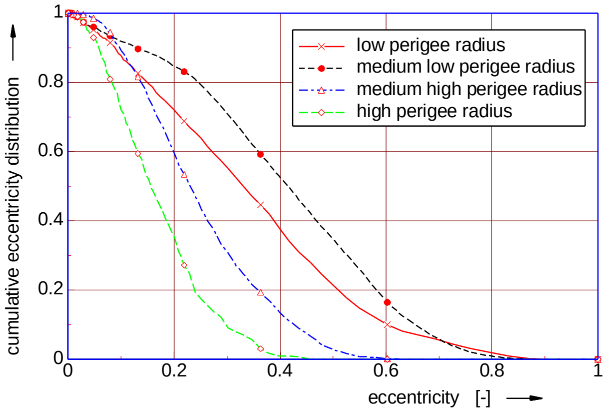

Fig. 3: Eccentricity distributions of the fragment population, diameter class: 10^{-4}m < d < 10^{-3}m

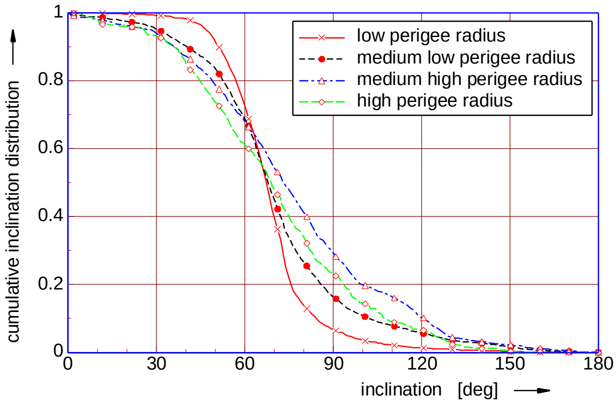

Fig. 4: Inclination distributions of the fragment population, diameter class: 10^{-4}m < d < 10^{-3}m

The perigee radius ranges used to derive the distributions given in Fig. 3 and Fig. 4 were

low perigee:	6500 km < r_p < 8000 km
medium low perigee:	8000 km < r_p < 25000 km
medium high perigee:	25000 km < r_p < 38000 km
high perigee:	38000 km < r_p < 45000 km

The described population definition strategy takes into account cross-coupling effects between the orbital elements of the particles in the specified size class. In the current implementation of the model 151 probability tables have been extracted to render the MASTER ’99 reference population.

4.2 Results

Fig. 5 shows the spatial density as a function of altitude for all sub-populations of the MASTER '99 reference population. The solid lines indicate the spatial density results of the new model, while the dotted lines represent the results obtained on the basis of a deterministic computation of the residential probability of all representative objects in each altitude band.

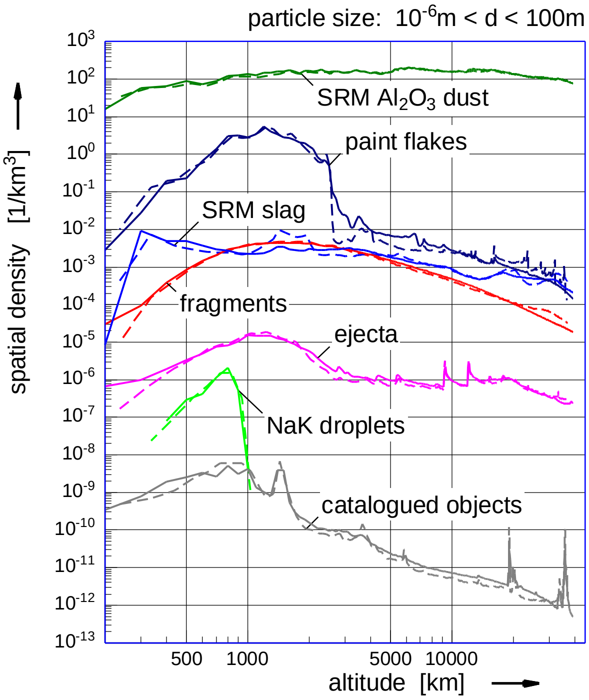

Fig. 5: Spatial density vs. altitude

Generally, it can be stated that the correspondence of the spatial density distributions derived with completely different approaches is excellent. The remaining deviations are a consequence of cross-couplings between the orbital elements which are not considered (e.g. dependency of the perigee radius and inclination distribution on the inclination range as observed in case of the SRM slag particles population). These deficiency will be resolved in the next version of the model (see section 6.1). The advanced population derivation process by means of an automated statistical analysis of the population characteristics will also decrease the remaining deviations in the flux analysis results to less than 25% compared to the results of the deterministic MASTER Analyst application.

5 CLOUD MODEL

As mentioned in Table 1, DIADEM offers the ability to evaluate the flux contribution from particle clouds. Particle clouds are defined as groups of orbiting objects with the same origin. This leads to well defined distributions of their orbital parameters. Especially the inclinations of the cloud particles should have a narrow distribution, and it may be assumed, that the initial nodal lines of all cloud object orbits cover a certain limited range.

5.1 Implementation

Fig. 6 gives a simplified illustration of the cloud model.

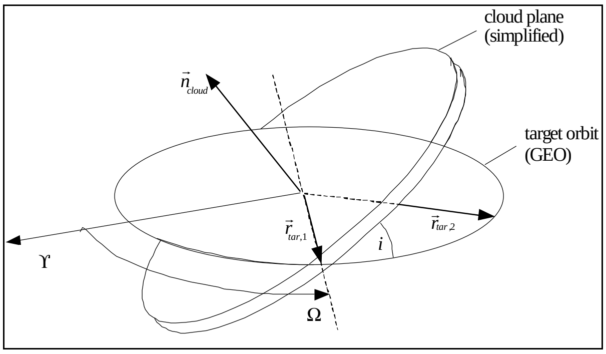

Fig. 6: Geometry of flux contribution from a cloud

The implementation of the cloud model takes place within the evaluation of the inclination integral (equation (3)). For each inclination step (inclination i) the dot product between the current target position vector and the cloud normal vector is calculated in order to establish the cloud flux contribution condition:

$flux \neq 0$, if

$$\vec{r}_{tar} \circ \vec{n}_{cloud,1} \leq 0 \quad \text{and} \quad \vec{r}_{tar} \circ \vec{n}_{cloud,2} \geq 0 \tag{8}$$

where the cloud normal vector is given by

$$\vec{n}_{cloud} = \begin{pmatrix} \sin i \cdot \sin \Omega \\ \sin i \cdot \cos \Omega \\ \cos i \end{pmatrix} \tag{9}$$

The indexes 1 and 2 indicate the maximum and minimum values of the (current) right ascension of the ascending node Ω. The nodal line position is given in the cloud population input file by means of the initial maximum and minimum Ω values, which are propagated forward in time from the cloud creation epoch to the flux analysis epoch using an analytical approach which accounts for the J_2-term of the Earth's gravitational potential:

$$\Delta\Omega = -9.96° \frac{(R_e/a)^{\frac{7}{2}}}{(1-e^2)^2} \cdot \cos i \tag{10}$$

where R_e is the Earth's radius, a the particle orbit semi-major axis, and e the particle orbit eccentricity. The particle orbital elements are taken from the distributions D_1, D_e, D_i of the cloud.

Since the evolution, and finally the disappearance of a cloud in the background population is a highly dynamic process, a second time-dependency was introduced in the model. In order to consider the decay of a large number of the smaller cloud particles due to radiation pressure a so called 'cloud decrease rate' r_{cloud} has been implemented. The decrease factor to be applied to the cloud particle number is determined for each particle size class (s. section 4.1) under the assumption of a simple exponential law:

$$f_{cloud} = (1 - r_{cloud})^{t - t_{cce}} \tag{11}$$

where t is the analysis epoch and t_{cce} is the cloud creation epoch as specified in the cloud population input file.

5.2 Results

The evaluation of the flux contribution resulting from particle clouds has been applied to the SRM firing of the object with the USSPACECOM satellite number 25358 (Thor III). The results were correlated with data measured by the GORID detector, which is operating on-board a Russian geostationary EXPRESS spacecraft since 1997 (refer to [8]).

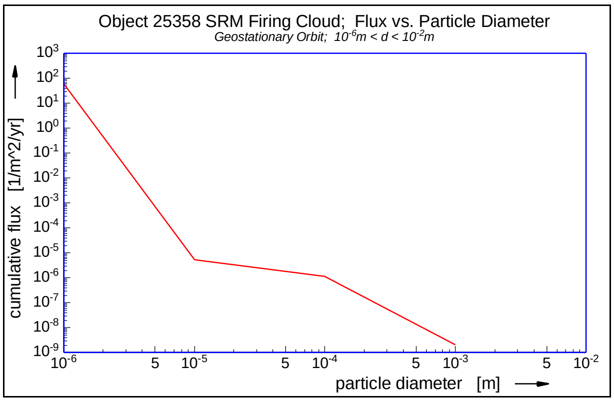

Fig. 7: Cloud flux with respect to particle diameter

Fig. 7 indicates that the cloud flux is dominated by the small particles. The contribution of particles with d > 1micron is larger by about 7 orders of magnitude.

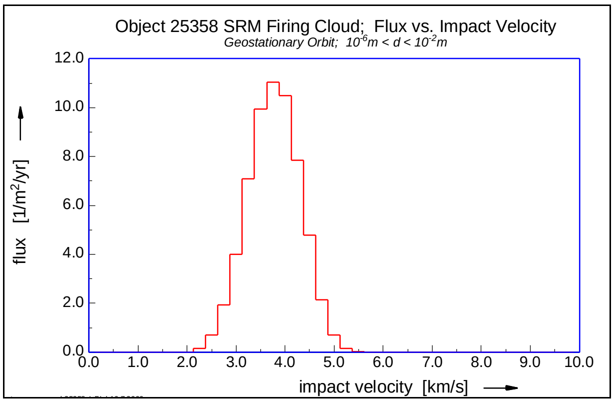

Fig. 8: Cloud flux with respect to impact velocity

The cloud particle impact velocities range from 2.1km/s to 5.4km/s with an average of 3.8km/s. This, and the impact azimuth angle spectrum (Fig. 9) is a consequence of the cloud particle inclination distribution which ranges from 60° to 120° with the maximum at about 90° [8].

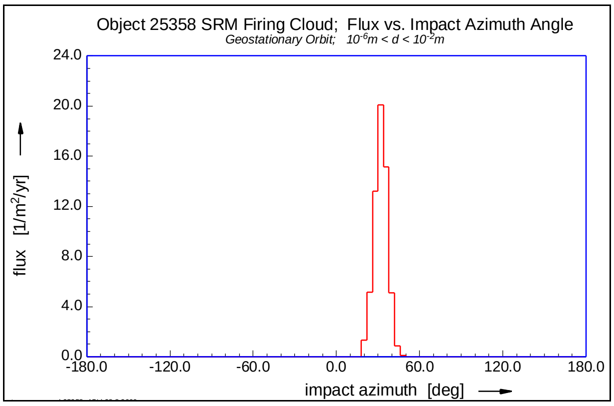

Fig. 9: Cloud flux with respect to impact azimuth

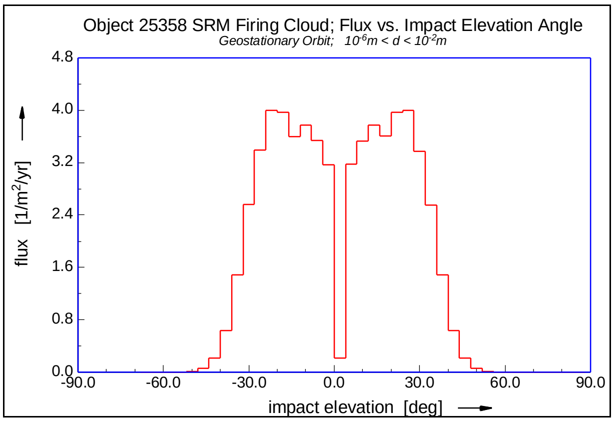

Fig. 10: Cloud flux with respect to impact elevation

Both, the impact azimuth and elevation distributions allow the visibility of the examined cloud for the GORID detector, since the azimuth of its normal vector is 65°, the elevation is –5°, and the viewing cone is 140° [8].
It should be noted that the flux values given in Fig. 7 to Fig. 10 have to be multiplied by a factor of approximately 100, taking into account the reduced reference volume of the particle cloud. After application of the given factor the flux values are well above those of the background population (i.e. the SRM dust population in the investigated size range). Consequently, the perceptibility of the cloud by GORID is given.

6 FURTHER DEVELOPMENT AND FUTURE APPLICATIONS

6.1 Further Development

Several model extensions are intended to be implemented within the ongoing ESA/ESOC contract "Upgrade f the MATER Space Debris and Meteoroid Environment Model":

- establishment of a tool for the automated and optimised establishment of the probability density distribution which describe the population,
- consideration of asymmetric population characteristics by means of the distributions of the nodal line position and the perigee position,
- consideration of meteoroids (background & streams),
- calculation of flux on oriented surfaces.

The main extension of the model will be its ability to provide a "quick-look flux and spatial density analysis" for historic, present and future debris environments. For this purpose the above mentioned development of a "population pre-processor" is essential, since a large number of populations has to be provided for historic and future epochs. Different future scenarios cause a further increase of the number of populations to be provided.

The MASTER Standard application will be able to generate time dependent spatial density and flux distributions.

6.2 Applications

Apart from the extensions of the model within the MASTER upgrade, its incorporation to the ESABASE/Debris software is intended.

In order to perform parametric investigations using the cloud model feature of DIADEM (e.g. in the context of an application to various 'real' SRM firing clouds), the further improvement of the cloud modelling capabilities would be highly appreciable.

A recommendation for another future applications of the model is its inclusion into a 'closed loop tool' allowing the derivation of population characteristics from measurement data (either data derived from impact damage or data derived from detectors such as GORID or the DEBIE detectors). The expected large amount of data resulting from an increased number of in-situ measurements requires an automated evaluation and "translation" of the data to appropriate debris populations. Due to its fast computation the new model would restrict the required iterative population definition process to reasonable computation times.

7 SUMMARY AND CONCLUSION

The Divine approach, which originally has been developed for meteoroid flux calculation was successfully adapted to the needs of a debris environment model. Two versions of the new analytical flux/spatial density browser were established:

1. The model was implemented within the MASTER '99 model as so called 'Standard' application. It makes use of the MASTER '99 reference population, which was implemented by means of distributions of the particle diameter and the particle orbital elements perigee radius, eccentricity and inclination.
2. The same kernel was used in DIADEM, an analytical GEO debris model. DIADEM uses the same implementation of the MASTER '99 reference population, and thus produces consistent results. This application was extended with respect to the processing of particle cloud populations (e.g. SRM slag and dust clouds).

Both, the flux and spatial density calculation, and the population implementation were extensively tested and validated. Since the model is generally able to process any population defined by the a.m. distributions, it may serve as an universal flux and spatial density browser.

Additional major advantages of the model are the low storage needs (less than 1Mbyte including the executable, the input files, and the population data), and the fast computation (less than 8 CPU-seconds on a Pentium II PC for a LEO target considering all population sources, the complete size range, and output of twelve 2D flux spectra).

The Divine-based debris model offers a high potential for further applications and extensions.

8 ACKNOWLEDGEMENTS

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