MODELLING OF THE SPACE DEBRIS EVOLUTION BASED ON CONTINUA MECHANICS

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ABSTRACT

The paper is aimed at mathematical modelling of long term orbital debris evolution taking into account mutual collisions of space debris particles of different sizes. The present model is based on the continua mechanics approach [1]. The space debris environment containing fragments differing greatly in mass, velocity and orbital parameters the multiphase continua approach was introduced distinguishing classes of fragments possessing similar properties. Under this approach the evolution equations contain a number of source terms responsible for the variations of different fractions of orbital debris population due to fragmentations and collisions. Those source terms were developed based on the solution of a high velocity collision and breakup problem. The Russian Space Debris Prediction and Analysis (SDPA) model [2, 3, 4] developed using the continua approach served the basis for the present study. The model used the averaged description for the sources of space debris production and took into account collisions of debris fragments of different sizes (including non-catalogued ones) that could lead not only to debris self-production but also to a self-cleaning of the Low Earth Orbits.

1. MATHEMATICAL MODEL

Essential differences in debris particles sizes and thus collision consequences brings to a necessity of introducing a number of "phases" or "mutually penetrating continua" into the model, each phase being characterized by its own density of distribution. The particles could be assembled into groups ("phases") due to the following attributes: their characteristic size d_j; the perigee altitude of the orbit $h_{\rho j}$, eccentricity e_j; inclination of the orbit i_j; ballistic coefficient. The number density of particles of the j-th phase per volume unit ρ_j evolution can be determined by the following equation [1]:

$$\frac{\partial \rho_j}{\partial t} + div\ \rho_j \vec{v}_j = \sum_{k=1}^{N} \psi_{jk} + n_{j\ op} + n_{j\ ex} - \nu_j\ , \quad (1)$$

where ψ_{jk} is the number of particles transferred from the k-th to the j-th phase per time unit due to fragmentation in collisions; $n_{j\ op}, n_{j\ ex}, \nu_j$ - the rates of particles number growth and/or decrease due to external sources, $\vec{v}_j$ is the local velocity of the j-th phase.

Averaging the equation (3.22) in longitude and latitude gives the following form of the model equation:

$$\frac{\partial N_j}{\partial t} = -W_j \frac{\partial N_j}{\partial r} - N_j \frac{\partial W_j}{\partial r} + \dot{N}_j\ , \quad (2)$$

where $N_j(t,r)$ is the number of debris particle of the j-th phase per altitude spherical layer of the thickness Δh; $W_j(t,r) = \left(dr/dt \right)_j$ - radial velocity of particles (sedimentation velocity); $\dot{N}_j$ - the rate of variation of particles number per altitude layer due to external sources.

$$\dot{N}_j(t,r) = \int_r^{r+\Delta h} \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \left(\sum_{k=1}^{N} \psi_{jk}(t,r,\Omega,\theta) + n_{j\ op}(t,r,\Omega,\theta) + \right.$$
$$\left. + n_{j\ ex}(t,r,\Omega,\theta) - \nu_j(t,r,\Omega,\theta) \right) \cdot r^2 \cos\theta\ d\Omega d\theta dr$$

The sedimentation velocity can be determined by formula [1]:

$$\dot{r} = W_j = -\frac{3}{8} \frac{c_f^j \rho_a(r,t)}{\rho^0{}_j d_j} \sqrt{\gamma M r}\ , \quad (3)$$

where γ is the gravity constant and M is the mass of the Earth, ρ_j^0 - actual density of material of a particle, the function $\rho_a(r,t)$ could be obtained from one of the models of a standard atmosphere, or from its approximations:

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$$\rho_a(r,t) = \rho_a(r_0,t)\exp\left(-\int_{r_0}^{r}\frac{dr}{H(r,t)}\right), \qquad (4)$$

where $H(r, t)$ is the scale height of a uniform atmosphere at the altitude r; $\rho_a(r_0,t)$ is a known density at the altitude r_0. Substituting Eqs. 3 and 4 in the Eq. 2 one obtains

$$\frac{\partial N_j}{\partial t} = -W_j\frac{\partial N_j}{\partial r} + \frac{N_j W_j}{H}\left(1-\frac{H}{2r}\right) + \dot{N}_j. \qquad (5)$$

Introducing positive sedimentation velocity $V_j(t,r) = -W_j(t,r) > 0$ and assuming $\frac{H}{2r} << 1$ brings the Eq.5 to the form used in the SDPA model

$$\frac{\partial N_j(t,h)}{\partial t} = V_j\left(\frac{\partial N_j(t,h)}{\partial h} - \frac{N_j(t,h)}{H}\right) + \dot{N}_j, \qquad (6)$$

where h is the perigee altitude.

2. EVALUATION OF COLLISIONS PROBABILITY

The average number of collisions of spherical-shaped spacecraft (SC) with small-sized space debris particles is determined as follows [2-4]:

$$\frac{dN}{dt} = S\cdot\rho(t)\cdot\int_{A=0}^{2\pi} p(t,A)\cdot V_{rel}(t,A)\cdot dA = S\cdot\rho(t)\cdot\bar{V}_{rel}(t). \qquad (7)$$

Here $\boldsymbol{S}$ is the satellite cross-section, ρ *is* the spatial density of particles, *p(t,A)* is their azimuthal distribution at time moment $\boldsymbol{t}$ and V_{re} is the relative velocity of a particle with respect to the given satellite. The integral has a meaning of the mean relative velocity of a space object (SO) at this point.

The averaging of SO flux through a unit cross-section of a space vehicle is performed for one revolution (for the time interval equal to the SC period $\boldsymbol{T}$). This mean value is calculated by the formula

$$\bar{Q} = \frac{1}{T}\cdot\int_{t=0}^{T}\rho(t)\cdot\int_{A=0}^{2\pi}p(t,A)V_{rel}(t,A)\cdot dA\cdot dt. \qquad (8)$$

The study and estimation of probabilities of mutual collisions of objects belonging to different groups - large-size (catalogued), medium-size (from 1 up to 20 cm) and small-size (for example, from 0.1 up to 1 cm) etc. - is of considerable interest. We shall assume that the space debris can have various sizes including those, which can not be neglected. A possible size of particles will be characterized by the distribution density *p(d)* of their mean diameter *d*. Taking account of the distribution density *p(d),* we can introduce into equation (7) modifications, which take into consideration the variability of particles' sizes. It is convenient to express the spatial density of particles, sizing larger than the arbitrary quantity *d*, as a product of some dimensionless factor *k(d)* by the spatial density of particles sizing larger than some specified value d_0:

$$\rho(d,t) = k(d)\cdot\rho(d_0,t) \qquad (9)$$

Here coefficient *k(d)* is supposed to be independent of time. We designate the derivative of coefficient *k(d)* as $f(d) = dk(d)/dd$. Then the averaged number of collisions of a SC, having size *D*, with particles whose size lies in the range of (d_1, d_2), can be expressed as

$$N(D,d_1,d_2) = F_d\cdot\bar{Q}(d_0,t)\cdot(t-t_0), \qquad (10)$$

where $F_d = \left[-\frac{\pi}{4}\int_{d_1}^{d_2}(D+d)^2\cdot f(d)\cdot dd\right]$.

Having estimated the value $N(D,d_1,d_2)$ - the averaged number of collisions of a single spacecraft of diameter *D* with particles sizing in the range of (d_1,d_2) one could develop the average number of collisions of a group of objects, having size in the range of (D_1,D_2) and situated in some altitude region *(h,h+Δh),* with all SOs having size in the range of (d_1,d_2) (this estimate is designated as $N(h,h+\Delta h)_{Dd}$ below). It is necessary to sum up the estimates $N(D,d_1,d_2)$ for all SOs of the given size lying in the given altitude range. As a result, we obtain the following estimate:

$$N(h,h+\Delta h)_{Dd} = F_{Dd}\cdot n(h,h+\Delta h)_{cat}\cdot\bar{Q}(d_0,h,t_0)\cdot(t-t_0). \qquad (11)$$

where F_{Dd} is calculated by the formula

$$F_{Dd} = \left[\frac{\pi}{4}\int_{D_1}^{D_2}\int_{d_1}^{d_2}(x+y)^2\cdot dk(x)\cdot dk(y)\right]/2. \qquad (12)$$

This value has a meaning of the mean cross-sectional area of collisions of objects sizing in the range (D_1,D_2) with particles from the range of (d_1,d_2), where $n(h,h+\Delta h)_{cat}$ is the number of the catalogued objects within the altitude range $(h,h+\Delta h)$.

Evaluating components of F_{Dd} matrix for SOs of different sizes drives to the following conclusion. The number of collisions of small-sized particles (sizing smaller than 1 cm) between each other, as well as with larger objects, is much higher, than the number of mutual collisions of catalogued objects (sizing larger than 10 - 20 cm). This result testifies the necessity of taking into account mutual collisions of space debris of different sizes.

3. FRAGMENTATION MODEL FOR OLLISIONS OF SPACE DEBRIS PARTICLES

The developed model allows to evaluate collision probabilities, relative velocities, masses and sizes of colliding objects within all the altitude ranges. The results of collisions could be evaluated using the fragmentation model [5, 6]. The basic relationships for this model modified to meet the requirements of LEO debris self-production modeling are presented briefly below.

The high-velocity collision of particles of mass M_1 and M_2 is considered, the velocities of particles being equal V at the time of collision. The angle between velocity vectors is equal to 2β (in the inertial space). We use the designations: $M=M_1+M_2$, $k_1=M_1/M$, $k_2=M_2/M$. Then the mean velocity of fragments after collision will be:

$$V_M = V \cdot \sqrt{1 - 4 \cdot k_1 \cdot k_2 \cdot \sin^2\beta} \ . \qquad (13)$$

The amount of energy generated in collisions is characterized by the density of internal energy ***u***, which is uniformly distributed within particles. This specific internal energy can be determined as follows:

$$u = U/M = \frac{1}{2} \cdot k_1 \cdot k_2 \cdot (2 \cdot V \cdot \sin\beta)^2 = \frac{1}{2} \cdot k_1 \cdot k_2 \cdot (V_{rel})^2 \qquad (14)$$

In deriving this formula we used the assumption, that after collision all fragments (of total mass M) are moving with the same velocity (13). Indeed, with a great difference in size (and mass) between a target and a particle ($M_1 << M_2$) the energy of collision is absorbed only by a small fraction of target's mass (by individual components of the SC structure in our case). In this case in applying formula (14) only some portion of masses of colliding bodies should be used as a reference mass, rather than their total mass, i. e. $M_2 = \tilde{A} \cdot M_1$, as in paper [7].

The density of internal energy is supposed to be sub-divided into elastic (e) and inelastic (dissipation) (÷) components, i.e. $u = e_\alpha + \chi_\alpha$ (α=1,2). The entropic criterion of limiting specific dissipation is used here as a macrodestruction criterion, i. e. $\chi \le \chi^*$, where χ^* is the limiting specific dissipation, which depends on particle's material and is assumed to be known. It is also assumed, that a part of elastic energy, accumulated in particles after collision, is spent for destruction of particles. Then the energy spent for destruction can be calculated by formulas $e_\alpha^f = k \cdot e_\alpha$, where k is some factor, which is considered to be known. If the internal energy of a particle is found to be $u < \chi_\alpha$, then the destruction of particle α does not occur.

For description of fragments distribution in mass, the modification of Weibull distribution is used:

$$N(<m) = N_0 \cdot \left[1 - exp\left(-\left(\frac{m - m_{min}}{m_*}\right)^{Ë}\right)\right], \qquad (15)$$

$$m_{min} \le m \le m_{max}.$$

Here Λ is the parameter, whose value depends on particle's material. It characterizes the degree of "compactness" of destruction fragments' distribution in masses. For the discrete spectrum of particles' masses $m_1^á, m_2^á, ..m_{K_á}^á$ the number of fragments of ensemble $m_j^á$ is

$$N_j^\alpha = N_0^\alpha \cdot \left(b_j^\alpha - b_{j+1}^á\right),$$

$$b_j^\alpha = exp\left(-\left(\frac{\sqrt{m_{j-1}^\alpha \cdot m_j^\alpha} - m_{min}^\alpha}{m_*^\alpha}\right)^{Ë_\alpha}\right). \qquad (16)$$

The system of K_α equations (16) is supplemented by the two following equations:

$$\sum_{j=1}^{K_\alpha} m_j^á \cdot N_j^\alpha = M_\alpha \ , \qquad (17)$$

$$\sum_{j=1}^{K_\alpha} \gamma_\alpha \cdot \frac{s_j^\alpha}{2} \cdot N_j^\alpha = M_\alpha \cdot e_\alpha^f \ , \qquad (18)$$

where γ_α is specific energy required for producing a destruction surface unit, $s_j^á$ is the area of arising destruction surface for fragment $m_j^á$. Equation (17) expresses the condition that the total mass of particle's fragments is equal to the initial mass of the particle, and equation (18) expresses the equality of elastic energy, accumulated in a particle, to the energy spent for producing a destruction surface.

The solution of the system of equations (16), (17) and (18) allows us to determine the number and masses of fragments. The known values (Eq. 13) of fragments velocity at a collision, and the velocity increments

$v_j^{\alpha} = \sqrt{2 \cdot (1-k) \cdot e_{\alpha}}$, acquired by particles after collision, and the relation between the area of particle's surface and its size: $d_j \approx \sqrt{s_j^{\alpha} / \pi}$ make it possible to determine:

- The number and mass of breakup fragments remained in orbit;
- The number and mass of breakup fragments sizing larger than 0.1 cm;
- The distribution of breakup fragments sizing larger than 0.1 cm over the perigee altitude.

As an example, we consider the results of modeling the collision of two balls: steel one of mass of 2g and aluminium one having mass of 20g. The altitude of a circular orbit of objects before collision was 950 km. The values of $sin\beta$, used in formulas (8) and (9), were taken to be equiprobable over the interval of 0 - 1.0. Table 1 below presents the data on the average number of objects of different sizes - both remained in orbit and deorbited. In total, 1 797 954 fragments of different sizes were formed. 943 717 (52 %) of them have continued orbital motion, the remaining 48 % have deorbited. It is seen, that the maximum of the size distribution lies in the range of 0.025--0.05 cm. Table 2 below presents also the distribution of a number of objects of different sizes over the perigee altitude.

4. ACCOUNTING FOR COLLISIONS IN SPACE DEBRIS ENVIRONMENT MODELING

Various space objects (SO) were considered, whose perigee altitudes did not exceed 2000 km. We choose the perigee altitude (h_p) from the vector of SO orbital elements. It is supposed that among all variable SO parameters only the perigee altitude essentially influences the evolution of the altitude distribution of SO number. The other orbital elements will be designated by E. We subdivide the whole set of objects with different elements E into some finite number of sub-sets (groups) with elements E_j, j=1,2,..., l_{max}.

The technique for semi-analytical solution of equations (6) was developed for the SDPA model.

The use of statistical distributions of SOs vs altitude, ballistic factors and velocity allows us to determine the averaged consequences of one collision of SOs of different sizes (36 versions) beyond the solution of a forecasting problem. Though the calculations of these consequences are rather time consuming, they are executed only once - at a preparatory step to the forecasting procedure. Further on, during the integration of equations (6), the matrix of probabilities of collision of SOs of different sizes is calculated at each time step. In this symmetrical matrix P_{Dd} (8 x 8) 36 values are meaningful. Their multiplication by a priori calculated characteristics of collision consequences allows us to determine the component of sum $\dot{N}_j$, which relates to collision consequences, as well as to some other sources.

The space debris environment was modeled on a preceding time interval: from the year 1960 to 2000. The forecasts were made with and without mutual collisions of SOs sizing larger than 0.1 cm. Besides, the version of "partial collisions" was considered, in which the collisions of all SOs except catalogued ones were taken into account. For these versions the data on a number of SOs of different sizes in 2000 are presented in the upper two lines of the Table 3. Naturally, in case the collisions were taken into account, the number of small-sized space debris particles happened to be greater than that obtained in forecasts disregarding collisions (the third line of Table 3). The considerable changes were observed only for particles sizing larger than 0.1-0.50 cm. Accounting for collisions the estimated number of particles was 18 - 22 % greater, than without collisions. In the case of "partial collisions" the estimates have had intermediate values. The data of Table 3 testify, that the consequences of collisions of SOs of different sizes on a preceding time interval resulted in 11-12% increase of the number of particles sizing 0.1 - 0.5 cm. The influence of this source on the population of large-sized space debris is insignificant. The estimation of the contribution into the level of contamination by particles sizing smaller than 0.1 cm requires additional analysis.

Figure 1 gives the comparison of altitude distributions of a number of SOs in the 100-km altitude layer in 2000, obtained using the model with and without collisions of SOs of different sizes, as well as in the intermediate case (without collisions of objects sizing larger than 20 cm). These data indicate that the maximum contribution of collision consequences is achieved in the altitude range of 800 – 1000 km with allowance for all mutual collisions and is equal now to 33 % of the total level of the altitude layer contamination with particles of the regarded size. The growth accounts 16% as compared to the intermediate case. This shows that the contribution of mutual collisions of catalogued objects is slightly greater, than the contribution of all other collisions under consideration. Nevertheless, the contribution of collisions of smaller SOs between each other and with large-sized objects is rather significant – it is equal to 14 %.

Analysis of the probabilities of collisions of fragments on the preceding time interval (Table 4) shows that the total (accumulated) expected number of collisions by nowadays is the greatest for the particles sizing 0.1 - 0.25 cm colliding with the catalogued Sos (5490). The accumulated number of collisions for the catalogued SOs between each other is relatively low – it is equal to 0.96.

The estimates of the total number of fragments generated in collisions for all possible collisions of SOs sizing larger than 0.1 cm by the year 2000 and sizing 0.1-0.25, 0.25-0.5, 0.5-1.0 and 1.0-2.5 cm (including deorbited objects) show that the total mass of fragments is equal to 432 kg. About a half of those fragments (43 % in mass) was deorbited at the time of collision. The other set of fragments (47 % in mass) relates to small-sized particles sizing smaller than 0.1 cm. And only a small part of mass (9 %) relates to particles sizing larger than 0.1 cm..

The total number of collision fragments sizing 0.1-0.5 cm makes 45-55 % in relation to the current number of fragments of this size estimated disregarding collisions. Thus, the contribution of collisions to a current population of particles sizing 0.1 - 0.5 cm, in our opinion, is essential.

CONCLUSIONS

The estimations of the mean contribution of SO collisions into catalogued SD environment are made. The objects sizing larger than 0.1 cm at altitudes up to 2000 km are considered. It is found that the maximal contribution of collisions is reached in the altitude range of 800 - 1000 km with account of all mutual collisions and is equal now to 33 % of the general contamination level of this high-altitude layer by particles sizing 0.25-0.5 sm.

Small sized particles generated in collisions are strongly influenced by the upper atmosphere thus contributing to the self-cleaning of LEO. Taking into account this effect the role of collisions in the cascade effect of Space Debris growth in LEO should be thoroughly reconsidered.

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Table 1

	Range of fragments' size(the lower boundary only, cm)								
Par-tic-	.0025	.005	.010	.025	.050	.100	0.25	0.50	1.0
les	Remained on orbit:								
ja=1	5	486	28401	43053	1256	121	12	1	0
ja=2	4427	39976	217271	547138	7387	3716	452	16	1
	Deorbited:								
ja=1	5	444	25884	38870	1069	96	9	1	0
ja=2	4033	36416	197674	495929	50402	3086	346	10	1

Table 2

Alti-tude, km	Range of fragments' size(the lower boundary only, cm)								
km	.0025	.005	.010	.025	.050	.100	0.25	0.5	1.0
450	36	328	2010	5036	652	60	10	1	0
550	36	328	2010	5036	652	60	10	1	0
650	36	328	2010	5036	652	60	10	1	0
750	36	328	2010	5036	652	60	10	1	0
850	36	328	2010	5036	652	60	10	1	0
950	4253	38825	235620	565013	55383	3539	414	14	1
Sum	4433	40465	245670	590193	58643	3839	464	19	1

Number of particles of different sizes in 2000 **Table 3**

Version	Size of particles, cm							
	0.1-0.25	0.25-0.5	0.5-1.0	1.0-2.5	2.5-5.0	5.0-10	10-20	>20
All collis.	77.7E+6	7.57E+6	1.58E+6	203000	81850	32500	16780	7699
Partial col.	66.2E+6	6.75E+6	1.56E+6	201000	81730	32480	16780	7700
No collis.	65.7E+6	6.21E+6	1.55E+6	200000	81710	32480	16780	7700

Matrix of accumulated probabilities (the average number) of collisions of SOs of different sizes **Table 4**

	jd=1	jd=2	jd=3	jd=4	jd=5	jd=6	jd=7	jd=8
jd=1	41.0	18.3	12.5	4.90	7.9	11.4	20.60	5490
jd=2		1.5	1.6	0.50	0.73	1.00	1.78	463
jd=3			0.32	0.16	0.20	0.25	0.42	105
jd=4				0.01	0.026	0.026	0.041	9.24
jd=5					0.009	0.015	0.019	3.58
jd=6						0.004	0.009	1.18
jd=7							0.003	0.52
jd=8								0.96

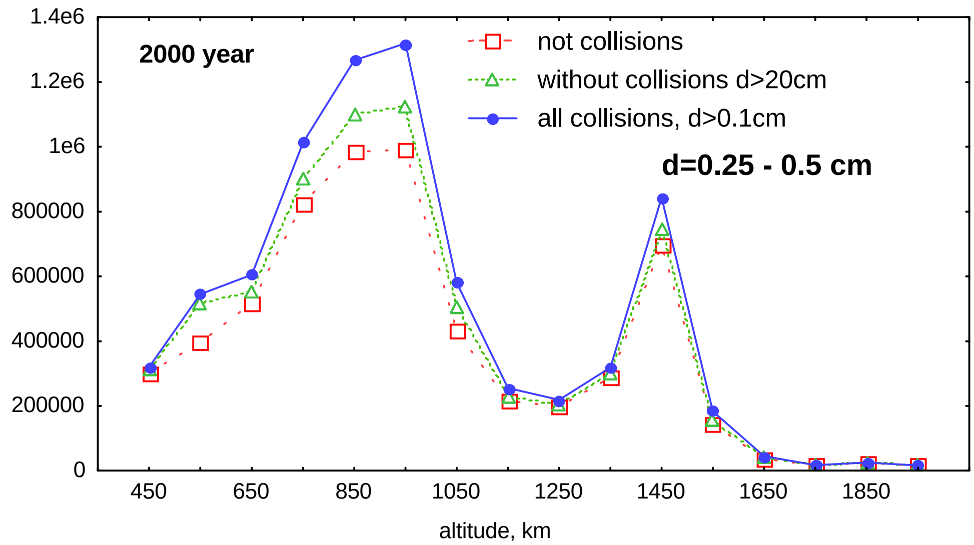

Figure 1. Comparison of the altitude distribution of SO number for different prediction strategies