THE ANALYTIC BASIS FOR DEBRIS AVOIDANCE OPERATIONS FOR THE INTERNATIONAL SPACE STATION

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ABSTRACT

A debris avoidance process based upon collision probability has been developed for the International Space Station (ISS) using covariance information supplied by the United States Space Command (USSPACECOM). Given the debris flux and the distribution of debris covariances for conjuncting debris, it is possible to estimate risk reduction, fractional residual risk and maneuver rate for the ISS as a function of chosen maneuver threshold collision probability.

1. BACKGROUND

Orbiting objects larger than about 10 cm in size are tracked both by radar and optically by a number of agencies, which maintain the orbital elements for these objects. The flux of tracked objects is small compared with the flux of objects large enough that space vehicle shielding is ineffective, but too small to be tracked. However the monetary value of the collision risk, about 10^{-3} annually, between tracked debris and the International Space Station is sufficiently large that the development and maintenance of debris avoidance process is more than economically justified. Further the flux of tracked debris is expected to increase significantly in coming years. With a debris avoidance system in place, debris avoidance is economically justified for other vehicles.

2. PROBLEM DEFINITION

The simplest debris avoidance method is that of an exclusion volume in which a maneuver is considered or performed if a conjunction is expected to occur within some predefined volume about a space vehicle. Such a procedure has been adopted by the space Shuttle Program. If a conjunction is predicted to occur within a 4x10x4 km box aligned along the Shuttle velocity vector, an avoidance maneuver is performed if it is allowable within all other Shuttle operational constraints. The box size was determined by performing Monte Carlo calculations which indicated that the collision probability for any object projected to pass outside the box was 10^{-5} or less [1]. However, based on the known tracked flux, this procedure would result in from 10 to 15 maneuvers per year for the International Space Station (ISS), too many for a micro gravity without knowledge of the uncertainty distribution of the tracked conjuncting objects.

In this work, we show the consequences of a debris avoidance process based on collision probability. This process minimizes the number of maneuvers and allows assessment of risk reduction, but requires a state vector for both ISS and the conjuncting object and an accurate estimate of the state vector uncertainties of the two objects.

3. PROCESS

USSPACECOM maintains Cartesian state vectors for all objects with perigees below 600 km, on a special Astrodynamics Workstation (ASW). These objects are tracked with a slightly elevated level of tasking. At the same time the sensors are routinely calibrated using satellite laser ranging data.

If a conjunction is predicted within a 40x80x80 km pizza box about ISS, conjunctor tasking is increased and the object is watched. The Johnson Space Center (JSC) flight controllers are notified 72 hours prior to a conjunction if the object is predicted to come within a 4x50x50 km box. If the collision probability is greater than 10^{-4} and the conjunctin geometry has been stable for three of the last four state vector updates, an avoidance maneuver is performed if it is possible.

4. COLLISION PROBABILITY

For conjunctions with non-zero relative velocity [2], the seemingly difficult task of calculating a collision probability reduces, in general, to a simple two-dimensional problem.

At some time t near conjunction, let the vector separation between the objects and the relative velocity be $\vec{r}_0$ and $\vec{v}_r$. The vector separation as a function of time is given by $\vec{r}_r = \vec{r}_0 + \vec{v}_r t$. Time of conjunction is determined by use of

$$\frac{d}{dt} \vec{r}_r \cdot \vec{r}_r = 0 = +2 \vec{r}_r \cdot \vec{v}_r + 2 \vec{v}_r \cdot \vec{v}_r t. \quad (1)$$

Proceedings of the 3rd European Conference on Space Debris, ESOC, Darmstadt, Germany, 19 - 21 March 2001
(ESA SP-473, August 2001)

The time to conjunction is given by

$$t = -\frac{\vec{r}_r \cdot \vec{v}_r}{v_r^2} \; . \qquad (2)$$

When $t = 0$, $\vec{r}_0$ is perpendicular to $\vec{v}_r$, if $\vec{v}_r$ or $\vec{r}_r$ are not zero. Thus the conjunction will take place in a plane perpendicular to the relative velocity vector, called the collision plane. The three dimensional position covariances of the conjuncting objects can be rotated into a common coordinate frame and added. If the contributions of the velocity uncertainties over the time of the conjunction do not contribute materially to the combined position uncertainty, the position of the conjunction on the collision plane will not change over the time of the conjunction. The problem is reduced to two dimensions with the effective covariance being the projection of the three-dimensional combined covariance on to the collision plane. The spatial density of the uncertainty in the debris position relative to the space vehicle is

$$f(\bar{r}) = \frac{1}{2\pi\sqrt{|C|}} e^{-\frac{1}{2}(\bar{r}-\bar{r}_d)^T C^{-1} (\bar{r}-\bar{r}_d)} \; . \qquad (3)$$

The ISS is sufficiently large compared to C that it is necessary to integrate over its area and the collision probability becomes

$$P_C = \frac{1}{2\pi\sqrt{|C|}} \int_{\otimes} e^{-\frac{1}{2}(\bar{r}-\bar{r}_d)^T C^{-1} (\bar{r}-\bar{r}_d)} d\bar{r} \; . \qquad (4)$$

Here $\vec{r}_d$ is the debris position in the collision plane and $\vec{r}$ is another point in the collision plane within the ISS area. Currently, a 60-meter radius circle is used to approximates the plane projection of the ISS on to the collision plane.

5. ORBIT DETERMINATION PROCESS

USSPACECOM uses a batch weighted least squares method for orbit determination solutions used in debris avoidance. A nominal 36-hour fit interval is used for low to moderate drag objects, with a shorter fit interval for high drag objects when possible. No process noise is used over the orbit determination fit interval. The covariance is an a priori covariance based on observation geometries. Sensor noise and bias values for individual sensors are based upon sensor calibration using satellite laser ranging. The propagated covariance is in agreement with observation statistics for objects experiencing low atmospheric drag, but the covariances of higher drag objects are too small due to poor atmospheric drag modeling. A noise term (consider parameter), 12% of the energy dissipation rate due to atmospheric drag, is included over the epoch to conjunction propagation interval.

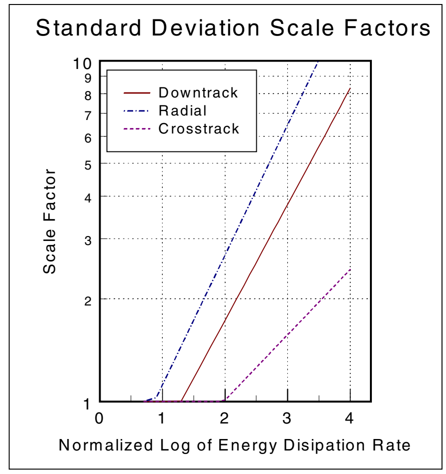

Fig. 1. Standard Deviation Scale Factors

For use in debris avoidance, the covariances are scaled, using scale factors developed from observation statistics, which depend only on the atmospheric drag energy dissipation rate (EDR). Fig. 1 shows the scale factors for the radial, down track and cross track standard deviations. The covariance would be effectively scaled by the square of these values as

$$C_{scaled} = \begin{bmatrix} f_u & 0 & 0 \\ 0 & f_v & 0 \\ 0 & 0 & f_w \end{bmatrix} C \begin{bmatrix} f_u & 0 & 0 \\ 0 & f_v & 0 \\ 0 & 0 & f_w \end{bmatrix} . \qquad (5)$$

The scale factors were determined at hourly intervals out to 72 hours and found to be nearly independent of propagation time. This indicates that the remaining problem is poor modeling over the orbit determination fit interval and that the 12% figure was a good global value, at least for the time period of the data.. In the debris avoidance software the down track component is used in lieu of the other two components, producing conservatively large radial and cross track variances. Thus the covariance matrix undergoes a simple scalar multiplication by the square of the down track scale factor.

6. COVARIANCE DISTRIBUTION

The average covariances of 63 objects over 105 days, for 8 and 24-hour propagation, were determined from observation statistics. The standard deviation data was then fit to a function of the form

$$\sigma = \rho \, [B_1 \cdot TPD^{-0.5} + B_2 \cdot TPD^{-2.5} + B_3 \cdot EDR] \qquad (6)$$

Here $\rho(EDR)$ is the standard deviation scale factor, while *TPD* is tracks per day. The radial, down track and cross track values of the B coefficients and of ρ are different. The coefficients B_1, B_2, B_3 are determined by the least squares process:

$$\vec{x}\vec{B} = \bar{y} \qquad \vec{x}^T\vec{x}\vec{B} = \vec{x}^T\bar{y} \qquad B = (\vec{x}^T\vec{x})^{-1}\vec{x}^T\bar{y} \quad (7)$$

where

$$\vec{x} = \rho[TPD^{-0.5}, TPD^{-2.5}, EDR] \qquad (8)$$

and

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \qquad (9)$$

are a matrix and a vector, while $\bar{y}$ is the vector or position uncertainties of the individual objects. The radial, down track, and cross track data were fit separately and the coefficients B_1, B_2, B_3, were determined for each. Figs. 2 and 3 show the normalized fits for the low earth orbit (LEO) radial and down track uncertainties.

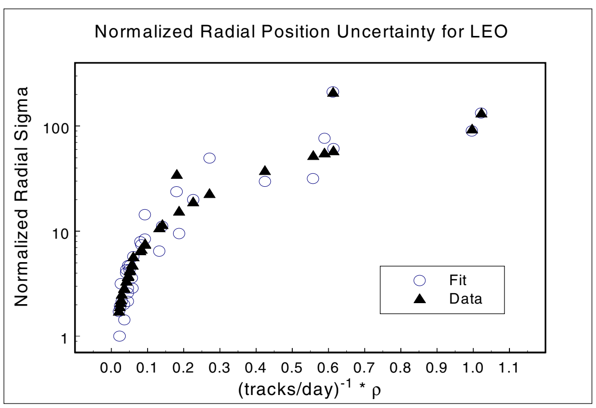

Fig. 2. LEO Radial Position Uncertainty

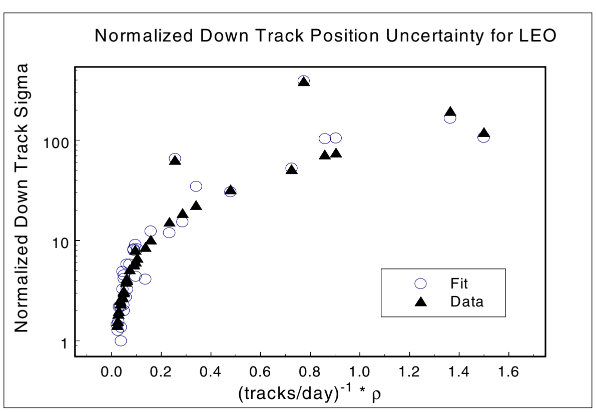

Fig. 3. LEO Down Track Position Uncertainty

Figs. 4 and 5 show the normalized fits for the Deep Space object radial and down track un-certainties The uncertainties are about a factor of ten larger than those for LEO, and the data is far more dispersed about the fits.

All of these figures are for an 8-hour state vector propagation for orbit determination to time of conjunction. A restricted amount of data for a 24-hour state vector propagation allows an estimate of the 24-hour uncertainties.

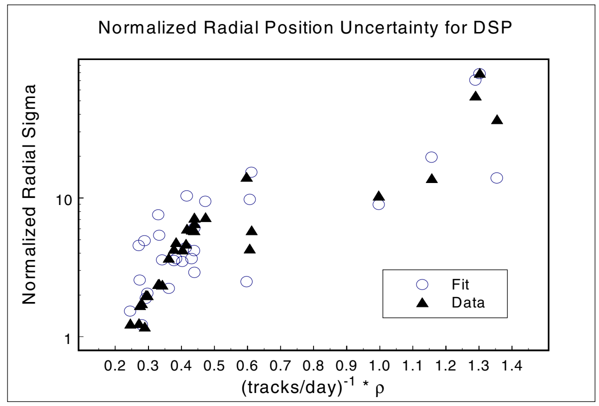

Fig. 4. Deep Space Radial Position Uncertainty

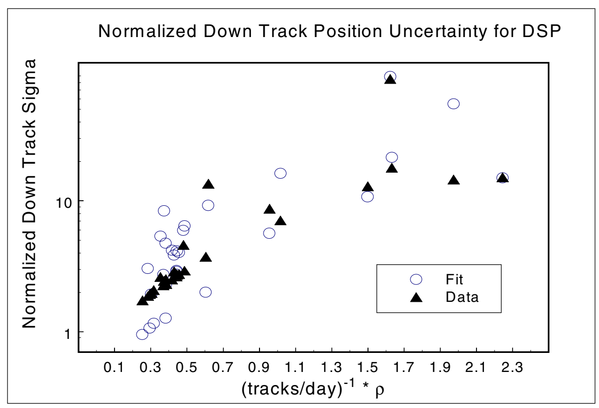

Fig. 5. Deep Space Down Track Position Uncertainty

7. KESSLER DEBRIS FLUX MODEL

The Kessler Debris Flux Model [3]. Was used to calculate a flux for every object crossing the ISS altitude band. This model averages argument of perigee and right ascension of ascending node, over extended time.

7.1 Spatial densities

For orbiting object j, with orbital inclination, perigee q_j and apogee q_j', the spatial density at altitude R and latitude β is

$$S=\left[\frac{1}{4\pi^2 a_j R\sqrt{(R-q_j)(q_j'-R)}}\right]\left[\frac{2}{\pi\sqrt{\sin^2 i_j-\sin^2\beta}}\right] \quad (10)$$

For orbiting object 0 in a circular orbit (e.g. ISS) with orbital inclination i_0, perigee q_0 and apogee q_0' the spatial density over an altitude interval ΔR about altitude R and at latitude β is given by

$$S=\left[\frac{1}{4\pi\, a_0^2\,\Delta R}\right]\left[\frac{2}{\pi\sqrt{\sin^2 i_0-\sin^2\beta}}\right] \quad (11)$$

The average flux over altitude range ΔR from object j, seen by object 0 over a prolonged period of time can be written as

$$F=\int_\beta S_0\, S_j\, V_{rel}\, d\beta\ . \quad (12)$$

7.2 Directionality of flux

The relative velocity between object j, and object 0 is

$$\vec{V}_{rel}=\vec{V}_j-\vec{V}_0\,. \quad (13)$$

Taking the square

$$V_{rel}^2=\vec{V}_j\cdot\vec{V}_0=V_j^2+V_0^2-2V_jV_0\cos\phi_\beta\,, \quad (14)$$

and the angle between the velocity vectors is

$$\phi_\beta=\cos^{-1}\left[\frac{V_j^2+V_0^2-V_{rel}^2}{2V_0\,V_{rel}}\right]. \quad (15)$$

If one of the objects is in a circular orbit, there are only 2 equally probable values of V_{rel}, V_{rel+} and V_{rel-} , for a given value of β . For each object which can conjunct with ISS we take a weighted average over β for $\bar{\phi}_+$ and $\bar{\phi}_-$. Then for every object crossing the ISS altitude band we have an average flux, two average incident directions from each side of the velocity vector, and the covariance C.

8. MANEUVER RATE AND RESIDUAL RISK

We choose a collision probability Pc = Pm for which, if the collision probability is exceed, the vehicle will always maneuver but otherwise will not. Then, given a position error covariance, an average flux and directionality for each object, it is straightforward to calculate the anticipated maneuver rate, the anticipated risk reduction and the anticipated residual risk for a given choice of collision probability maneuver threshold, Pm.

Fig. 6 shows the, approximately elliptical, contours of debris position for constant collision probability about ISS from a given direction. The ISS is represented as a 60-meter radius sphere at the center of the figure.

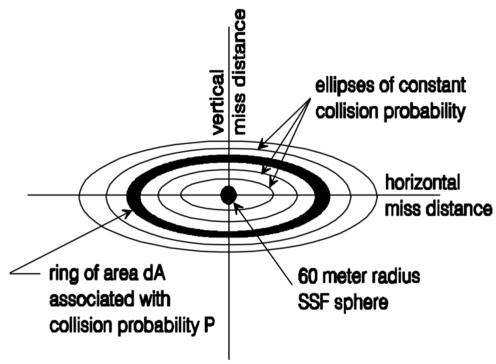

Fig. 6. Contours of constant collision probability about ISS.

Clearly the total annual collision probability for a space vehicle is the sum of the fluxes of the individual conjuncting objects, multiplied by the area of the space vehicle. The total annual collision probability must also be the integral out to infinity of the collision probability per unit area associated with constant collision probability contours. The annual Collision Probability, P_A, for ISS is

$$P_A = \sum_j F_j\, A_\otimes = \sum_j \int_{A=0}^{\infty} PF_j\, dA \quad (16)$$

where $A_\otimes$ = Area of target sphere, and F_j is the annual flux for jth object.

The annual ISS maneuver rate for a single object is the area within the Pm contour for that object times the associated flux. The total annual maneuver rate, M_A, is the sum of the single object maneuver rate over all objects.

$$M_A=\sum_j\int_{A=0}^{A(Pm)} F_j dA\,. \quad (17)$$

The Annual Collision Probability is

$$\sum_j F_j\, A_\otimes=\sum_j\int_{A=0}^{\infty} PF_j dA= \sum_j\int_{A=0}^{A(Pm)} PF_j dA+\sum_j\int_{A(Pm)}^{\infty} PF_j dA\,. \quad (18)$$

That is, P_T = Total Risk = Risk Reduction + Residual Risk. Letting Q = Risk Reduction and R = Residual Risk:

$$P_T = Q + R\,, \qquad \frac{Q}{P_T} + \frac{R}{P_T} = 1\ . \tag{19}$$

Identifying R/P_T as Fractional Residual Risk (FRR) and R/P_T as Fractional Risk Reduction FRD

$$\text{FRD} + \text{FRR} = 1. \tag{20}$$

If FRR << 1, and the maneuver rate is tolerable then the adopted strategy is effective. Fig. 6 shows the fractional residual risk as a function of annual maneuver rate for 8 and 24-hour propagation from the orbit determination epoch.

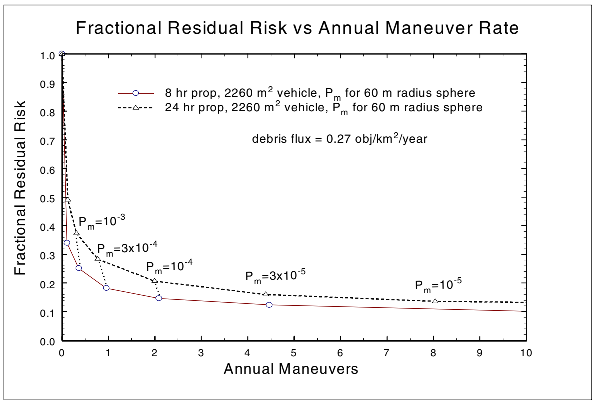

Fig. 6. Fractional residual risk versus annual maneuver rate for 8 and 24-hour state vector propagation from epoch.

For a maneuver threshold of Pm = 10^{-4}, based on the January 1999 space debris catalog, we anticipate two maneuvers per year either for 8 or 24 hour state vector propagation. The fractional residual risk for an 8-hour propagation is about 0.15. The value for a 24-hour propagation is about 0.22

9. CONCLUSIONS

A debris avoidance system for ISS has been implemented which allows assessment of risk reduction, minimizes maneuvers, and is very cost effective. The method requires accurate estimates of the state vector and state vector covariance for both the space vehicle and the conjuncting object. The covariance data from the orbit determination software matches observation statistics for low drag objects. The orbit determination software covariance is scaled for high drag objects. With the current debris avoidance system, there is a risk reduction of from 80 to 85 percent with about 2 maneuvers per year. Improvements continue to be made to the system.

10. ACKNOWLEDGMENTS

W. Taft Devere, Nicholas L. Johnson and Joseph H. Frisbee provided critical reviews of the manuscript and made a number of valuable suggestions.

11. REFERENCES

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2. Foster, J. L. Jr. and Estes, H. S., A Parametric Analysis of Orbital Debris Collision Probability and Maneuver Rate for Space Vehicles, JSC25898, NASA Johnson Space Center, May 1992.

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