DETECTION OF AN IMPACT SITE USING THE LEAST SQUARE METHOD

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ABSTRACT

The method of the determination of the penetration site at the SC's cover by a space debris fragment or a meteoroid using sensors recording elastic waves coming from the impact point. The method based on the cover material's homogeneity when the disturbances from the impact with SD/M particle spread. The method of least squares is used for processing the information from the recording sensors.

When the SD/M impacts the SC's cover depressurization can occur. There are some reasons that the pressure decreasing will last from several minutes to tens minutes. During this time we should to determine the impact site coordinates and to carry out measures to mend the hole and prevent gas leakage from the SC.

Let the sensors recording acoustic disturbances from the impact site are located on the SC's protective cover and their coordinates are known in the orthogonal frame. Also we assume that the acoustics waves from the impact site can spread along the cover in all directions with the same velocity. Then the distance from the impact point (disturbances origin's location) to the k^{th} sensor $\vec{r}_{distur}$ is equal

$$\mathbf{D}\vec{r}_{\hat{e}} = \vec{r}_{\hat{e}} - \vec{r}_{distur} \quad . \tag{1}$$

If the sound velocity is C the time required the acoustic wave to spread from the impact site to the k^{th} sensor can be determined as

$$t_{\hat{e}} = \frac{/\ \mathbf{D}\vec{r}_{\hat{e}}\ /}{C} . \tag{2}$$

The sensor nearest to the impact site first records the acoustic wave. The spreading time to this sensor is minimal and equal

$$(t_?)_{min} = \tau \quad . \tag{3}$$

Other sensors record time differences, i.e.

$$\Delta t_? = t_? - \tau \ . \tag{4}$$

Using (4) equation (2) can be rewritten as

$$\frac{/\ \mathbf{D}\vec{r}_{\hat{e}}\ /}{C} - t = \mathbf{D}t_{\hat{e}} . \tag{5}$$

In the left part of equation (5) there are unknown coordinates of the impact site and time τ, and in the right part of it - $\Delta t_?$ – is measured .

To be short denote the left part of equation (5) as

$$h_{\hat{e}} = \frac{/\ \mathbf{D}\vec{r}_{\hat{e}}\ /}{\tilde{N}} - t \ . \tag{6}$$

Then the values of $\Delta t_?$ - $(k = i, j)$ - are the measured magnitudes of parameters, which have true values $\eta_?$. Hence

$$\Delta t_? = \eta_? + \delta_? \quad , \tag{7}$$

where $\delta_?$ - is a stochastic error of the measurement of $\eta_?$

Assuming the statistical errors $\delta_?$ are distributed normally with mathematical expectation of zero and the measurements $\Delta t_?$ of are equally precise with mean square root error σ and using the maximal verisimilitude principle we obtain the condition for estimation of minimum of sum of the squares of the residual errors [1]

$$\mathop{\text{å}}_{\hat{e}} e_{\hat{e}}^2 = \mathop{\text{å}}_{\hat{e}} \left[\mathbf{D}t_{\hat{e}} - h_{\hat{e}}\left(\mathbf{D}\vec{r}_{\hat{e}}, t\right)\right]^2 . \tag{8}$$

Equating the partitioned derivatives on the components of the vector $\mathbf{D}\vec{r}_{\hat{e}}$ a on the time τ one finds normal system of equations

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$$\begin{cases} \sum_{k} \dfrac{\partial h_k}{\partial \vec{r}_{distur}} \cdot h_k\left(\Delta \vec{r}_k, t\right) = \sum_{k} \dfrac{\partial h_k}{\partial \vec{r}_{distur}} \cdot \Delta t_k \\ \sum_{k} \dfrac{\partial h_k}{\partial t} \cdot h_k \cdot \left(\Delta \vec{r}_k, t\right) = \sum_{k} \dfrac{\partial h_k}{\partial t} \cdot \Delta t_k \end{cases} . \quad (9)$$

The system (9) is nonlinear. To simplify the task let do the linear system as follows.

Let represent the local estimations for unknown values as

$$\vec{r}_{distur} = \vec{r}^{\,(1)}{}_{distur} + \Delta \vec{r}^{\,(1)}{}_{distur} \ , \ \tau = \tau^{(1)} + \Delta\tau^{(1)}, \quad (10)$$

one write

$$h_k\left(\Delta \vec{r}_k, t\right) = h_k\left(\Delta \vec{r}^{\,(1)}{}_k \ , t^{(1)}\right) +$$
$$+ \frac{\partial h_k}{\partial \vec{r}_{distur}} \cdot \Delta \vec{r}^{\,(1)}{}_{distur} + \frac{\partial h_k}{\partial t} \cdot \Delta t^{(1)} + \ldots , \quad (11)$$

where the partitioned derivates are functions of $\vec{r}^{\,(1)}{}_{distur}$ and $\tau^{(1)}$.

Neglecting in right part of equation (11) with terms of second and higher orders and substituting (11) into (9) one can write the linear equations system for unknown $\Delta \vec{r}^{\,(1)}{}_{distur}$, $\Delta\tau^{(1)}$.

The full system is

$$\left\| a \right\| \cdot \left\| \begin{matrix} \Delta \vec{r}^{\,(1)}{}_{distur} \\ \Delta t^{(1)} \end{matrix} \right\| = \left\| \begin{matrix} \sum_{k} \dfrac{\partial h_k}{\partial \vec{r}_k} \left(\Delta t_k - h_k\right) \\ \sum_{k} \dfrac{\partial h_k}{\partial t} \left(\Delta t_k - h_k\right) \end{matrix} \right\| . \quad (12)$$

Solving the system (12) one find $\Delta \vec{r}^{\,(1)}{}_{distur}$ and $\Delta\tau^{(1)}$ using them find the second approximation for the to-be-determined parameters

$$\vec{r}^{\,(2)}_{distur} = \vec{r}^{\,(1)}_{distur} + \Delta \vec{r}^{\,(1)}{}_{distur} , t^{(2)} = t^{(1)} + \Delta t^{(1)} . \quad (13)$$

Using $\vec{r}^{\,(2)}_{distur}$ and $t^{(2)}$ for determination of the coefficients in system (12) we find $\Delta \vec{r}^{\,(2)}{}_{distur}$, $\Delta t^{(2)}$, i.e.

Calculations are repeated until the needed relative error for values $\vec{r}_{distur}$ and τ. is gained.

CONCLUSION

The calculations are quickly convergent, the calculated coordinates of the penetration site agree the real their values very accurately.

In conclusion the advantage of the proposed method should be noted that consist in possibility to process of surplus information to get extra stability of the calculations in the case of malfunctions of some components of measuring system.

The method and the appropriate software can be proposed for earth-based on-board systems for processing of information on the shock waves and other physical processes in space technical elements.

REFERENCES

1. Agekjan T.A. Foundations of theory of errors for astronauts and physicists. Nauka, Moscow, 1972.