DYNAMICS OF LARGE OBJECT REMOVAL SYSTEMS

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ABSTRACT

Due to the permanent extension of space activities, the danger generated by the larger space debris objects is increasing. Mechanisms and procedures to remove those objects should be developed in the next future. It is to be thought that the objects in LEO would be removed by putting them in into an earth atmosphere entry orbit, while those in GEO could be placed into an orbit 300 km or more higher than the nominal geostationary orbit.
This paper deals with the general problem of dynamics for a system formed of a remover vehicle and a removed object. The trajectory and the attitude motion are considered for the remover and the debris. The number of degrees-of-freedom of the system is reduced, considering the joint system.
Particular cases of tethered systems can be in correlation with the TERESA (Tethered Remover Satellite) concept, which has been developed by P. Eichler and A. Bode or with TOT (Tumble Orbit Transfer), initially proposed by T. Yasaka.
Lagrange's equations for holonomic systems with dependent variable are used in the case the constraint equations depend of generalised coordinates. In the case the constraints equations are dependent of generalised coordinates and generalised velocities, the Lagrange's equations for non-holonomic systems are used.
A model for a remover vehicle can be proposed. The spacecraft will be provided with a remote controlled tethered subsystem to be connected with the removed object. The risk of cutting the tether by small particles is considered. The remover spacecraft will be provided with several elements of tethers. In the case the tether is cut, the joint is disconnected from the removed object, and self connects to a new unit of tethers. A remover vehicle provided with two such joint spacecraft will diminish the debris collision probability of the space mission.

1. CHANGING OF DEBRIS ORBITS

Orbits of the useless cosmic bodies can be modified through active methods. In the case of debris localised on equatorial orbits it is necessary "pushing" of these bodies in a 200 – 300 km higher orbits, and in a case of low and medium orbits solution can be changing of these orbits in a very low orbits localised in the higher atmosphere.
Orbits' changing can be analysed on the basis of Hohmann transfer theory between circular orbits.
On the basis of total velocity jump, fuel relative consumption can be calculated with

$$\frac{\Delta m}{m_0} = 1 - e^{-\frac{\Delta v}{I_{sp} g}} \qquad (1)$$

where I_{sp} it is specific impulse of the manoeuvre thruster.
In the table 1 relative fuel consumption values for Δr geostationary orbits radius increasing, for different values of the specific impulse are presented.

Δr [km]	200	400	600	800	1000
Δv_1 [m/s]	3,635	7,249	10,842	14,414	17,964
Δv_2 [m/s]	3,631	7,232	10,804	14,346	17,859
I_{sp}=300 s	0,25	0,49	0,73	0,97	1,21
$\Delta m/m_0$ [%] I_{sp}=400s	0,18	0,37	0,55	0,73	0,91
I_{sp}=500 s	0,15	0,29	0,44	0,58	0,73

Table 1 Relative fuel consumption for changing of geostationary orbits

Can be observed, it is necessary fewer fuel for the transfer from the geostatioanary orbits to the few hundred km higher orbits.

2. DYNAMICS OF HOLONOMIC SYSTEMS WITH DEPENDENT VARIABLES

For a non-holonomic rheonomic system, Lagrange's equations for h coordinates are given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_k}\right) - \frac{\partial E}{\partial q_k} = Q_k + \sum_{i=1}^{p} \lambda_i a_{ik}, (k = 1,2,\cdots,h) \qquad (2)$$

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and are completed by constraints:

$$\sum_{k=1}^{h} a_{ik}\dot{q}_k + b_i = 0, \quad (i = 1,2,\cdots p) \tag{3}$$

Solving the h equations system, given by relation (2), and the p equations, given by relation (3) leads to the determination of the h coordinates q_k and p multiplicators λ_i.

From the equation (2) can be obtained holonomic systems through the changing of the a_{ik} functions in these equations.

In the case of the holonomic system, constraints are give by relations:

$$\Phi_i(q_1,\cdots,q_h,t) = 0, \quad (i = 1,2,\cdots p). \tag{4}$$

Equations (2) became:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_k}\right) - \frac{\partial E}{\partial q_k} = Q_k + \frac{\partial U_\Phi}{\partial q_k}, \quad (k = 1,2,\cdots,h) \tag{5}$$

where

$$U_\phi = \sum_{i=1}^{p} \lambda_i \Phi_i \ . \tag{6}$$

From the above written h equations and from the p constraints equations given by relation (4) function for the h generalised coordinates q_k and the p multiplicators λ_i will be determined .

3. MOTION OF THE SYSTEM OF TWO PARTICLES

In the case of a system formed by two material points which are connected through a negligible mass tether, force function is given by

$$U = \frac{\mu m_1}{r_1} + \frac{\mu m_2}{r_2}, \tag{7}$$

and

$$U_2 = \frac{\mu m}{r_v} - \frac{\mu m^* S^2}{2r_v^3}\left(1 - 3\cos^2\theta\cos^2\varphi\right) \tag{8}$$

is obtained,

where $S = S_1 + S_2$ is the distance between the two points of the system, $m = m_1 + m_2$ is the system mass, and

$$m^* = \frac{m_1 m_2}{m_1 + m_2} \tag{9}$$

Using the relation for the force function U_2, motion equations for the studied system in S, θ, and φ coordinates can be obtained. In the case of a circular orbit the following equations can be written:

$$\begin{gathered} S'' - S\left[\varphi'^2 + (1+\theta')^2\cos^2\varphi - 1 + 3\cos^2\theta\cos^2\varphi\right] = \frac{1}{m^* n^2}Q_\rho^* \\ \frac{\mathrm{d}}{\mathrm{d}v}\left[S^2(1+\theta')\cos^2\varphi\right] + 3S^2\sin\theta\cos\theta\cos^2\varphi = \frac{1}{m^* n^2}Q_\theta^* \\ \frac{\mathrm{d}}{\mathrm{d}v}\left(S^2\varphi'\right) + S^2(1+\theta')^2\sin\varphi\cos\varphi + 3S^2\cos^2\theta\sin\varphi\cos\varphi = \frac{1}{m^* n^2}Q_\varphi^* \end{gathered} \tag{10}$$

In the figure 1 variation of the ratio between length of the tether and initialy length of the tether with respect to number of mass center rotations in the orbit are presented . Comparation between results obtained in this case and results obtained from equation 2 on the basis of a linear model, show a more rapid deployement of the tether than in a case of a linear model. Diference betweent these two cases is espected because linearization of equations leads to loosing of the inertial forces which favorise a rapid deployement of the tether. In the figure 2 deployement with a constant strenght in the tether is presented.

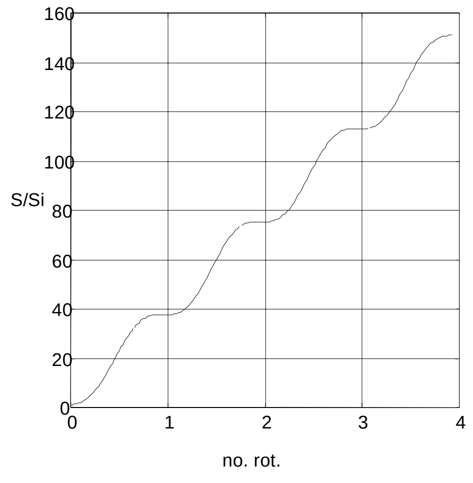

Fig. 1 Relative length variation of the tether (without strength)

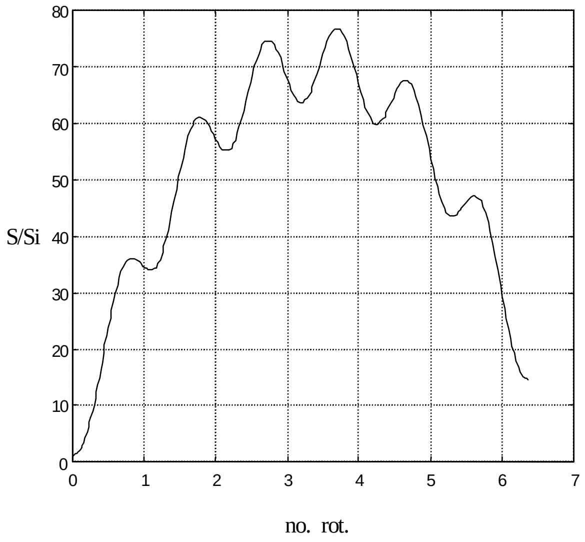

Figure 2 Relative length variation of the tether (constant strength)

4. BODIES SYSTEMS RELATIVE MOTION

In the case of an orbital station with considerable dimensions, considered like a bodies system, the motion is described using a reference frame which have the origin in the centre of the Earth. Integration of the equation leads to difficult problems because some coordinates, like vector radii, have big values and another coordinates, like distances between bodies, have small values. Some of these difficulties can be avoided if the relative motion of the bodies system is studied in a reference frame with known motion.
For two tethered bodies (figure 3) constraint equation is

$$\left|\overline{\mathrm{O}_1\mathrm{O}_2}\right| = l = const. \tag{11}$$

Using the projections of the vector

$$\overline{\mathrm{O}_2\mathrm{O}_1} = \overline{\mathrm{CM}_1} + \overline{\mathrm{M}_1\mathrm{O}_1} - \left(\overline{\mathrm{CM}_2} + \overline{\mathrm{M}_2\mathrm{O}_2}\right) \tag{12}$$

on the axes of the Cx_cy_c coordinate system, the constraint

$$\begin{aligned}\Phi \equiv\ & \rho_1^2 + \rho_2^2 - 2\rho_1\rho_2\cos(\theta_1-\theta_2) + 2\rho_1 a_1\cos\varphi_{31} + \\ & + 2\rho_1 b_2\cos(\theta_1-\theta_2-\varphi_{32}) - 2\rho_2 a_1\cos(\theta_1-\theta_2+\varphi_{31}) - \\ & - 2\rho_2 b_2\cos\varphi_{32} + 2a_1 b_2\cos(\theta_1-\theta_2+\varphi_{31}-\varphi_{32}) + \\ & + a_1^2 + b_1^2 - l^2 = 0\end{aligned} \tag{13}$$

is obtain,

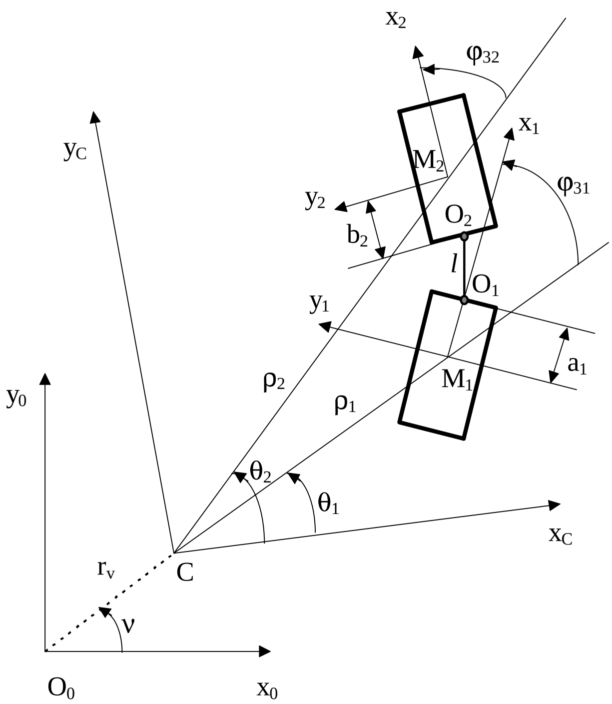

Figure 3 System of two tethered bodies

which is used in the function

$$U_\Phi = \lambda\Phi. \tag{14}$$

On the basis of the equations (5) and of constraints, the motion equations for a system formed by two bodies connected by tether can be obtain. Equations (5) are right if the strength in the tether is it positive one. In the case of a negative strength, the connection is not valid, and the bodies have independently motions. In this case, motion equations are obtain from equation (5) where λ parameter is considered zero. The strength in tether is calculated on the basis of an isolated body motion equation.
The studied case is of two cylindrical bodies connected by a tether without mass. The bodies are considered the same dimension or very close dimensions.
In the figure 4 radii ρ_1 and ρ_2 are presented and in the figure 5 angles θ_1 and θ_2 are presented.

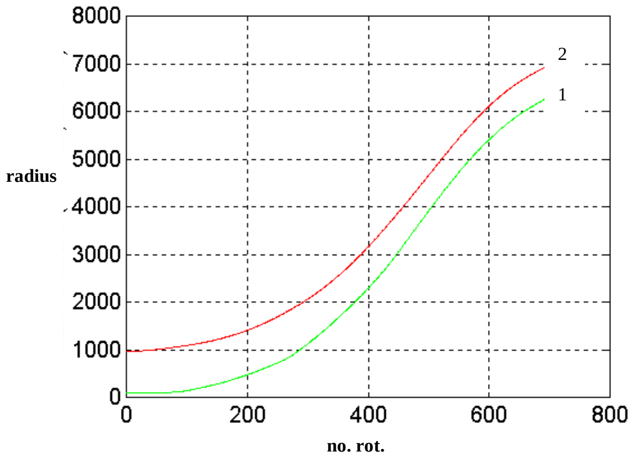

Fig. 4 Radius variation

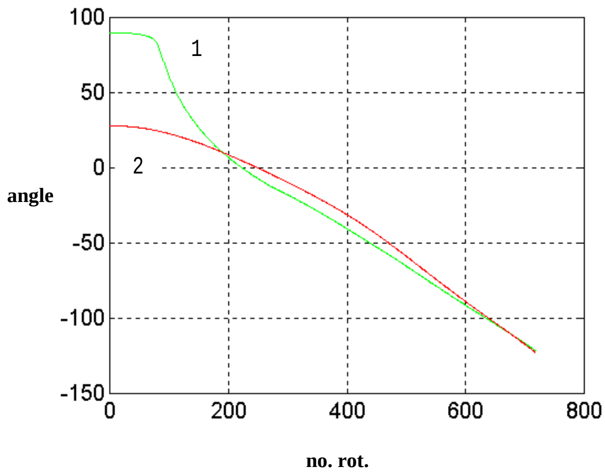

Fig. 5 Radius angle variation

5. CONCLUSIONS

Using Lagrange equations for holonomic and non-holonomic systems the motions of systems of particles and of rigid bodies are studied in this paper.
Motion of deployment of a system of two tethered particles and the relative motion of tethered bodies are analysed and numerical results are presented.

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