THE DEVELOPMENT OF THE STATISTICAL THEORY OF A SATELLITE ENSEMBLE MOTION AND ITS APPLICATION FOR SPACE DEBRIS MODELLING

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ABSTRACT

The statistic theory of satellite ensemble motion is developed. Unlike traditional approach of astrodyna-mics, which considers the individual motion of every satellite, this approach is based on a statistic description of satellite motion, as continuous environment. Five basic problems of the satellite ensemble motion are solved,namely:

1. Statistical description of space debris (SD) sources and disturbing factors;
2. Mid- and long-term forecast of the SD environment;
3. Construction of the altitude-latitude distribution of both spatial density and SD velocity vector characteristics;
4. Estimation of satellite collision probability and velocity collision parameters;
5. Collision probability determination with regard to the shape and orientation of typical spacecraft modules.

We shall consider briefly each of listed problems.

1. STATISTICAL DESCRIPTION OF SPACE DEBRIS SOURCES

The main reason of small- and medium-size SD formation is the orbital fragmentation of space objects. We apply the averaged approach to the description of SD sources. Its characteristic feature is that we apply, instead of the data on specific launches and breakup cases, the following averaged data: (a) the altitude distribution $\mathbf{dp(t,h_p,d)}$ of a number of annually formed objects sizing larger than **d** (here **t** is time, $\mathbf{h_p}$ is the perigee altitude), and (b) statistical distributions of their eccentricities **p(e,d)** and inclinations **p(i, d)**. The substantiation of such an approach is as follows:

- The SD number varies insignificantly during a year (a few percents only). Therefore, the more detailed (in time) modeling of SD sources is excessive: it strongly complicates a model, practically not influencing the accuracy.

The note. This statement does not exclude the possibility and expediency of detailed modeling of breakup consequences at short time intervals, when the SD "cloud" remains rather compact. However, the cloud scattering process is known to proceed rather quickly, as a rule. The duration of this process is about 1 month.

- The reasons and conditions of satellite fragmentation, which resulted in formation of the majority of small SD fragments, are extremely diverse. Therefore, it is difficult to expect that the results of modeling of consequences of all known fragmentations (the number of particles, the fly-away velocity) are accurate enough. The level of errors of such a modeling is unknown. Hence, the approach, based on averaged data, seems to be not only not worse, but even more preferable.
- The previous statement is even more valid for the future moments of time, for which the reasons and circumstances of fragmentation are unknown. The use of averaged data on the intensity of new SD formation is optimum at fulfilling the forecast.

The dependencces of initial distributions on objects' size are constructed based on the natural assumption, that all small-size SD were formed as a result of large (catalogued) objects fragmentation. Such an approach is based on using: (a) the Monte-Carlo method, (b) statistical distributions of annual surplus $\mathbf{dp(h_p,d_{cat}),p(e,d_{cat}),p(i,d_{cat})}$ constructed from the real data on catalogued objects, and (c) a priori data about the dependence of SD fly-away velocity on their size. The less the size of a particle, the greater velocity increment it receives at the formation time. In such a manner the initial distributions are updated by using the fragmentation modeling data.

2. FORECAST OF THE SPACE DEBRIS ENVIRONMENT [1]

Let us consider various space objects (SO), whose orbit perigee altitude does not exceed 2000 km. Choose the perigee height (h_p) from the vector of SO orbital elements. We shall suppose that among all variable SO parameters only the perigee height essentially influences the evolution of height distribution of SO number. The other orbital elements will be designated by **Э**. We sub-divide the whole set of objects with different elements **Э** into some finite number of sub-sets (groups) with elements $Э_l$, $l=1,2,...,l_{max}$. Let **p(t,h)** be the density of the perigee height distribution for objects from the selected group at time **t**. Then we state the problem of sudying the laws of density variation in time. Index **l** will be omitted hereafter in analyzing the distribution evolution for some particular SO group.

The partial derivative equations are derived for describing the evolution of height distribution of SO number:

$$\frac{\partial p(t,h)}{\partial t} = V(t,h)\left[\frac{\partial p(t,h)}{\partial h} - \frac{p(t,h)}{H(t,h)}\right] + dp(t,h,...) \quad (1)$$

Here **V(t,h)** is the perigee lowering velocity; **H(t,h)** is the atmosphere scale height; **dp(t,h,...)** is the rate of SD density increment due to various reasons.

Explanations. In calculating the evolution of height distribution of SO number the following factors are taken into account:
- the atmospheric drag at heights of up to 2000 km;
- the sub-division of all SO by parameters into the groups which differ in size **d**, eccentricity **e** and ballistic factor **S**;
- the initial height distribution of SO of various types;
- the expected intensity of formation of new SO of various types as a result of launches and explosions: **dp(t,h,...)** is the increment of SO number at various heights per time unit;
- the non-stationary of factors taken into account, namely, the atmospheric density in connection with solar activity variation during the 11-year cycle and the intensity of new launches.

The algorithm for numerical solution of equations (1) is developed. The compromise between the analysis detailization of and the simplicity of an algorithm is required in choosing a number of SO sub-divisions into groups: when the sub-division is too detailed, the memory will be not enough and the calculation time will increase. Besides, one should take into account, that the initial data for the environment forecast have rather high uncertainty, which makes the excessive algorithm detailization senseless.

3.1. CONSTRUCTION OF THE ALTITUDE-LATITUDE DISTRIBUTION OF SPATIAL DENSITY [2]

The developed technique has some similar features with well-known D. Kessler's technique [3]. Characteristic features of our technique are as follows: the problem of the total SD concentration construction is solved on the basis of histograms of distributions $p(h_p)$, **p(e)** and **p(i)**.

The algorithm basis. We denote the altitude-latitude distribution of the SD number in a volume unit (the spatial density) as $\rho(h,\varphi)$. Here **h** is the altitude of a point over the Earth surface, φ is the geographic latitude of this point.

The following formula was deduced for function $\rho(h,\varphi)$:

$$\rho(\mathbf{h},\varphi)=\frac{\mathbf{F}(\varphi)}{2\pi^2(\mathbf{h}+\mathbf{R})^2\cdot\Delta h}\cdot$$
$$\cdot\int_{\mathbf{h_p}}\int_{\mathbf{e}}\Delta\tau(\mathbf{h_p},\mathbf{e})\Phi(\mathbf{h_p},\mathbf{e},\mathbf{h})p(\mathbf{h_p})p(\mathbf{e})\mathbf{dh_p de} \qquad (2)$$

Here **R** is the Earth radius,

$$\Phi(\mathbf{h_p},\mathbf{e},\mathbf{h})=\frac{(1-\mathbf{e})^2}{\sqrt{1-\mathbf{e}^2}}\left(\frac{\mathbf{h}+\mathbf{R}}{\mathbf{h_p}+\mathbf{R}}\right)^2, \qquad (3)$$

$$\mathbf{F}(\varphi)=\int_{\mathbf{i}}\frac{p(\mathbf{i})\cdot\mathbf{di}}{\sqrt{sin^2\mathbf{i}-sin^2\varphi}}, \quad \text{for } sin\,\mathbf{i}\ge sin\varphi. \qquad (4)$$

$\Delta\tau(\mathbf{h_p},\mathbf{e})$ is the normalized (in fractions of a period) time interval, during which the SD with elements $\mathbf{h_p}$, **e** is situated in the altitude range of $(\mathbf{h},\mathbf{h}+\Delta\mathbf{h})$.

It follows, that the altitude range should be subdivided into intervals, and formulae (2, 3 and 4) should to applied the for spatial density calculation at points of space with various altitudes and latitudes. Here it is convenient to use the same subdivisions of arguments, which are applied in histograms $p(h_p)$, **p(e)** and **p(i)**. Then the integrals in (2) are replaced by the appropriate sums.

3.2. CONSTRUCTION OF THE VELOCITY VECTOR CHARACTERISTICS

The principles of the technique given below were presented at the Space Mechanics Symposium [4], but were not published.

The problem statement. Let us consider the arbitrary point in the near-Earth space with spherical coordinates **r**, φ, λ. Assuming the SD spatial density $\rho(\mathbf{r},\varphi)$ to be known, we determine the number of objects, which pass in this point's vicinity through the cross-section of size $\delta\mathbf{F}$ per the time of one period (one revolution), and construct the azimutal distribution of frequency (probability) of such passages **p(A)**.

Basis of the algorithm. The total number of objects being in the spherical layer $(\mathbf{r},\ \mathbf{r}+\delta\mathbf{r})$ is $\delta\mathbf{n}=\mathbf{p(h)}\cdot\delta\mathbf{r}$, where

$$\mathbf{p}(\mathbf{h})=2\pi\cdot\mathbf{r}^2\int_{-\pi/2}^{\pi/2}\rho(\mathbf{h},\varphi)\cdot cos\varphi\cdot\mathbf{d}\varphi. \qquad (5)$$

Only a small fraction of $\delta\mathbf{n}$ passes in the given point's vicinity. The problem is: to determine the number of objects which pass in such a manner, that the shortest distance $\delta\mathbf{b}$ (along the binormal) from the given point satisfies the condition:

$$|\delta\mathbf{b}|\le\delta\mathbf{r}/2. \qquad (6)$$

Taking into account the above considerations, all possible trajectories are determined by two elements: inclination **i** and ascending node longitude Ω (that will be counted clockwise from the given point longitude). For the trajectories passing strictly through the given point these elements satisfy the relation

$$\mathbf{tgi}\cdot\mathbf{tg}\Omega=\mathbf{tg}\varphi. \qquad (7)$$

If for any value of Ω and $\mathbf{i}=\mathbf{f}(\Omega)$ we determine such deviations $\delta\Omega$ and $\delta\mathbf{i}=\mathbf{f}(\delta\Omega)$, that the condition (6) is valid, then it will become possible to determine quantitatively the sought fraction of objects from a priory specified distributions **p(i)** and $\mathbf{p}(\Omega)$ and from the relation

$$\delta\dot{\mathbf{n}}(\delta\Omega)=\mathbf{p}(\mathbf{i})\cdot\mathbf{p}(\Omega)\cdot\delta\mathbf{i}\cdot\delta\Omega. \qquad (8)$$

This is just the fraction of objects from the $\delta\mathbf{n}$ number, which have the node longitude within the $(\Omega,\ \Omega+\delta\Omega)$ interval and pass through the $\delta\mathbf{b}$-vicinity of the given point. There exists a single-valued correspondence between the elements and velocity vector direction azimuth, namely

$$\sin \mathbf{A} = \frac{\cos \mathbf{i}}{\cos\varphi} = \frac{\sin\Omega}{\sqrt{\sin^2\Omega\cdot\cos^2\varphi + \sin^2\varphi}}. \qquad (9)$$

The value of azimuth relates to the same quadrant, as the ascending node longitude value.

The basic problem consists in finding, for the set of trajectories $\mathbf{i}=\mathbf{f}(\varphi, \Omega)$, such $\delta\mathbf{i}$ and $\delta\Omega$ values (the "tube" of trajectories), that the condition (6) be satisfied. Once this region **S** is constructed, the curvilinear integral of the first kind

$$dn(\delta b) = \int_S p(i)\cdot p(\Omega)\cdot dS. \qquad (10)$$

will determine the fraction of objects from the $\delta\mathbf{n}$ number, which are situated inside the "tube" mentioned above. As a result of this "tube" construction, the curvilinear integral is presented as an ordinary integral

$$\delta\mathbf{n}(\delta\mathbf{b}) = \frac{\delta\mathbf{b}}{\mathbf{r}\cdot\sin\varphi}\int_0^{2\pi}\mathbf{p}(\Omega)\cdot\mathbf{p}(\mathbf{i})\cdot\sin\mathbf{i}\cdot\mathbf{d}\Omega. \qquad (11)$$

The inclination **i** in the integrand is associated with the ascending node longitude by relation (7). The total number of objects, which pass in the given point's vicinity through the cross-section of area $\delta\mathbf{F}=\delta\mathbf{r}\cdot\delta\mathbf{b}$ per one period time, is equal to

$$\mathbf{p}(\mathbf{h})\cdot\delta\mathbf{r}\cdot\delta\mathbf{n}(\delta\mathbf{b}) = \frac{\delta\mathbf{F}}{\mathbf{r}}\cdot\frac{\mathbf{p}(\mathbf{h})}{\sin\varphi}\cdot\int_0^{2\pi}\mathbf{p}(\Omega)\cdot\mathbf{p}(\mathbf{i})\cdot\sin\mathbf{i}\cdot\mathbf{d}\Omega. \qquad (12)$$

So, the first problem stated above is solved. Now we shall pass to the construction of objects' density distribution in azimuth **p(A)**. This density satisfies the relation

$$\int\mathbf{p}(\mathbf{A})\mathbf{dA} = 1. \qquad (13)$$

We shall make use of expression (11). The quantity

$$\frac{\delta\mathbf{b}}{\mathbf{r}\sin\varphi}\mathbf{p}(\Omega)\cdot\mathbf{p}(\mathbf{i})\cdot\sin\mathbf{i}\cdot\Delta\Omega = \Phi(\Omega)\cdot\Delta\Omega \qquad (14)$$

that corresponds to specific values of **i**, Ω elements and to the discrete increment $\Delta\Omega$, characterizes the number of objects (the fraction of $\delta\mathbf{n}$), which fall inside the $\delta\mathbf{b}$-vicinity of the point under consideration. All these objects pass through the azimuthal sector (**A**, **A**+Δ**A**), where

$$\Delta\mathbf{A} = \frac{\mathbf{dA}}{\mathbf{d}\Omega}\cdot\Delta\Omega. \qquad (15)$$

The derivative is determined from the relation (9). It follows from (14) and (15), that the azimuthal SO distribution density will be

$$\mathbf{p}(\mathbf{A}) = \mathbf{k}\cdot\Phi(\Omega)/\left(\frac{\mathbf{dA}}{\mathbf{d}\Omega}\right) = \mathbf{k}\cdot\frac{\sin\mathbf{i}}{\sin\varphi}\cdot\frac{\mathbf{d}\Omega}{\mathbf{dA}}\cdot\mathbf{p}(\Omega)\cdot\mathbf{p}(\mathbf{i}). \qquad (16)$$

Here some simplification was applied, that does not have any principal significance. The constant parameters are replaced here by some normalized factor **k** which is determined from the condition (13).

Explanation. The Monte-Carlo method is applied to construction of the discrete distribution **p (A)** (the histogram) on the basis of formula (16). The ascending node longitude value gets out on cast lots. The inclination and azimuth values are calculated by formula (7) or (9), respectively, for chosen values of ascending node longitude and latitude. The particular azimuth sub-division interval corresponds to this azimuth value. The summation of density estimations is made for each of sub-division intervals in accordance with the right-hand part of expression (16). The normalization of obtained histograms is made on the basis of condition (13) after completing the random choice.

For obtaining rather stable distributions **p(A)** the number of implementations in applying the Monte-Carlo method should be not less than 10 000.

4. THE ESTIMATION OF SATELLITE COLLISION PROBABILITY AND VECTOR VELOCITY PARAMETERS [5]

The problem statement. It is assumed that the spacecraft has spherical shape and moves over the known orbit. The concentration of SD particles of different sizes at any near-Eatth space (NES) point is supposed to be known as well. The SD velocity at the given altitude can be calculated to sufficient accuracy by the elementary formula for circular orbits. The statistical distribution of possible SD velocity directions at any NES point is also considered to be known. Such a description of velocity assumes, that the flux of particles is planar. This assumption is valid, because the most part of orbits are close to circular ones. Thus, the complete statistical information about SD - their concentration and field of velocities - is available.

The SC are also assumed to be essentially larger in size than the fragments, the danger of collision with which is estimated. It is required to estimate the average number of SD which cross the SC surface per time unit (a year).

The basis of an algorithm. The technique of estimating the average expected number **N** of collisions of spherical-shaped SC with SD is based on integrating the following differential equation:

$$\frac{dN}{dt} = F\cdot\rho(t)\cdot\left[\int_{A=0}^{2\pi} p(t,A)\cdot V_{\mathbf{rel}}(t,A)\cdot dA\right] \qquad (17)$$

Here: **t** is the time; **F** is the cross-section area of a sphere; **ρ(t)** is the space debris density (the number of SD in a volume unit); **A** is the direction azimuth of possible collision of SC under consideration with other objects; **p(t,A)** is the density of distribution of objects' flux directions at the given point of space; $V_{rel}(t,A)$ is the azimuthal dependence of collision velocity at the given point. The relative velocity is equal to the vector difference between SD and SC velocities. The relative velocity direction is characterized by its deviation ΔA from the tangential component of SC velocity only. This angle is equal to

$$\Delta A = \pi - (A_{SC} - A_{SD})/2. \quad (18)$$

Here indices SC and SD denote the azimuths of the velocity vector direction of the given SC and some SD, respectively. The relative velocity value V_{rel} is determined by the formula

$$V_{rel} \approx 2 \cdot V_{\tau} \cdot \cos(\Delta A). \quad (19)$$

The right-hand part in formula (17) has a meaning of instant value of the SD flux through the surface of a given sphere. This right-hand part is almost a periodic function with the period, equal to the time of one revolution. Therefore, to determine the average flux value it is necessary to integrate equation (17) over one revolution interval interval and to divide the result by the SC period. In so doing the calculation of the integral in the right-hand part of expression (17) is replaced by summation of integrand function values with an azimuthal step equal to the discreteness of histogram specification **p (t, A)**.

The above technique allows to take into account in detail the variability of SD flux as a function of orbital elements of SC under consideration and its position in space. The technique uses the condition, that variable functions **ρ(t), p(t,A)** and V_{rel}, appeared in the right-hand part of expression (17), are known. The construction of these functions is carried out with the help of methods mentioned above.

The proposed technique also allows to calculate rather simply various characteristics of expected SC/SD collisions: the distribution of directions of SD approach at possible collisions as well as the density of distribution of relative velocity values. These characteristics are rather simply constructed on the basis of formulas (18) and (19), which allow to establish the relation between the instant value of a flux in the elementary sector **dA**

$$F \cdot \rho(t) \cdot p(t, A) \cdot \mathbf{V_{rel}}(t, A) \cdot dA \quad (20)$$

and appropriate values of **A** and V_{rel}. Then the summation of estimations (20) in corresponding "boxes" of histogram of distributions **p(ΔA)** and **p(V_{rel})** is made. The normalization of histograms is carried out after completing the integration.

5. THE ACCOUNT OF SHAPE AND ORIENTATION OF TYPICAL SPACECRAFT MODULES [6]

The average value of a flux determined on the basis of equation (17) for a spherical-shaped SC with the cross-section area of F_{KA} is designated by P_0. For other shapes of SC the SD flux through their surface can be calculated by simple formula

$$P = C_N \cdot P_0. \quad (21)$$

Factor C_N takes into account here the influence of SC shape and its axis orientation. This factor is determined by formula

$$C_N = \frac{\int_F \cos\gamma(\Delta A) \cdot \mathbf{p}(\Delta A) \cdot \mathbf{dF}}{F_{KA}}. \quad (22)$$

Here **γ(ΔA)** designates the angle between the external normal to an elementary area of SC surface and the arbitrary direction of a relative SD flux; F_{KA} is the characteristic area of SC under consideration; **p(ΔA)** is the distribution of directions of SD approach at possible collisions. The integral in the right-hand part of (22) is taken over the whole external surface of SC under consideration with regard to the SC shape and orientation (without account of the shadow).

6. THE APPLICATION OF SPACE DEPRIS MODELING

The algorithms and programs for solution of mentioned problems were developed. The software module was integrated into a complete set of programs, called "Space Debris Prediction and Analysis" (SDPA-model).

The first version of this model was ready at the end of 1992 [7]. Many studies were carried out with the help of this model [8,9,10,11,...]. The model has been used in a number of Russian organizations: Center for Program Studies, RSC "Enrgiya", TSNIIMASH, " Krasnaya Zvezda " and "Vympel". The second version is now in progress.

6.1. The reliability of results is provided by updating the parameters of above-mentioned algorithms from the known experimental data. The flux of fragments of various size, passing through the 50-km layer of Haystack radar's field of view at the 900 km altitude, was modeled.

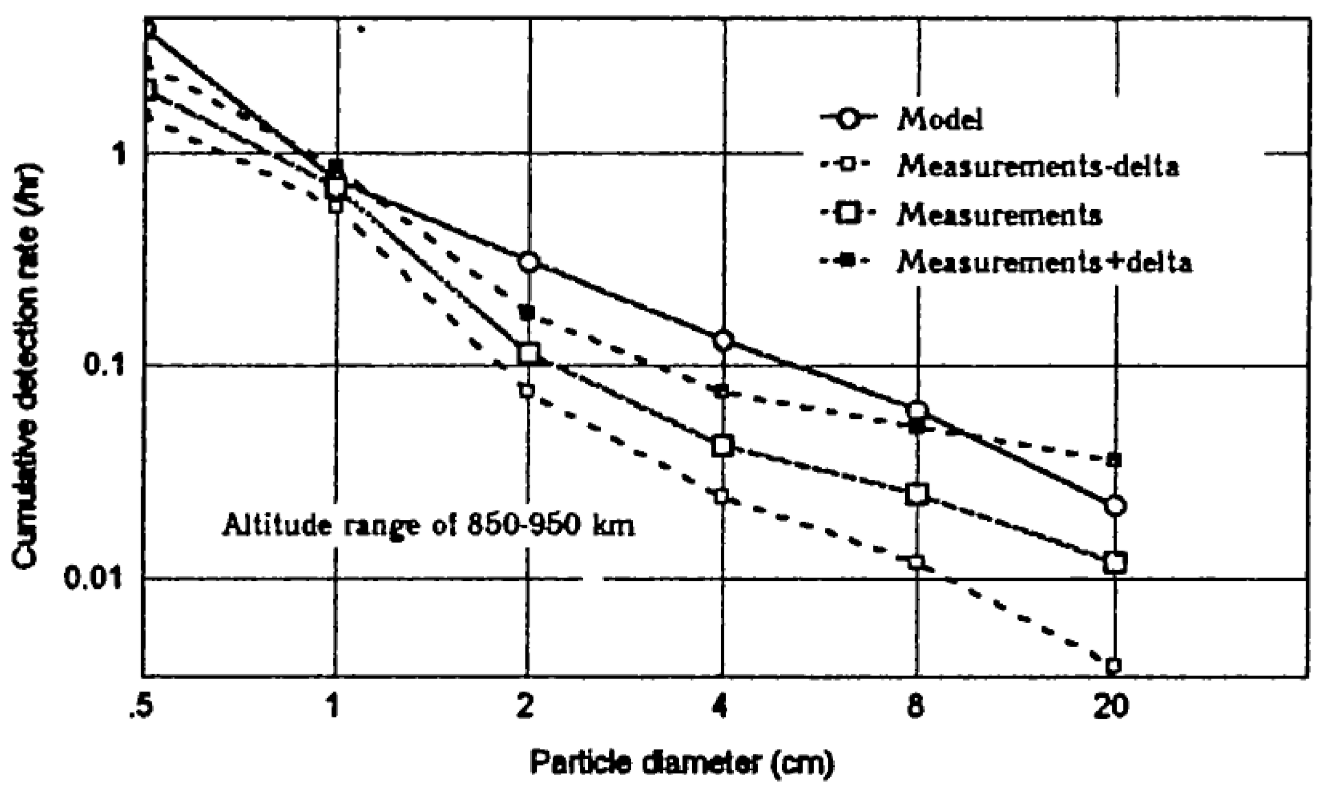


Figure 1. Cummulative measured and modeled flux of objects of various sizes for the altitude range of 850-950 km

The results were compared with the experimental data on a flux, published in paper [12] and related to the zenithal direction of radar's axis. These data are presented by the authors for observations in different years separately (5 plots for the years 1990 to 1994, respectively). For further use in the analysis we have taken the mean estimations, as well as the maximum and minimum values of a measured flux. One should note that for some sizes the authors attend their experimental estimations of the flux with corresponding standard deviations. These deviations grow with increasing of object's size and, as a rule, greatly exceed the actual scatter of estimations for different years. The standard deviations data were taken into account in the analysis.

The model and experimental data are presented in Figure 1. *They show that for fragments larger than 1 cm in size, as well as for fragments sizing larger than 8 cm, the model and experimental estimations of the flux are in a good agreement. For the other sizes of fragments the model data slightly exceed experimental ones, but in all cases they lie within the region of possible standard deviations.*

6.2. Current Space Debris Environment. Table 1 presents the total number of objects of various sizes with perigee heights in the altitude range of 400 to 2000 km.

Total SD number for the end of 1996 Table 1

No. of range, j	1	2	3	4	5	6
Interval, cm	from 0.5 to 1.0	from 1.0 to 2.0	from 2.0 to 4.0	from 4. to 8.0	from 8.0 to 20	more than 20
Number of SOs	1130000	152 900	64 500	26 600	14 300	6 000

The data on a current distribution of perigee heights for fragments of various sizes were recalculated into corresponding values of SOs concentration. The obtained results are presented in Fig. 2.

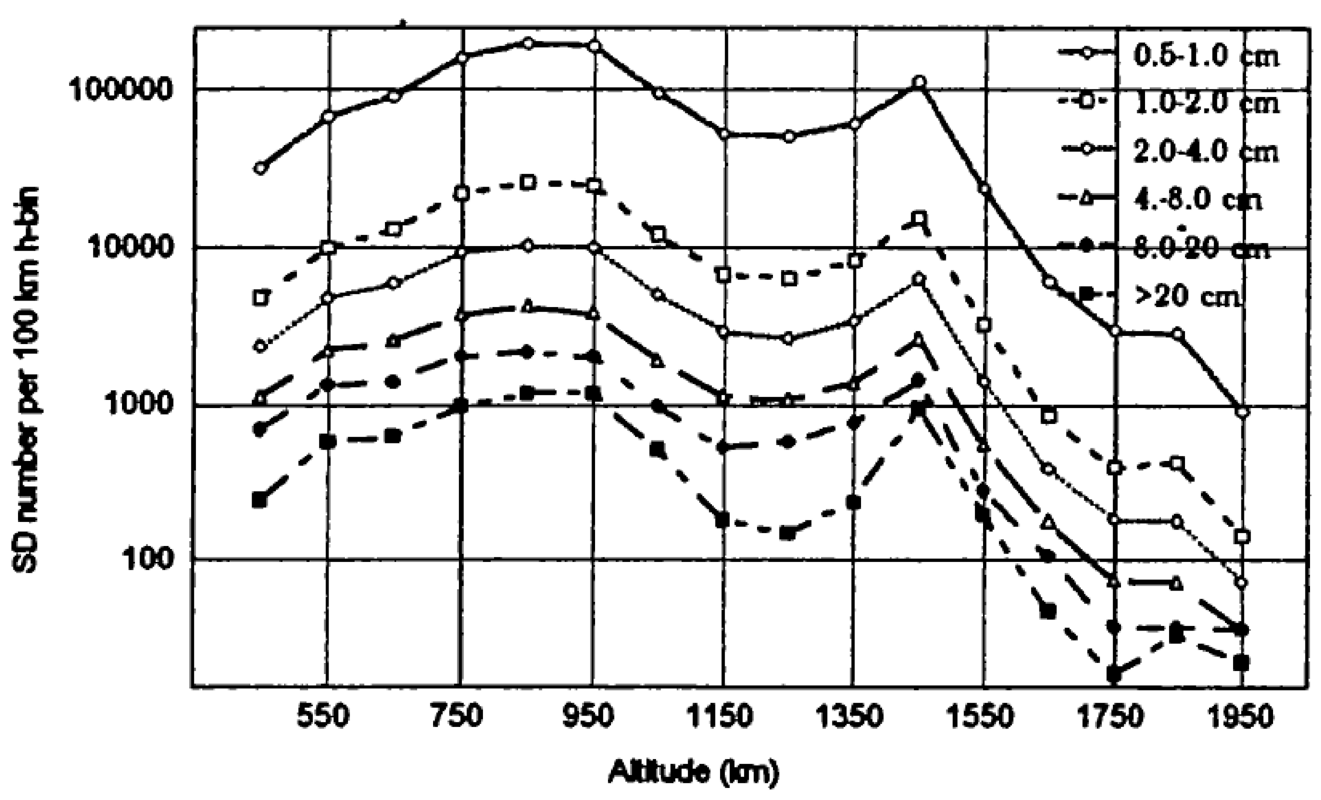


Figure 2. Distribution of perigee altitudes of SD various sizes, 1996

6.3. Prediction of the Environment. The analysis of the evolution of SD altitude distribution was carried out for particles sizing larger than 1 cm. 4 scenarios of future technical policy were considered here.

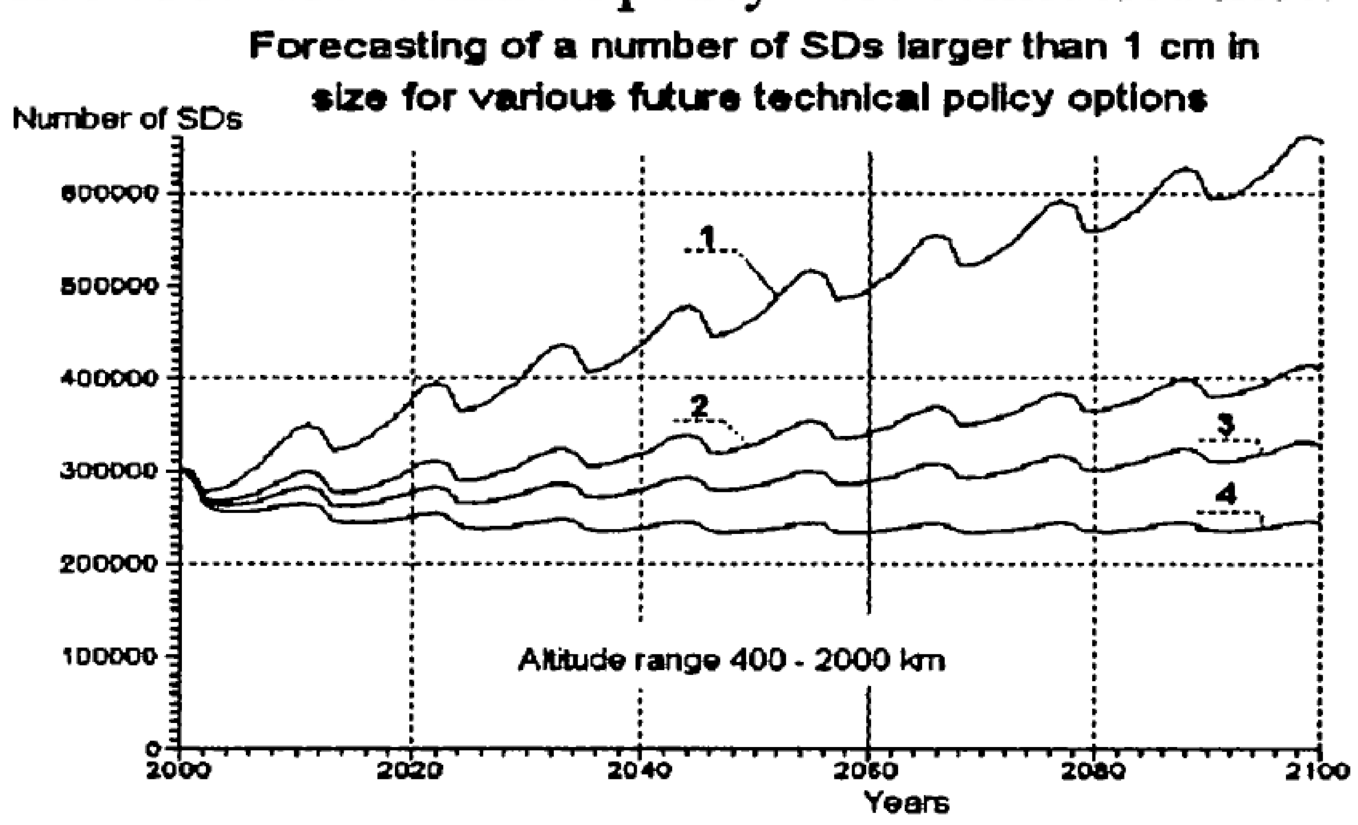


Figure 1.

Scenario 1. The intensity of launches and SD fragmentation is preserved the same as in 1991-1995 years ("Business-as-usual").

Scenario 2. The average number of explosions is 5 times reduced, whereas both the intensity of launches and the number of technological fragments are preserved.

Scenario 3. The average number of explosions is reduced down to 0, whereas both the intensity of launches and the number of technological fragments are preserved ("No-explosions").

Scenario 4. The number of technological fragments (remained in space) is 2 times reduced with the intensity of launches preserved; the explosions are completely excluded ("Deorbiting in 2000").

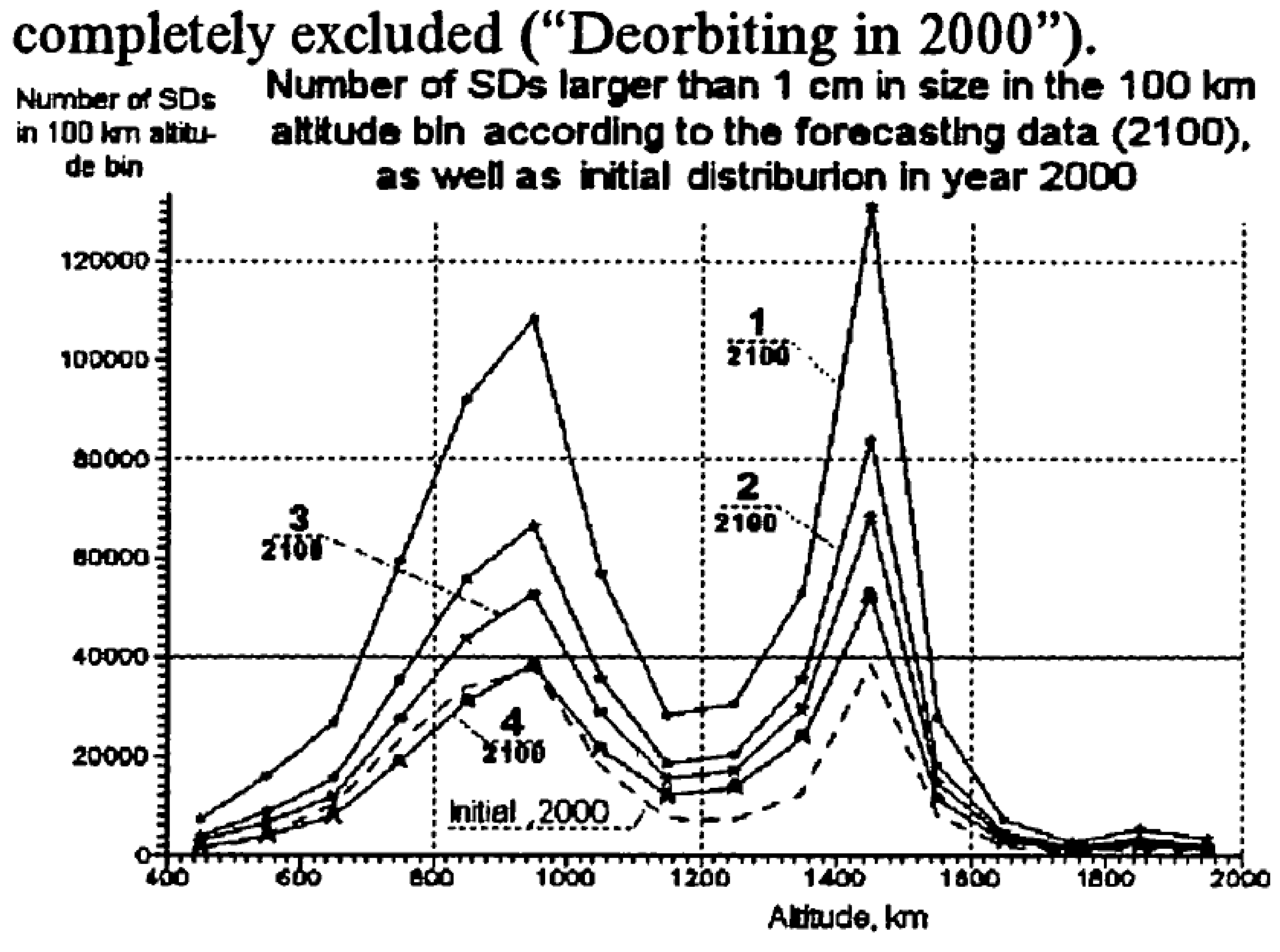


Figure 2

The environment forecasting was carried out up to the year 2100. The results are presented in Figures 3 and 4. The change of SD number for particles sizing larger than 1 cm in the altitude range of 400 - 2000 km is shown in the first of Figures. The resulting data indicate, that the SD number grows in all scenarios, except the latter. The greatest growth (more, than 2 times) takes place, naturally, under the first scenario conditions. For Scenario 4 some decrease of SD number will be achieved in comparison with the initial level of technogeneous pollution in 2000.

The expected altitude distributions of SD in 2100 and the initial distribution in 2000 are shown in Figure 4 for all scenarios considered. These data give rise to the important, in our opinion, conclusion, that *the growth of SD number at altitudes higher than*

1000 km will be kept even in case of the strongest decrease of SD formation intensity (Scenario 4).

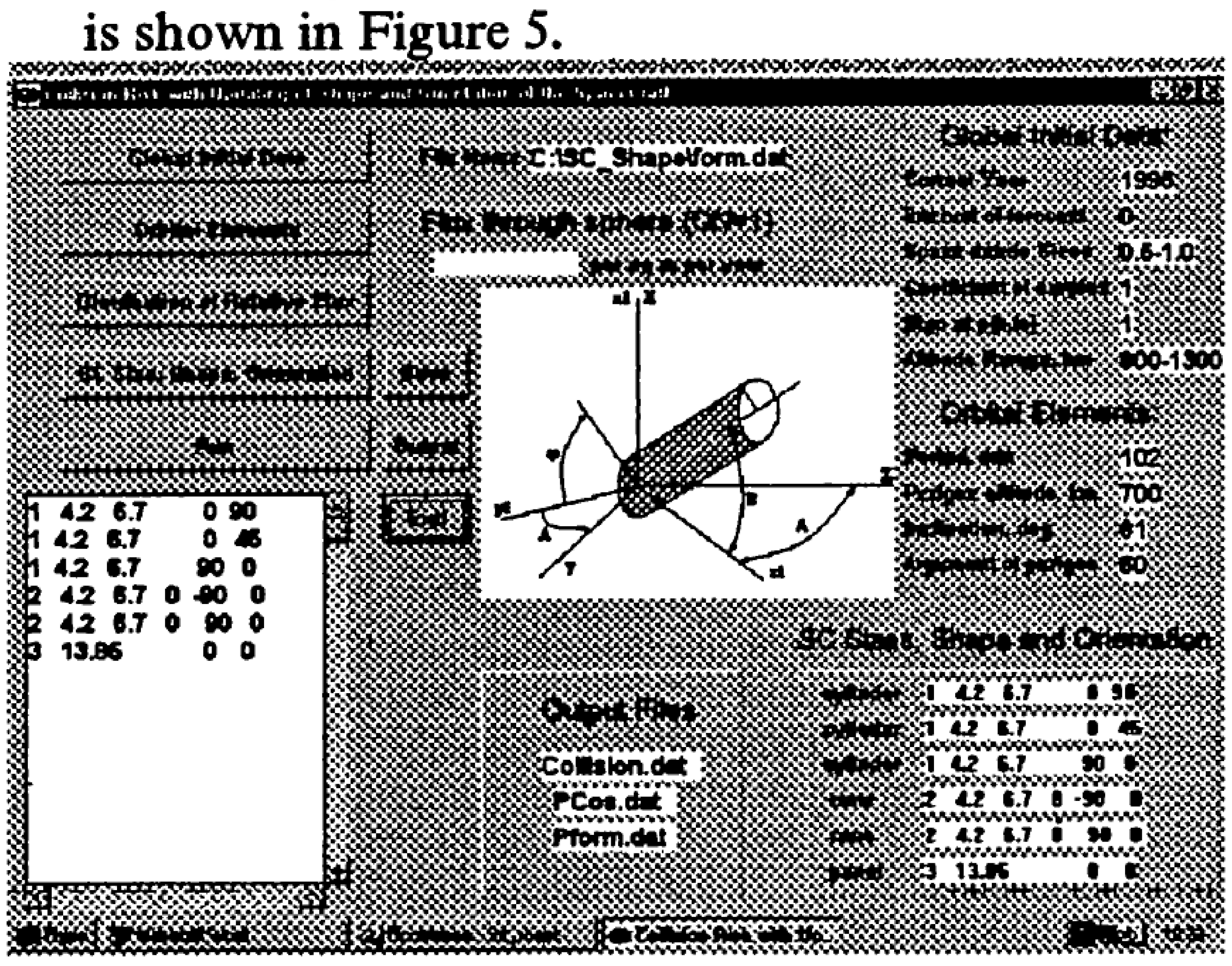
6.4. General characteristics of SDPA-model: this is a semi-analytical stochastic model for medium- and long-term forecast of the LEO debris environment, for construction of spatial density and velocity distributions as well as for the risk evaluation.

Principal characteristics of SDPA-Model

Table 2

No	Debris modeling characteristics	Comments
1	Range of altitudes	up to 2000 km.
2	Range of sizes	more than 0.6 mm.
3	Forecast of the environment	Fulfilled
4	Description of debris sources	Averaged
5	Construction of a spatial density distribution	Fulfilled
6	Construction of a statistical velocity distribution	Fulfilled
7	Estimation of possible collisions' characteristics	Fulfilled for the given spacecraft
8	Account of spacecraft shape and orientation	Fulfilled
9	Modeling method	Semi-analytical construction of statistical distributions
10	Users' possibilities for solving the above mentioned problems (items 5 –7}	Fulfilled
11	Information for updating the model parameters	Russian and US catalogue data, Haystack Radar measurements, the average flux function
12	Computer implementation	PC
13	Solution time	About 1 --2 minutes
14	Possibilities of using	Software

6.5. The example of one of sections of the model menu is shown in Figure 5.



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