LOW-PERIGEE SATELLITE CATALOG MAINTENANCE: ISSUES OF METHODOLOGY

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ABSTRACT

In Russia the main source of information on orbital and ballistic characteristics of man-made Earth satellites is the catalog of these objects, maintained by the Space Surveillance System (Ref.1). Maintenance of this catalog is performed in real-time scale by complex automatic system, which includes network of sensors and software tools for processing acquired data in automatic and interactive modes. General composition and characteristics of this system with regard to satellites in low orbits were described in Ref.2. Here we will treat the architecture of the algorithms, forming the basis of the system. Our attention will be concentrated mainly not on the algorithms by themselves, but on the basic initial conditions and assumptions for their development, key methodological notions, character of the main tasks to be solved and considerations of principal possibilities to find necessary solutions. Presented analysis is concluded by brief characterization of the main specific features of the algorithms employed in Russian Space Surveillance System for satellite catalog maintenance.

1. SENSORS

The main source of data for solving the task of satellites' catalogization are detection radars of the Early Warning and BMD systems. Certain characteristics of these sensors are presented in table 1 (Refs.3,4).

location	coordinates	type	system	sector in azimuth	band
Irkutsk, Russia	103° W, 53° N	Dnepr	EWS	$30^\circ \div 300^\circ$	VHF
Balkhash, Kazakh.	74° W, 45° N	Dnepr	EWS	$30^\circ \div 330^\circ$	VHF
Murmansk, Russia	40° W, 68° N	Dnepr	EWS	$295^\circ \div 355^\circ$	VHF
Riga, Latvia	22° W, 57° N	Dnepr	EWS	$220^\circ \div 310^\circ$	VHF
Sevastopol, Ukraine	33° W, 44° N	Dnepr	EWS	$140^\circ \div 260^\circ$	VHF
Uhzgorod, Ukraine	23° W, 48° N	Dnepr	EWS	$165^\circ \div 285^\circ$	VHF
Petchora, Russia	57° W, 65° N	Darial	EWS	$300^\circ \div 0^\circ$ $0^\circ \div 55^\circ$	VHF
Mingechaur, Azerb.	48° W, 41° N	Darial	EWS	$105^\circ \div 215^\circ$	VHF
Moscow, Russia	37° W, 55° N	Dunai	BMD	$255^\circ \div 305^\circ$ $65^\circ \div 120^\circ$	UHF

Table 1. Characteristics of the sensors

Note **the main specific features of this network.**

All sensors work in detection mode i.e. continiously observe the space within their fields of view and receive reflected signals, exceeding the threshold level, which form single measurements. Scanning is performed by each radar according to certain fixed programm. It means that on-site detection process is not under the control of the Center, responsible for catalog maintenance.

All the sensors are located within the territory of the former USSR in boundary and central regions. Their geographic locations cover 12% of latitude and 22% of longitude ranges and sectors of view are limited in azimuth, elevation angle and range. Thus **complete surveillance of low-perigee satellites is not ensured.** Maximal gaps in observations of low orbit objects of meter size, moving in circular orbits with inclinations exceeding 30°, are 12 hours a day (Ref.1).

Only one of the sensors (Dunai radar, located in Moscow region) is a UHF radar, the others are VHF radars. Observation conditions for objects with sizes less than 20-30 cm[1] are essentially different for UHF and VHF radars. Some of these satellites can be observed only by UHF radars. Thus, **satellites, observed by only one sensor, do exist.** In this case the range of latitude arguments u of observed parts of the orbit does not exceed $5^\circ \div 7^\circ$, i.e. covers $1.5 \div 2\%$ of the range of u. If observation conditions do not let the radar observe the satellite in ascending and descending revolutions, the total range of observed latitude arguments δu_{tot} may deviate from this value insignificantly.

2. MEASUREMENTS

Performing a set of single observations of the target the radar obtains the "marks" - **single measurements** of certain parameters along the track of a satellite. Usually a radar measures range D , azimuth ε , elevation angle γ and sometimes radial velocity $\dot{D}$ in local radar's coordinate system (Ref.5).

Acquired single measurements are "smoothed" for the interval t_{tr} ($t_{tr} \leq 50 \div 100$s (Ref.1)), producing as a result estimation of object's parameters or **measurement (observation)** $\mathbf{x} = (D, \varepsilon, \gamma, \dot{D}, \dot{\varepsilon}, \dot{\gamma})$.

Any radar measurement $\mathbf{x}$ contains data on orbital parameters $\mathbf{a}$ of the satellite, that produced it, since

$$\mathbf{x} = \mathbf{f}(\mathbf{a}) + \delta\mathbf{x}, \qquad (1)$$

where $\mathbf{f}(\mathbf{a})$ - functional relationship between orbital parameters and parameters of the measurement, $\delta\mathbf{x}$ - observation's error.

Design of measurements' processing procedures most essentially depends on the accepted **model of measurements' errors.** The following model is used to describe real errors.

- The value $\delta\mathbf{x}$ is presented as
$$\delta\mathbf{x} = \delta\mathbf{x}_{nr} + \delta\mathbf{x}_{an}, \qquad (2)$$
where $\delta\mathbf{x}_{nr}$ and $\delta\mathbf{x}_{an}$ - are **normal** and **abnormal** components respectively,
$\delta\mathbf{x}_{nr} = (\delta D_{nr}, \delta\varepsilon_{nr}, \delta\gamma_{nr}, \delta\dot{D}_{nr}, \delta\dot{\varepsilon}_{nr}, \delta\dot{\gamma}_{nr})$,
$\delta\mathbf{x}_{an} = (\delta D_{an}, \delta\varepsilon_{an}, \delta\gamma_{an}, \delta\dot{D}_{an}, \delta\dot{\varepsilon}_{an}, \delta\dot{\gamma}_{an})$.
- **Normal** component $\delta x_{i,nr}$ is present in each radar parameter x_i. It has Gaussian distribution with the mean m_i and root-mean-square deviation σ_i ($i = 1, \ldots 6$). Parameters m_i and σ_i usualy are unknown and satisfy the conditions $m_i \leq m_{i,max}$, $\sigma_i \leq \sigma_{i,max}$, where $m_{i,max}$, $\sigma_{i,max}$ - known constants.

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[1] The US satellite catalog comprises more than 2000 such objects.

- **Abnormal** component $\delta x_{i,an}$ satisfies the conditions $\delta x_{i,an} \gg \delta x_{i,nor}$, $|\delta x_{i,an}| \le \delta x_{i,max}$, where $\delta x_{i,max}$ - known constants. Its probability distributions usually are unknown. The mistake $\delta x_{i,an}$ is present only with certain probability $p_{i,an}$ and $p_{i,an} \ll 1$.

Let us note some features of this model.

- Systematic derivations m_i of normal component may be substancially greater than those of mean square σ_i.
- Probability of abnormal error in any of radar components does not exceed 0.1 as a rule.
- 21 combinations (out of 64 possible) of the six-dimensional vector $\mathbf{x}$ components can be abnormal simultaneuosly.
- The number of abnormal components is not more than three.
- Model's parameters $\mathbf{m}$ and σ depend on time and radar coordinates.
- Values $m_{i,max}, \sigma_{i,max}, \delta x_{i,max}$ for each sensor were managed to be chosen so that the resulting error does not exceed 0.001[2].

Regarding the **magnitudes of observation errors** the following can be mentioned.

Different parameters are not equally accurate. The most accurate is the range. Errors in azimuth and elevation angle (transformed to linear measure) are as a rule much greater. Similar ralationship is valid for velocity components.

Probabilities of arrival of abnormal errors are different for different parameters. The most consistent is the range. The least accurate components - elevation angle and its rate are subjected to distortions to the most great extent.

Among six orbital parameters three are determined most accurately on the basis of measurement: orbital inclination i, longitude of ascending node Ω and orbital period T. Approximate ralationships for the errors $\delta i\{degree\}$, $\delta\Omega\{degree\}$, $\delta T\{minute\}$ of i, Ω, T determination are as follows[3]:

$$\begin{aligned} \delta i &\approx 0.0075D|\delta\mathbf{D}_0| + 0.6(D\delta\ddot{D} + \\ &\quad + 0.008D|\delta\mathbf{D}_0|)\cos u \\ \delta\Omega &\approx 0.0075D|\delta\mathbf{D}_0| + 0.6(D\delta\ddot{D} + \\ &\quad + 0.008D|\delta\mathbf{D}_0|)\sin u \sin i \\ \delta T &\approx 6\dot{D}\delta\dot{D} + 3D\delta\ddot{D} + 0.025D|\delta\mathbf{D}_0|, \end{aligned} \tag{3}$$

where u - argument of latitude for the point in orbit, in which vicinity the measurement is fulfilled; i - orbital inclination; $|\delta\mathbf{D}_0|$ - error of evaluating direction to the object on the basis of measurement (error of the most roughly measured angular component); $\delta\ddot{D}$ - error in determining acceleration using the range, its upper limit evaluated using either the formula $\delta\ddot{D} = 2\cdot\tilde{\delta}\dot{D}/t_{tr}$, in case the radar measures $\dot{D}$ ($\tilde{\delta}\dot{D}$ - maximal error of single measurements of $\dot{D}$ not taking into account systematic component) or using the formula $\delta\ddot{D} = 16\cdot\bar{\delta}D/t_{tr}^2$, in case $\dot{D}$ is not measured ($\bar{\delta}D$ - maximal error of D single measurements, not taking into account the component, linearly evoluting with time).

It follows from (3) that the magnitude of errors of i, Ω, T determination on the basis of measurement for objects of meter size is ≈ 1% of the measured value, i.e. ≈1° in i and Ω and ≈1 minute in T.

3. TRACKING

Key aspect is the notion of the trackability of a satellite. This notion formally is not to be dependent on the radars, since their operation is not under the control of the Center, maintaining the catalog.

We consider the object **trackable**, in case it is

- **observable**, i.e. its observations are fulfilled regularly;
- **separable**, i.e. its observations can be selected on the background of other measurements.

Thus, tracking of observed satellite is influenced by other also observed objects.

If the object is trackable, then it is principally possible to determine its parameters using all attributed observations' data, thus providing calculation of its position for arbitrary moment (with certain accuracy).

Regular tracking of a satellite means periodically repeating procedures of correlating new measurements with this satellite and updating its orbital parameters using these measurements. This will be called **tracking process**.

Now we will formulate **condition of trackability (tracking)** of a satellite. Its significance requires rather detailed consideration.

Assume, that all the measurements for given object are separated and on their basis the orbit is determined. Let a new observation $\mathbf{x}$ is acquired for this object. Than to obtain correct correlation decision this measurement must be closer to the "own" object than to the others also having the orbits determined on the basis of measurements. "Distance" between the orbit and the measurement is determined by the differences between the measured and calculated using orbital parameters components of the observation.

Calculation of measurement's components on the basis of the orbit means propagation of the elset to the moment of measurement $\mathbf{x}$ and transformation of obtained data to parameters of the measurement.

If all the measurements and propagated elsets were not subjected to errors, the residuals of measurements with "own" orbits would be equal to zero and allocation of measurements to the objects won't pose any problems. Thus the residuals are determined by the errors of measurements and predicted orbit. Measurements' errors at least for one of measured parameters are essentially smaller than distances between neighbouring satellites, otherwise satisfactory selection of single measurements in course of on-site tracking and acquisition of measurements would become impossible. Thus, **the cause of mistakes in allocation of radar measurements to orbits of tracked satellites can be only the errors of predicted orbit.**

Orbital parameters for any object are obtained on the basis of the set of measurements, previously performed on this object by all the radars, participating in surveillance process. The employed for orbits' determination algorithms seek for the orbit, in certain sense, best to "inscribe" into these measurements.

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[2] I.e. 99.9% of observations have errors, corresponding to accepted model.

[3] These evaluations are obtained for satellites with orbital altitudes $h \le 3000$ km and eccentricities $e \le 0.1$, and also for observed by the radar at ranges $D \le 2000$ km and radiation directions, deviating from the main (normal to antenna's plane) not more than for 30°. Their derivation uses known geometrical and dinamical relationships (Refs. 5,6).

To calculate residuals with future measurements this orbit is propagated to their moments. Thus the errors of predicted orbit depend on

- errors of determinating orbital parameters on the basis of measurements,
- errors of propagating this orbit.

These two sources of errors interact with each other, producing the **error of orbit's determination and prediction.**

So, condition of trackability assumes **small errors of orbits' determination and prediction in comparison with distances between them.** In other terms **alien measurements are not to "insribe" into satellite orbit.** We will call it **informativity condition.**

Let us evaluate **accuracy of orbit's determination and prediction, ensuring fulfillment of informativity condition.**

On one hand, maximal density of satellites, regularly tracked by US and Russian sensors, is $\approx 10^{-7}$ objects within km^3 (Ref.7), i.e. ≈ 1 one satellite within a volume of 10^7 km^3, and density variations within altitude range 400-3000 km is three orders of magnitude. On the other hand, uncertainty domain, resulting from orbit's propagation, has the shape stretched along velocity vector, since the error in directions, normal to velocity vector is approximately e times smaller, where e - orbital eccentricity. For 90% of tracked satellites $e < 0.1$. Not taking into account objects with $e > 0.1$ and assuming that the volume of produced by propagation uncertainty domain is to be of the order of magnitude smaller than the volume, where one satellite can be found, we have the following condition for acceptable along the track prediction error δ_v: $0.1{\cdot}10^7 = (0.1{\cdot}\delta_v){\cdot}(0.1{\cdot}\delta_v){\cdot}\delta_v$. And hence $\delta_v \approx 500$ km. Transformed to time this is ≈ 1 minute along the track of a satellite. This estimation is obtained for most "populated" altitudes $\approx 800 \div 1000$ km and $\approx 1400 \div 1500$ km. For other altitudes acceptable errors are greater.

Let us evaluate **how informativity condition is satisfied.**

For propagating the orbit for M revolutions the error δ_v is evaluated using the formula:

$$\delta_v = M\delta T + 0.5M^2\delta(\Delta T), \qquad (4)$$

where δT and $\delta(\Delta T)$ - errors of determination of orbital period T and its rate (per revolution) ΔT.

Understanding of the accuracy of T determination using one measurement is provided by formula (3). As we have mentioned earlier typical value of δT is 1 minute. In case we have several measurements, distanced in general for N revolutions, the upper limit for δT can be estimated, using the formula $\delta T = 2{\cdot}\delta t_u/N$, where δt_u - maximal error of determinating the moment t_u of satellite's passing the point with latitude argument u, calculated using one measurement. Usually the object is observed in different revolutions with close latitude argument, thus the value of u can be chosen, for which δt_u is of the order of 1 s.

The value of ΔT depends on satellite's altitude over the Earth surface h and its ballistic coeffficient k_b. The values of ΔT (in minutes) can be given by approximate formula

$$log_{10}|\Delta T| \approx log_{10}\left(\frac{k_b}{0.01}\right) - \frac{h}{100}, \qquad (5)$$

where h - altitude in km, k_b - ballistic coefficient in m^2/kg (typical values are ≈ 0.01). The value of ΔT principally can not be determined as accurately as we desire since $\delta(\Delta T)$ is defined by the error of the used model of atmospheric density for prediction interval, that ranges from several percent up to several times (typical values are $\approx 10\%$) (Ref.8).

Analysis of this situation brings to the following.

- The orbit, determined using one measurement, allows to track the object in case it is observed in each revolution. The sensors do not provide this possibility. Hence **estimation of orbital parameters for any satellite requires to combine the data acquired for different revolutions.**
- **The orbit, determined using several measurements for different revolutions, allows to track satellites observed daily (with $h >$ 300 km and $k_b < 1$ m^2/kg).**
- Observation conditions for satellite in near circular orbit depend on its altitude h. With increase of h the range of object's pass through radar's field of view usually increases and observation conditions become worse. On the other hand with the increase of h prediction errors decrease and acceptable prediction interval increases. In altitudes greater than 600 km (and $k_b < 1$ m^2/kg) a satellite can be tracked even in case only one measurement a month is obtained. That is why **regularly (however, may be seldom) observed satellites in near circular orbits can be tracked.**
- In case a satellite is in orbit with eccentricity, that can change observation conditions for various values of perigee argument, the measurements on this object may be irregular and long gaps in observations become possible. In this case tracking of satellite using the measurements may become impossible and **break of tracking** occures. However this situation is temporary. When new data on this satellite arrive after a long gap, trackability condition for the set of measurements after the gap may be satisfied and the object principally, may be tracked again.

Above considerations treated "stationary" situation, when no new observed objects arrive in space.

Launches, separations, break-ups as well as arrival of new sensors and modernizations of existing ones also produce disturbances of informativity condition. Here disturbance of informativity condition is also of local (only for certain domain of parameters) and temporary (only for limited intervals) character. After certain time new objects will depart for significasnt distances and acquired observations will principally allow to determine their orbits with accuracy, required for tracking.

Here one more important notion is to be introduced. It will be treated in the next section.

Conclusions follow.

- **Informativity condition is indispensable condition for tracking a satellite. For observed objects it is normally satisfied.**
- **For certain satellites informativity condition may be disturbed for some intervals. Breaks of tracking may occure in this case for tracked satellites.**
- **In case observations' data on new or not tracked (but tracked previously) satellite arrive, it must be stored until informativity condition is satisfied.**

4. PRIMARY DETERMINATION OF ORBITS

We already mentioned that the orbit determined using one measurement does not ensure satisfactory separation of future observations' data on this object. To obtain more accurate orbit we are to combine several measurements. In case the object is tracked this orbit is available. However its accuracy decreases with time. Therefore it is to be updated using new received observations. If the object is not tracked, principally new task is to be solved - **primary determination of the orbit** or **orbit's detection**.

Informativity condition must be satisfied to solve this task. This condition is satisfied in case the data, present in measurements of this object, principally allow to determine its orbit with accuracy making the errors of determination and prediction essentially smaller than the distances to other observed objects so that "alien" measurements won't "inscribe" into the orbit.

For primary determination of orbit two measurements for different revolutions are not enough, since the chances to take two measurements produced by two objects are too great[4]. We need to have at least **three measurements for three different revolutions (triplet)**. The orbit determined using these measurements is much more accurate than the orbit produced by one measurement and the chances that three measurements inscribed into one orbit are produced by different objects are essentially smaller than for two measurements.

In reality for primary determination of orbits we need:

- to find associating triplet among the measurements, not correlated to tracked objects;
- using discovered triplet, determine the orbit.

Let us analyze **principal possibilities to solve this task**[5].

The main parameter, defining this possibility is the **uncertainty in the number of revolutions N between the boundary (with regard to reference moments) measurements of the triplet**.

Let $k_{ud} = [N\frac{|\delta\tilde{T}|}{T}] < 0.5(p+q)$, where $\delta\tilde{T}$ - the error in period $\tilde{T}$, determined using one measurement; $[A]$ - entire of A, rounded-off; p and q - minimal integer positives, satisfying the condition $p/q = P/Q$; P - the number of revolutions between boundary and intermediate measurements, $Q = N - P$. Then no uncertainty exists. This condition will be called **unambiguity condition**.

When the unambiguity condition is satisfied for the set $\mathbf{S}_N$ of possible values of N (taking into account uncertainty coefficient k_{ud})[6] the unique value $\hat{N}$ exists, when parameters $t_u^{(1)}$, $t_u^{(2)}$, $t_u^{(3)}$ are matched ($t_u^{(1)}$, $t_u^{(2)}$, $t_u^{(3)}$ - calculated using the measurements of the triplet moments of satellite's arrival at a certain latitude argument u for the revolutions, corresponding to these measurements).

The value $\hat{N}$ is **true number of revolutions** between boundary measurements of the triplet and corresponds to the estimation $\hat{T}$ of T, given by $\hat{T} = |t_u^{(1)} - t_u^{(3)}|/\hat{N}$, which accuracy exceeds accuracy of $\tilde{T}$ by 1-2 orders of magnitude.

Consider the example. Let $|\delta\tilde{T}| < 1$ minutes and $T = 100$ minutes. Then for $N < 50$ ($|t_u^{(1)} - t_u^{(3)}| < 3.5$ days) $k_{ud} = 0$ and $\tilde{N} = \hat{N}$. Unambiguity condition is satisfied irrespectively of relative position of the intermediate measurement with regard to the boundary ones. Let now $N = 200$ (two weeks between boundary measurements) and $P = 100$. Then $k_{ud} = 2$, $p = q = 1$ and unambiguity condition is not satisfied. In this case $\mathbf{S}_N = \{\tilde{N}-2, \tilde{N}-1, \tilde{N}, \tilde{N}+1, \tilde{N}+2\}$, step of uncertainty $p+q$ is two (corresponds to the step in period ≈1 minute) and within the set $\mathbf{S}_N$ two or three possible values of $\hat{N}$, when $t_u^{(1)}, t_u^{(2)}, t_u^{(3)}$ are matched, do exist. Each of them correspond to its own estimation $\hat{T}$ with error $|\delta\hat{T}| \approx 0.01$ minute.

If uncertainty in the number of revolutions between boundary measurements of the triplet exists several primary orbits can be obtained, equally well inscribing into the measurements. In this case none of them can be found preferable and orbits' detection task won't be solved. Solution may be found some time after arrival of new measurements on the object. To be successful we need:

- informativity condition is to be satisfied for the interval of these observations;
- additional observations' data is to remove the uncertainty in the number of revolutions between boundary measurements.

Conclusions.

- **Possibility of solving the task of orbit's detection for certain satellite depends on its observability.**
- **Detection of daily observed satellites is possible in case they are observed by at least one sensor.**
- **Rarely (less than one time a day) observed objects principally can be detected provided certain arrangement of measurements within the interval of observations and fulfillment of informativity condition.**

5. IDENTIFICATION

After primary determination of orbit the origin of detected satellite is to be determined. This is the task of **identification**. Obtained orbit (in case it is enough reliable) may be either the orbit of new satellite, not previously tracked, or the orbit of the object tracked before, but lost due to break of informativity condition caused by absence of observations for a long time. In the first case the sourse of the object is to be determined, for example launch, break-up or separation. In the second case predecessor is to be identified.

Let us evaluate **the possibilities to solve this task**.

Identification task **is principally solvable**, in case informativity condition is satisfied for the set of possible hypotheses for the origin of the new object. In other words, regarding parameters used for identification the "distance" to the true hypothesis is to be significantly smaller than the distances to alternative ones.

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[4] Especially for multi-element launches and break-ups, when the orbits of different elements are close.

[5] For the task of primary orbits' determination the computation's aspects are also important, since decision making procedures are rather time-comsuming. We won't deal with these issues here. Assume that arbitrary complex procedures are realizable.

[6] $\mathbf{S}_N = \{\tilde{N} + i,\ i = 0, \pm 1, \pm 2, ..., \pm k_{ud}\}$, where $\tilde{N} = [|t_u^{(1)} - t_u^{(3)}|/\tilde{T}]$.

Accuracy of identification depends on delay $\tau_d = t_{det} - t_{in}$, where t_{in} - the time of newly detected object arrival in space or the time when previously tracked satellite was lost; t_{det} - the time of object's detection and start of its tracking.

If τ_d is small, identification with the origin does not pose serious difficulties, since first, rather complete information on space operations is published and second, the orbital data allows simple and acurate determination of satellite's parameters for the moment of its arrival t_{in} (for identification task parameters i, Ω and spatial position for this moment are of major interst).

With the increase of τ_d the error of determination of satellite's parameters for the moment t_{in} increases and we approach the moment, when correct determination of the origin immediately or after a short time after detection becomes difficult or impossible.

If **detected object is a "prolongation" of the old one**, then for making correct identification decision we need:

- parameters of detected object "inscribe" into the orbit of the old one;
- with regard to objects' parameters the "distance" (taking into account propagation errors) between the old and detected object is essentially smaller than the distance between various previously tracked satellites.

Computation of this distance is fulfilled propagating parameters of the old satellite (it is more accurate since it was tracked for longer period) to the epoch of detected object. Prediction intervals may reach several years and we are to have prediction procedures good enough for such intervals. Principally this task is solvable in case old object was tracked for sufficiently long period and within the interval from its first detection up to its last loss rather complete pattern of orbital parameters' variations is available. In this case we can approximate temporal evolution of orbital elements with fragments of Fourier-Taylor series and perform predictions for the old object using this approximation. To fulfill these operations **global archive of orbital data** for all satellites, for which breaks of tracking are possible, is needed.

If **detected object is not identified with any of the old ones and it can not be affiliated to recent launches and break-ups**, most likely it is a fragment of one of old launches or break-ups. Possibilities of identification in this case mostly depends on accuracy of determination of Ω for the moment of assumed arrival (moments of launch or break-up). In its turn this accuracy is determined by duration of detected object's tracking. Sometimes several years of tracking are needed to determine the origin and the moment of satellite's arrival, that took place 20-30 years before the moment of detection. Here long-term prediction of Ω, similar to the cases of objects' loss, is performed using approximating functions, calculated on the basis of orbital data for new satellite for the whole interval of its tracking.

Conclusions.

- **To be successful in identification of detected orbits, corresponding to satellites, recently (from several days up to several months) arrived in space, we need the data on major orbital elements of the parent and on the time of arrival.**
- **In case detected orbit is the orbit of satellite, residing in space for a long time, in addition we need global archive of orbital data for this satellite, stored for the interval, comparable to the interval when the satellite was not tracked.**

6. SPECIFIC FEATURES OF THE ALGORITHMS

Treated in the above sections initial conditions, input data and approaches to solving the task of satellite catalog maintenance lead to the following specific features of the algorithms used for this purpose (Refs.9,10).

Analysis of data sources and measurements of Russian network revealed the following. Certain limitations in observed altitudes, inclinations and orbital points do exist. Permanent control over low-perigee satellites is not ensured. Among the measurements of well observed meter-sized objects $\approx$10% of abnormal and rough are present and for small-sized satellites this percent is greater. The orbit determined using one non-abnormal measurement is not enough accurate for trackability.

However, using temporally parted measurements and existing accurate and consistent components, integral amount of data, present in all the measurements principally allows to track the major part of observable objects regularly or with certain gaps.

The algorithms must use this principal possibility, i.e. extract this information from the measurements and "incorporate" it into the orbits without essential losses.

The algorithms, based on **statistical decisions theory** (Ref.11) are informationally efficient.

Characteristic feature of considered task is uncertainty in statistical description of observations' errors and the "noise" of the system[7]. Thus we are to use special methods of statistical decisions theory, developed for this case (Ref.12).

In the scope of statistical decisions theory the task of catalog maintenance is formulated as a task of estimating the amount of observed satellites and their parameters using measurements' data. In case informativity condition is satisfied unique solution of this task adequate to real situation exists. Let us treat some features of the algorithm, realizing this solution.

- Final decisions on allocation of measurements to objects and estimation of orbital parameters (for tracking as well as for primary determination) are made using analysis in points of minimum of quadratic functionals $\Phi(\mathbf{a})$, constructed on normalized residuals of measured and computed values in "radar" parameters D, ε, γ, $\dot{D}$, $\dot{\varepsilon}$, $\dot{\gamma}$ for various objects. Together with estimation of "main" parameters[8] "interfering" parameters[9] are evaluated. Here **adaptive Bayes approach** is used.
- Selection of abnormal components of measurements is fulfilled using multi-pass minimization of $\Phi(\mathbf{a})$ with separation (in each pass) of abnormal components of all measurements using normalized residuals and with corrections of their weights[10] in $\Phi(\mathbf{a})$. Using results of this selection hypothesis of

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[7] errors in prediction of orbital parameters caused mainly by atmospheric "noise".

[8] satellites' orbital parameters.

[9] unknown parameters of measurements' errors for various sensors and propagation errors for various objects, subjected to essential atmospheric drag.

[10] coefficients of residuals' squares

non-predictable variations of orbital parameters[11] is tested.

- Decision regarding affiliating any measurement to a satellite is made using **minimax decision function**, providing acceptable probability of miss[12] under maximal values of errors in possible abnormal components. Parameters of this function are tuned to provide that in case informativity condition is satisfied probability of false decisions won't exceed the accuracy of the employed model of measurements' errors, i.e. 0.001. In particular, not greater than 0.001 in probability of unreliable decision[13] in tracking and in probability of miss.
- For motion predictions we use procedures with methodical errors not exceeding the maximum of two values: stochastic components of observations' errors and potentially achievable real errors of prediction.
- Minimization of $\Phi(\mathbf{a})$ functional uses specially developed techniques for making initial approximation and search for minimum, taking into account specific features of real measurements and providing determination of the minimum point under conditions of tracking and primary determination not less than in 99.9% and 90% of cases respectively.
- For primary determination of orbits we realized the procedure ensuring (for a set of up to 20000 non-correlated with tracked objects measurements), **exhaustive search** of observations' triplets with analysis (for each triplet) of all possible values of coefficient of uncertainty in the number of revolutions between its boundary measurements, not exceeding 40.
- Identification of detected orbit with lost objects is based on comparison with their last elsets. False identification of different objects and miss of identification are ensured to be of the level less than 0.1% and 10% respectively.
- Various computation techniques , reducing CPU-time without losses in quality of decision making and allowing to have computer code capable of processing arriving observations in real time scale are employed. The major are as follows:
  - preliminary allocation of arriving measurements to tracked satellites using **rough selection** of surely not correlating pairs (satellite-measurement) according to special three-step three pass scheme and **residuals with orbits** prior to their updating using these measurements ("input" residuals[14]);
  - preliminary selection of triplets of non-correlated measurements, surely not produced by one and the same object, using four parameters, most accurately calculated using individual measurements;
  - a set of techniques reducing CPU-time without loss of accuracy, employed in prediction procedures and minimization of functional $\Phi(\mathbf{a})$.

7. REFERENCES

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[11] These variations may be caused, for example by ignition of satellite's engines, break-up of the satellite, significant geomagnetic storms or others

[12] not affiliating of measurement to "own" object.

[13] The orbit is unreliable in case it is either not determined or it is determined but does not provide regular tracking of a satellite. Indispensable condition of reliability is the presence of at least three "inscribed" into the orbit measurements for different revolutions.

[14] contrary to residuals after minimization which can be called "output" ones and are used for making final decision regarding affiliation of measurements.