NUMERICAL SIMULATION OF FRAGMENTATION PROCESSES

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ABSTRACT

We present a new numerical scheme to simulate high–velocity impacts between solid bodies. The colliding objects are modeled as three–dimensional lattices of finite elements, which can interact with each other in both an elastic and a shock–wave regime. The finite elements are hard spheres with a given value for mass and radius, which can (i) scatter each other through inelastic collisions and (ii) influence the motion of their nearest neighbors by an elastic force, until the distance exceeds a given breakup threshold. Dissipative effects arise from a force term proportional to the velocity, with a given decay time. This model requires finding a plausible correspondence between the input parameters appearing in the force terms, and the physical properties of real solid materials. Despite the preliminary character of the code developed so far, we have obtained some interesting results, which appear to mimic in a realistic way the typical outcomes of actual impacts.

1. INTRODUCTION

High–speed collisions are the most important process affecting the evolution of the different populations of small solid bodies orbiting in the Solar System. This is true for both natural objects (asteroids, meteoroids, small satellites, ring particles, transneptunian comet–like objects) and the "cloud" of manmade debris orbiting our own planet, which is undergoing a collisional process with potentially troubling consequences for the future of space activities (Refs. 1,2). For different types of objects, impact velocities range from hundreds of m/s to several tens of km/s, but in all cases collisions can cause the catastrophic breakup of the target bodies, and the generation of swarms of new orbiting fragments.

Our current capability to model and predict in a quantitative way the outcomes of impact processes is still very limited. A common approach is the experimental one: high–speed impacts are carried out at laboratory sizes thanks to special guns, or simulated by suitable explosive techniques (Refs. 3 to 6). This method however is technically complex and costly both in time and resources, can explore a limited range of the relevant physical parameters (impact speed and angle, projectile and target size, composition and shape) and, most important, its results can be extrapolated to most problems of interest only through poorly known scaling relationships (see e.g. Refs. 3, 7).

Alternatively, one can resort to numerical simulations. A simple "semiempirical" numerical model, based on the observed geometry of the fragment ejection field, has been proposed by Paolicchi et al. (Ref. 8) and recently refined and extended (Ref. 9). This model has given encouraging results in reproducing experimental findings, but cannot be easily applied to targets having complex shapes and/or internal structures; moreover, the relationship between its basic assumptions and parameters and the detailed physical processes following a real hypervelocity impact (propagation and reflection of shock waves, formation of cracks, etc.) is only indirect, and some *a posteriori* parameter fit is required to match the observations from real impact events. Much more complex are the so–called "hydrocodes" (e.g. Refs. 10 to 12), and certainly closer to the ideal of a realistic physical modeling scheme, since they are based on the numerical integration of general hydrodynamical equations with suitable boundary conditions. However, in practice the hydrocodes have huge requirements in terms of computer time, and for targets dominated by solid–state strength, some empirical relationships allowing for the formation and propagation of flaws are required (e.g., those appearing in the widely used Grady–Kipp theory). Such relationships may oversimplify the behavior of real solid materials. Also, in general they contain numerical parameters which are known only in a very approximate way from experiments, and it is not clear how much the corresponding values may vary when different target materials, sizes and shapes are examined.

These considerations have prompted us to develop a new type of numerical model, which is based on very simple physical concepts, yet is capable of reproduc-

ing in a natural way all the main properties of solid bodies which are relevant in our context: the elastic behaviour resulting into the propagation of sound waves, a finite compressibility and a threshold beyond which deformation causes local fracturing, the generation of shock waves following impact, and energy dissipation effects. In practice, we have studied numerically the behaviour of a physical system having all the properties listed above, that is a lattice of finite mass elements having a hard–core radius (to take into account the effects of finite compressibility) and an inelastic scattering restitution coefficent (to properly simulate the shock wave regime), and interacting with their nearest neighbors through an elastic force supplemented by the hypotesis that whenever the amplitude of the oscillation exceeds a given value the bond disappears.

At present, our method is still in an early phase of development, and is not intended as a simulation tool for specific real cases, but rather as an instrument for examining the behaviour of the physical system described above to assess, first, whether it really resembles the observed processes caused by impacts at a qualitative level, and second, whether a reasonable choice of the input parameters can yield semi–quantitative agreement with the observed collisional outcomes. Once these conditions are met, our preliminary model can then be used to better understand the most important physical mechanisms which are at work in reality and to constrain the critical parameters. Following this methodology, as a first step we have studied two–dimensional systems (Ref. 13). Subsequently, the procedure has been extended to three-dimensional systems.

The remainder of this paper is organized as follows: in Sec. 2 the main physical assumptions underlying the model are summarized; in Sec. 3 we describe our current numerical code and present some results for a simple test case; and in Sec. 4 we draw some conclusions about the potential of this model and the refinements we are working to.

2. MAIN ASSUMPTIONS

In our scheme a solid body is modeled as a regular parallelepipedal lattice of finite mass elements interacting elastically (i.e., connected by springs) with their nearest neighbors. The elements are hard spheres of mass m and radius r_0, which can collide and scatter each other following the laws of inelastic scattering, with a given restitution coefficient η. In this way the finite compressibility of solid materials and the shock–wave regime occurring when the velocity of the elements is higher than the sound velocity can be reproduced.

The springs have an elastic constant k and an equilibrium length l_0. We recall that for such a system the sound velocity is $v_s = l_0\sqrt{k/m}$. Friction effects are taken into account in the equations of motion by means of a dissipative term proportional to the velocity and characterized by a decay time τ. Another parameter characterizing the springs is the breakup length $\bar{l}$. We assume that when the length of a spring exceeds this value the elastic bond disappears. The presence of pre–existing flaws in the material can be simulated by introducing a number of broken bonds at the starting time $t = 0$; in our current model we just assign the relative density of such broken bonds, whose location is then chosen at random.

We stress that this physical model is just the simplest one which we could devise in order to generate the main qualitative features which characterise the behavior of real solid objects following high–velocity impacts, as described above. With the same general method, other kinds of interaction between the mass elements can be investigated — in particular nonlinear, anelastic terms could be added to the elastic potential to better describe the behavior of real materials in the proximity of the breakup threshold.

Before fractures appear, the resulting expression for the force exterted by a mass element at position $\mathbf{r}_1$ on another at position $\mathbf{r}_2$ is:

$$\mathbf{F} = \begin{cases} \infty & \text{for } l \le 2r_0 \ , \\ -k(l - l_0)\,\mathbf{l}/l & \text{for } 2r_0 < l \le \bar{l} \ , \\ 0 & \text{for } l > \bar{l} \ , \end{cases} \qquad (1)$$

where $\mathbf{l} = (\mathbf{r}_1 - \mathbf{r}_2)$ and $l = |\mathbf{l}|$. Once a bond has been broken it disappears forever, even when the distance between the two elements falls again below $\bar{l}$. Moreover, we assumed that after a bond has disappeared the breakup length of all the neighbouring bonds may be reduced by some fixed factor, so that the next flaw is more likely to occur near a previous one.

The next step is the choice of the input parameters. Suppose that we want to simulate a sample of material whose density, sound velocity, and other "macroscopic" properties are known. First we have to assign the number of lattice elements in the x, y and z directions, N_x, N_y and N_z, respectively. It is clear that the model becomes more and more realistic as the number of elements grows (ideally, one should reach a regime in which any further increase in the number of elements does not affect the results any more); on the other hand, a simulation setup with too many elements involved may lead to unacceptable computational requirements.

The mass m of each element is related to the total mass M of the body by the straightforward relation

$$m = \frac{M}{N_x N_y N_z} \ , \qquad (2)$$

whereas the elastic constant k of the springs can be derived from the the sound velocity through

$$k = m\left(\frac{v_s}{l_0}\right)^2 \tag{3}$$

(typical sound velocities in solid materials range between 2 and 6 km/s — for example for steel $v_s \approx$ 5 km/s). If l_x, l_y and l_z are the dimensions of the body to be simulated, we can derive the equilibrium lattice spacing l_0 as:

$$l_0 = \frac{l_x}{N_x} = \frac{l_y}{N_y} = \frac{l_z}{N_z} \ . \tag{4}$$

Dissipative effects in the elastic regime are taken into account by assigning the decay time τ. A reasonable estimate for it can be obtained from the Q quality factor of the free vibrations of bodies made of the material under consideration; for instance, in the case of a piano string Q is $\approx 10^3$. More difficult is estimating a "realistic" value for the radius of the hard spheres. A possibility is that of considering the compressibility effects for solids, taking into account the relative volume reduction at the pressure for which a linear relationship ceases to be valid. We have

$$\Delta V = -\frac{1}{\varepsilon}\Delta P\, V \ , \tag{5}$$

where ΔV and ΔP are the increments in volume and pressure, respectively, and ε is the compressibility modulus. If one assumes that the linear relationship holds up to pressures of the order of $\bar{P}$, it is easy to show that the ratio r_l between the hard sphere radius r_0 and l_0 is:

$$\frac{r_0}{l_0} = \frac{1}{2}\sqrt{1 - \frac{\bar{P}}{\varepsilon}} \ . \tag{6}$$

A plausible value for $\bar{P}$ is $\bar{P} \approx 10^3$ N/cm^2, while the compressibility moduli for some materials are:

$$\varepsilon\left(\mathrm{N/cm^2}\right) = \begin{cases} 1.3 \times 10^7 & \text{(Al)} \\ 1.6 \times 10^7 & \text{(Fe)} \\ 1.0 \times 10^7 & \text{(Brass)} \\ 0.5 \times 10^7 & \text{(Glass)} \end{cases}$$

Of course the compressibility modulus is a quantity related to static measurements, while it is well known that in dynamical processes which bring the system far from thermodynamic equilibrium (as in the case of hypervelocity impacts) the actual compression factors may become much higher.

In order to establish a plausible value for the breakup length of the springs, we have considered the thermal dilatation at the melting temperature, i.e., we have assumed that

$$\frac{\bar{l}}{l_0} = 1 + \lambda T_f \ , \tag{7}$$

λ being the linear dilatation coefficient and T_f the melting temperature. Some measured values for λ are:

$$\lambda\left(^\circ\mathrm{C}^{-1}\right) = \begin{cases} 11 \times 10^{-6} & \text{(Steel)} \\ 24 \times 10^{-6} & \text{(Al)} \\ 1.2 \times 10^{-6} & \text{(C, diamond)} \\ 7.9 \times 10^{-6} & \text{(C, graphite)} \\ 19 \times 10^{-6} & \text{(Brass)} \\ 9 \times 10^{-6} & \text{(Glass)} \end{cases}$$

while the melting temperatures are:

$$T_f\left(^\circ\mathrm{C}\right) = \begin{cases} 660 & \text{(Al)} \\ 4027 & \text{(C, diamond)} \\ 1083 & \text{(Cu)} \\ 1539 & \text{(Fe)} \end{cases}$$

An estimate of $\bar{l}$ can also be obtained by comparing the density of potential elastic energy of the springs and the specific internal energy of the real materials. The corresponding results are in good agreement with those obtained using the dilatation coefficients.

Finally, there are some parameters for which we have not yet succeeded in finding a convincing physical argument supporting an *a priori* plausible estimate, so for the moment we have just tested a range of different values. This is the case for the restitution coefficient of inelastic scattering η, which is related to viscosity effects and the decay of shock waves. The same holds for the reduction factor of the spring breakup length near a pre–existing fracture. Of course this factor must exceed some minimum value, otherwise unphysical chain reactions of fractures are generated.

3. DESCRIPTION OF THE CODE AND NUMERICAL RESULTS FOR A TEST CASE

The model described above has been implemented in a numerical code based on a fully object–oriented approach, so that a large number of simulations with different features can be carried out without any change in the code. The code consists of the following parts: (i) the input preparation routine, which creates the input files for the target and projectile with the coordinates of the mass elements written in the proper format; (ii) the numerical integration routine, which computes the time evolution of the system and writes output files with "snapshots" of the colliding objects and data about the properties of the created fragments at the end of the simulation; (iii) a post–processing program to collect and summarize information on the outcome of the collision from the output files and generate mass, velocity and angular distributions for the fragments; and (iv) a visualization program to show the actual evolution of the system during the simulated impact.

Apart from the visualization program (for which only a PC version is currently available), all the codes are written for either PC or UNIX systems. In particular the results presented below have been obtained on a PC Pentium 166MHz with 64 MB RAM. The required computational time has been about 4×10^{-4} s per time step per element. The same calculations have been replicated on a RISC6000 workstation, and in this case the computing time has been reduced by about a factor 3.

As a test case, we have generated two parallelepipedal objects (with axes parallel to the x, y and z axes of the reference system) of a material with the same density and melting temperature of aluminum. We have set the sound velocity at 2.5 km/s, and comparatively small values were taken for the restitution coefficient and the damping constant, in order to simulate a material with high viscosity in the liquid state and good elastic properties in the solid state. The value for the radius of the spherical elements is somewhat smaller than that given by Eq. (6), in order to take into account the behavior of materials far from thermodynamical equilibrium. We have not implemented the options of simulating a pre–existing distribution of flaws and of decreasing the breakup lengths near the fractures. Here follows a complete list of the adopted input parameters:

— mass of one element: $m = 0.0216$ kg;

— lattice spacing: $l_0 = 0.02$ m;

— elastic constant: $k = 3.375 \cdot 10^8$ N/m;

— decay time: $\tau = 20$ (using l_0/v_s as a unit);

— radius of the rigid spheres: $r_0/l_0 = 0.45$;

— breakup length: $\bar{l}/l_0 = 1.016$;

— impact restitution coefficient: $\eta = 0.2$.

The projectile is a cube of $3 \times 3 \times 3$ elements ($M_p = 0.5832$ kg), whose center of mass moves in the xy plane at a velocity of 3.162 km/s and at an angle of 18° to the x-axis; the target has sides of 5, 10 and 10 elements along the x, y and z directions, respectively, so that its total mass is 10.8 kg. The fragmentation process is observed in the center–of–mass reference system. The integration time step is 10^{-6} s (short compared to the travel time of the shock wave over a lattice spacing) and the total time span of the simulation is 2×10^{-3} s. The computing time required by the simulation has been 416 s on the PC Pentium machine.

Since the ratio between projectile kinetic energy and target mass is about 2.7×10^5 J/Kg, well above the typical threshold for catastrophic breakup of metallic targets (several times 10^4 J/kg), we expected to observe an extensive fragmentation of the target. Indeed this is the case, as it can be easily seen from the visualization of the evolution following the impact. However, the disruption of the target is not really complete, and a large remnant body survives. This is due to the specific geometry of our simulated impact, and indicates that assuming a unique, geometry–independent breakup threshold certainly oversimplifies the relationship between impact parameters and outcomes.

The detailed mass distribution of the fragments is the following: 109, 51, 6 and 2 fragments consisting of 1, 2, 3 and 4 mass elements, respectively; then 3 fragments of 5, 8 and 9 elements and finally 2 large fragments of 63 and 205 elements. This mass distribution resembles the power–law trend observed in the experiments, but of course we need to improve the mass resolution of the model to get better statistics.

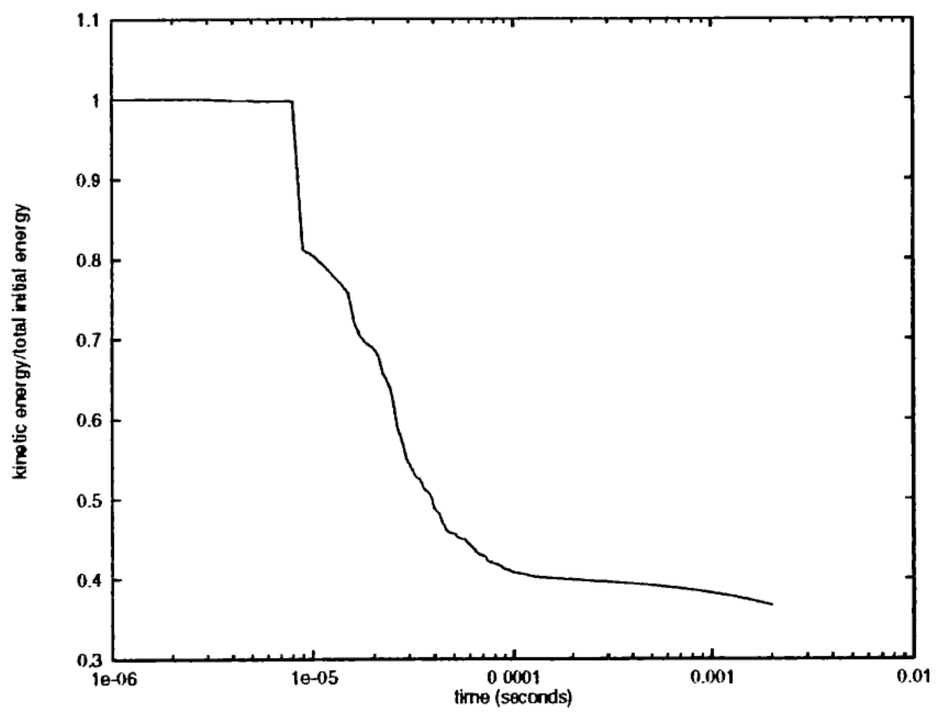

Figure 1. Ratio of the kinetic energy to the total initial energy as a function of time.

An interesting feature of the dynamics of the fragmentation process can be pointed out by monitoring the behavior of the ratio between the kinetic energy and the total energy of the system, as shown in Figure 1. Note that in the first $\approx 10^{-4}$ s after the impact the decrease of the kinetic energy is very fast, whereas later on the curve flattens and the dissipation is much slower. The reason for this behavior is that in the former phase inelastic collisions between the spherical elements are the dominant dissipation mechanism, and this corresponds to the propagation and rapid damping of shock waves, while in the latter phase only slower damping of elastic oscillations (e.g., sound waves) dissipates energy.

The ejection velocity distribution of the fragments (with respect to the center of mass of the system) is shown in Figure 2. The peak is at about 200 m/s, but there is a tail at speeds exceeding 1 km/s. This is also in good agreement with experimental results. As far as the geometry of the ejection velocities is concerned, there is a clear peak in the angular distri-

bution near the direction of the incoming projectile. Moreover, most fragments are ejected near the xy plane. This strong anisotropy is an interesting result, and one which may be important to model in a realistic way the outcomes of collisions between orbiting bodies.

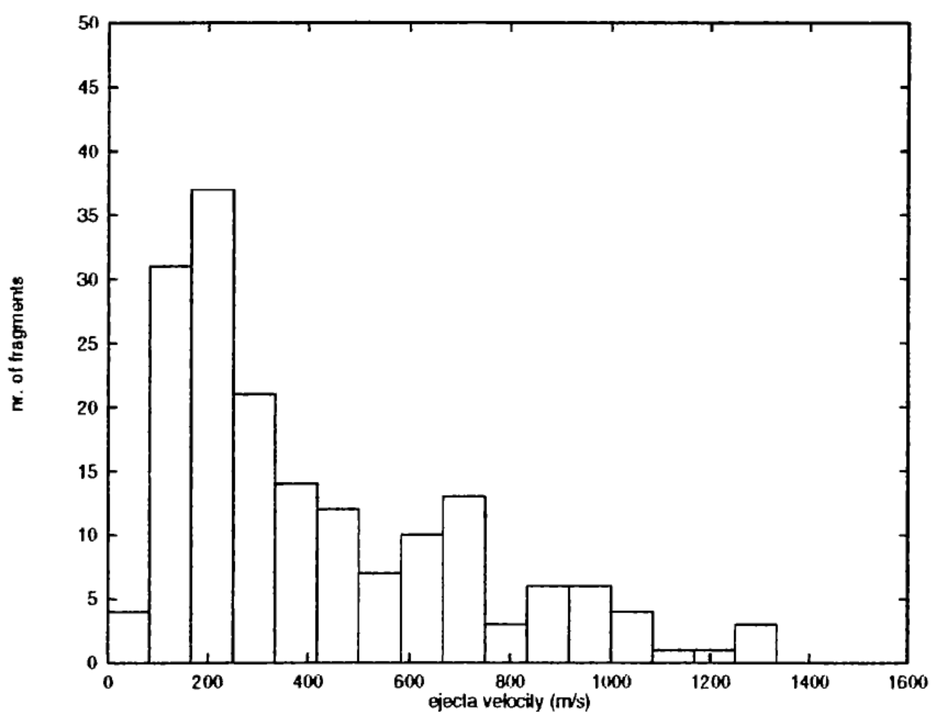

Figure 2. Velocity distribution of the fragments.

4. CONCLUSIONS

We have developed a new simple physical model for the behavior of solid bodies after hypervelocity impacts. Solid objects are considered as lattices of finite mass elements, modeled as hard spheres with a finite radius, which can collide inelastically with each other and interact through an elastic force, up to some breakup length. A number of input parameters can be chosen in such a way to be consistent with the observed properties of solid materials, such as sound velocity, dilatation coefficient and melting temperature. By running a number of test simulations similar to the one described in Sec. 3, we have found that several observed properties of real impact outcomes can be reproduced, such as: (i) an extensive disruption of the target whenever the ratio between kinetic energy of the projectile and target mass exceeds the commonly accepted values for the fragmentation threshold; (ii) the ejection of fragments at speeds about one order magnitude lower than the impact velocity (but with a "tail" of small, fast ejecta); (iii) a rapidly increasing number of fragments at smaller masses.

We are currently running further simulations to explore the input parameter space and to derive better statistics for the fragment masses and speeds, in order to check whether their distributions can be approximated by power laws (as observed in the experiments) and to determine the way these distributions depend on the collision energy and geometry. Work is also under way to better understand the relationship between some of our input parameters (such as the impact restitution coefficient and the damping time of elastic oscillations) and the macroscopic properties of the materials. Finally, we are going to improve the capabilities of our numerical code, especially for simulating impacts between objects of arbitrary shapes and compositions, and/or with cavities in their interiors.

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