COMPUTER SIMULATION OF THIN WALLED FUEL TANKS FRAGMENTATION UNDER THE ACTION OF EXPLOSION

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ABSTRACT

To analyze causes and consequences of fuel tanks explosions of last rocket stages on near-earth orbits there are first experiments on fragmentation of thin-walled shell structures under the action of internal explosion [1] and also attempts to solve the problem theoretically. In [2, 3] the simplest mathematical models are suggested that allow to calculate an average number of fragments, which are the result of breakup of thin cylindrical and spherical shells in explosion, and their initial velocity of scattering provided that all fragments have the same mass. This paper deals with numerical modelling of thin walled spherical shells fragmentation under the action of dynamic internal loading taking account of fragments' distribution in terms of mass.

1. PROBLEM STATEMENT ON DEFORMING A SHELL

The following assumptions on shell geometry and a character of its deforming and failure are made:

1. The shell is thin: $h/r \ll 1$ (h - the thickness, r - the shell radius).

2. The effect of an internal dynamic load is modelled by pressure $p = p(t)$ depending on time and being uniformly distributed along the internal surface of the shell. Approximated formule for pressure were obtained on basis of physical processes of detonation and combussion in gases.

3. The shell material is considered to be elastoviscoplastic and process of its deforming is adiabatic.

4. As a condition of the shell break-down beginning the entropic criterion of a limit specific dissipation [4] is assumed.

5. The shell break-down is assumed to occur as a result of the action of tensile ring stresses at the expense of consumption of elastic energy accumulated in the shell up to time $t = t_*$ of the breakup beginning; the work of external forces in breakup time is neglected; spalation fractures aren't considered [2, 3, 5].

Due to the first three assumptions the problem on deforming a thin shell can be considered as the one-dimensional spherical one.

Then the momentum equation has next form:

$$\rho \dot{v} = \frac{p(t)}{h} - 2\frac{\sigma_\theta}{r},$$

where ρ is the density, v is radial velocity, r is the current radius of the shell, σ_θ is ring stress (the averaged stress over the shell thickness); a dot over a symbol means a material derivative with respect to time.

The rate of a ring deformation is determined by

$$\dot{\varepsilon}_\theta = \frac{v}{r},$$

other deformations are absent due to the shell thinness.

The equation of mass conservation is reduced to the form

$$\rho = \rho_0 \exp(-2\varepsilon_\theta).$$

State equations of elastoviscoplastic materials are taken in the Perzina's form which, taking account of

$$\sigma_\theta = \sigma + S_\theta,\ \sigma_\phi = \sigma_\theta,\ \sigma_r = \sigma + S_r = 0,\ 2S_\theta + S_r = 0$$

for a spherical case, can be reduced to the form

$$\dot{S}_\theta = \frac{2}{3}\mu\dot{\varepsilon}_\theta - \frac{\mu}{\eta}S_\theta(1 - \frac{Y_0}{3}|S_\theta|)H(1 - \frac{Y_0}{3}|S_\theta|),$$

$$\sigma_\theta = 3S_\theta.$$

Here μ - shear modulus, η - dynamic viscosity of material, Y_0 - static limit of elasticity in simple tension, $H(x)$ -unit Heavyside function. Herewith the stress tension σ_{ij} divided into spherical $\sigma\delta_{ij}$ and deviator S_{ij} partials and it is assumed that rates of deformations can be divided into the rates of elastic and plastic ones and a plastic flow is incompressible:

$$\dot{\varepsilon}_\theta = \dot{\varepsilon}^e_\theta + \dot{\varepsilon}^p_\theta,\ \dot{\varepsilon}_\phi = \dot{\varepsilon}_\theta,\ \dot{\varepsilon}_r = \dot{\varepsilon}^e_r + \dot{\varepsilon}^p_r = 0,\ 2\dot{\varepsilon}^p_\theta + \dot{\varepsilon}^p_r = 0.$$

Specific (per mass unit) elastic energy E and mechanical dissipation D can be determined by integration of the formulae

$$\dot{E} = 2\frac{\sigma_\theta}{\rho}\dot{\varepsilon}^e_\theta, \qquad \dot{D} = 2\frac{\sigma_\theta}{\rho}\dot{\varepsilon}^p_\theta,$$

and rates of elastic and plastic deformations are determined by the formulae

$$\dot{\varepsilon}^e_\theta = \frac{\dot{S}_\theta}{2\mu} + \frac{2}{3}\dot{\varepsilon}_\theta, \qquad \dot{\varepsilon}^p_\theta = \dot{\varepsilon}_\theta - \dot{\varepsilon}^e_\theta.$$

Criterion of shell destruction is the entropic criterion of a limit specific dissipation [4] that for the used medium model is reduced to mechanical dissipation D:

$$D|_{t=t_*} = D_*,$$

where D_* is the constant of limit specific dissipation experimentally defined [4], t_* - fracture time.

2. CALCULATION OF FRAGMENTS' NUMBER

Fragments' number obtained in shell breakup can be found from the balance of elastic energy and work for breaking off a material. To describe fragments' distribution in terms of mass in explosive breaking of shells the Weibull distibution is most often used that is the special case of general probability distributions [6, 7]:

$$N(< m) = N_0\big[1 - \exp\big(-(m/m_*)^\Lambda\big)\big]. \qquad (1)$$

Here $N(< m)$ is the number of fragments with mass less than m; N_0 is the total number of fragments with mass more than 0 (a theoretical constant); m_* - the characteristic mass of distribution; Λ - the parameter of fragmentation quality.

As shown in [6], the unimodal distribution (1) describes satisfactorily spectra of break-down of metallic cylindrical shells of a medium-sized thickness ($h/r \simeq 0.1$). Better results can be given by the two-modal hyperweibul distribution suggested in [6] which includes two morphological collections: large fragments containing both shell surfaces (external and internal) and accompanying smaller fragments containing one of the surfaces.

For the case considered shells are very thin: $h/r \approx 0.001$ [1]. Therefore, evidently, we can restrict ourselves to unimodal distribution (1) and assume that all fragments contain both an internal surface and an external surface of the shell that is confirmed indirectly by experiments [1] conducted for such thin shells.

Let an area of an initial external Lagrangian surface of a fragment be equal to s, its internal surface be also equal to s (due to the shell thinness), and area of its lateral surface - $2ph$, where p is the semiperimeter of the contour s. The fragment mass is $m = \rho_0 hs$, therefore distribution in terms of mass (1) can be presented in the form of distribution in terms of areas s of fragments:

$$N(< s) = N_0\big[1 - \exp\big(-(s/s_*)^\Lambda\big)\big], \qquad (2)$$

where $s_* = m_*/\rho_0 h$ is the characteristic area of the fragment.

Number of fragments with areas $s' \le s \le s''$ will be equal to:

$$N(s' \le s \le s'') = N_0\big[\exp\big(-(s'/s_*)^\Lambda\big) - \\ -\exp\big(-(s''/s_*)^\Lambda\big)\big].$$

Suppose now that from the total spectrum of the shell fragments with areas $0 < s < \pi d_0^2$ (d_0 is the initial diameter of the spherical shell) one can extract K groups of fragments with areas $s_1, s_2, ..., s_K$: $s_{min} < s_1 < s_2 < ... < s_K < s_{max}$, s_{min} and s_{max} are certain minimal and maximal areas of fragments. Let all fragments with areas $s_{min} \le s \le 0.5(s_1 + s_2)$ prove to be within the group of fragments with area s_1, all fragments with areas $0.5(s_1 + s_2) < s \le 0.5(s_2 + s_3)$ prove to be within the group with area s_2, etc. up to the group with area s_K wherein there are fragments with areas $0.5(s_{K-1} + s_K) \le s \le s_{max}$.

Further in place of distribution (2) the following distribution will be used:

$$N(< s) = N_0\big[1 - \exp\big(-(\frac{s - s_{min}}{s_*})^\Lambda\big)\big], \\ s_{min} \le s \le s_{max}. \qquad (3)$$

Then the number of fragments of the group s_j ($j = 1, 2, ..., K$) will be equal to

$$N_j = N_0(\beta_j - \beta_{j+1}),$$

$$\beta_j = \exp\left(-\left(\frac{0.5(s_{j-1} + s_j) - s_{min}}{s_*}\right)^{\Lambda}\right). \quad (4)$$

Here $s_0 = 2s_{min} - s_1$, $s_{K+1} = 2s_{max} - s_K$.

System (4) of K equations for calculation of the fragments' number of the groups can be complemented by the following two equations:

$$\sum_{j=1}^{K} s_j N_j = \pi d_0^2, \quad \sum_{j=1}^{K} \gamma h p_j N_j = \pi d_0^2 h \rho_0 E_*. \quad (5)$$

The first equation (5) means that the summarized area of external surfaces of the fragments is exactly equal to the shell area, and the second equation (5) means that the elastic energy $\pi d_0^2 h \rho_0 E_*$ ($E_* = E|_{t=t_*}$) accumulated in the shell by time $t = t_*$ spends for formation of fracture surfaces. Herein γ is the specific energy, consumed for formation of the free surface unit, p_j is the semiperimeter of a fragment of the j-th group.

On using the first equation (5), the following equation can be derived from the second equation (5):

$$\sum_{j=1}^{K} N_j\,(p_j - \rho_0 E_* s_j/\gamma) = 0. \quad (6)$$

To find $(2K+1)$ unknown variables p_j, N_j, N_0 there are only $(K+2)$ equations (4), (5) and (6). Select the following particular solutions of equation (6):

$$p_j = \rho_0 E_* s_j/\gamma, \quad j = 1, 2, ..., K. \quad (7)$$

Physical meaning of (7) consists in the following. One-half of energy necessary for formation of breakdown surfaces around each j-th fragment with area s_j and mass m_j is extracted from mass m_j contained inside the fragment, and the second half - from the outside (from neighboring fragments). This is apparently a reasonable assumption.

The rest of $(K+1)$ equations (4) and the first equation (5) can be easily solved now. At first we find:

$$N_0 = \pi d_0^2 / \sum_{j=1}^{K} s_j(\beta_j - \beta_{j+1}), \quad (8)$$

and then according to (4) all N_j can be determined.

3. RESULTES

A pressure $p = p(t)$ approximated by the following way (Fig. 1).

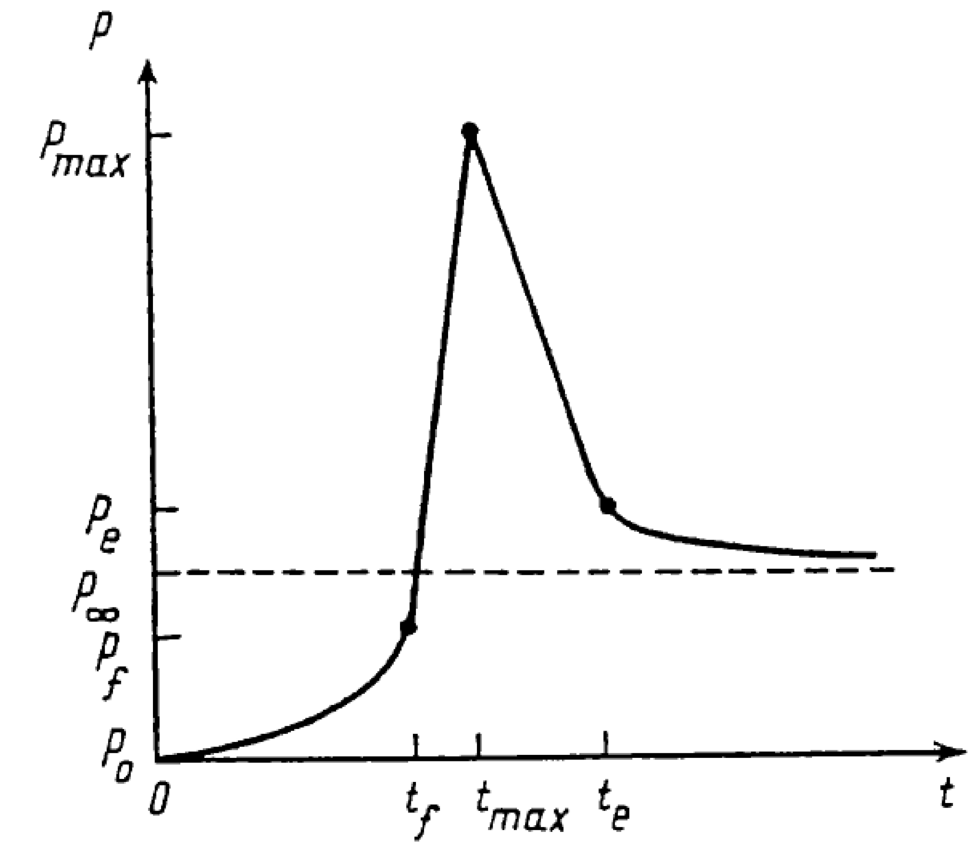

Figure 1. Relation graph of pressure depending on time.

Since initial pressure into a shell p_0 not too large, we suppose that a shell remained in elastic state in initial state. Then initial conditions are the same:

$$v = 0, \ \sigma_\theta = p_0 r_0/2h, \ S_\theta = \sigma_\theta/3, \ \varepsilon_\theta = \varepsilon_\theta^e = \sigma_\theta/2\mu,$$

$$\varepsilon_\theta^p = 0, \ w = r_0\varepsilon_\theta, \ r = r_0 + w, \ E = \mu[1 + (2\varepsilon_\theta - 1)$$
$$\exp(2\varepsilon_\theta)]/\rho_0, \quad D = 0, \quad \rho = \rho_0 \exp(-2\varepsilon_\theta).$$

Here r_0 - initial shell radius, w - displacement.

For the first group of the least fragments accepted that $s_1 = \pi(\gamma/\rho_0 E_*)^2$[7]; in all considered $K = 20$ groups of fragments, moreover $s_j = s_{j-1}\,10^{0.25}$, $j = 2, 3, ..., 20$; $s_{min} = 2\pi(\gamma/\rho_0 E_*)^2/(1+10^{0.25})$, $s_{max} = 10^5 s_{min}$.

The calculations were performed for a duralumin spherical shell having a radius $r_0 = 0.25m$ and thickness $h = 0.5mm$; $\rho_0 = 2700kg/m^3$, $Y_0 = 0.25GPa$, $\mu = 27GPa$, $\eta = 100KPa.c$, $\gamma = 100kJ/m^2$, $D_* = 30kJ/kg$. For approximation a pressure $p = p(t)$ it is accepted (Fig. 1): $p_0 = 0.41MPa$, $p_f = 4p_0$, $t_f = 10mks$, $p_{max} = 20MPa$, $t_{max} = 11mks$, $p_e = 6p_0$, $t_e = 13mks$, $p_\infty = 5p_0$, $\tau = 200mks$. Under this initial date we have $E_* = 79.6kJ/kg$, $t_* = 45.6ms$, $m_1 = 0.917mg$ ($s_1/h^2 = 2.72$), velocity of shell extension at moment of destruction $t = t_*$ equal $V_0 = 267m/s$.

Distribution of number of fragments n over the groups $m = m_j$ ($j = 1, 2, ..., 20$) under $s_* = 1500s_1$ and under the next parameters of fragmentation quality Λ: $\Lambda = 0.6$ (curve 1, $N_\Sigma = 475$), $\Lambda = 0.8$ (curve 2, $N_\Sigma = 635$), $\Lambda = 1$ (curve 3, $N_\Sigma = 719$) and $\Lambda = 1.2$ (curve 4, $N_\Sigma = 768$) presented on Fig. 2. Distribution for the case $\Lambda = 0.8$ under the next characteristic squares s_*: $s_* = 500s_1$ (curve 4, $N_\Sigma = 1921$), $s_* = 1000s_1$ (curve 3, $N_\Sigma = 955$), $s_* = 2000s_1$ (curve 2, $N_\Lambda = 473$) and $s_* = 4000s_1$ (curve 1, $N_\Sigma = 231$) presented on Fig. 3.

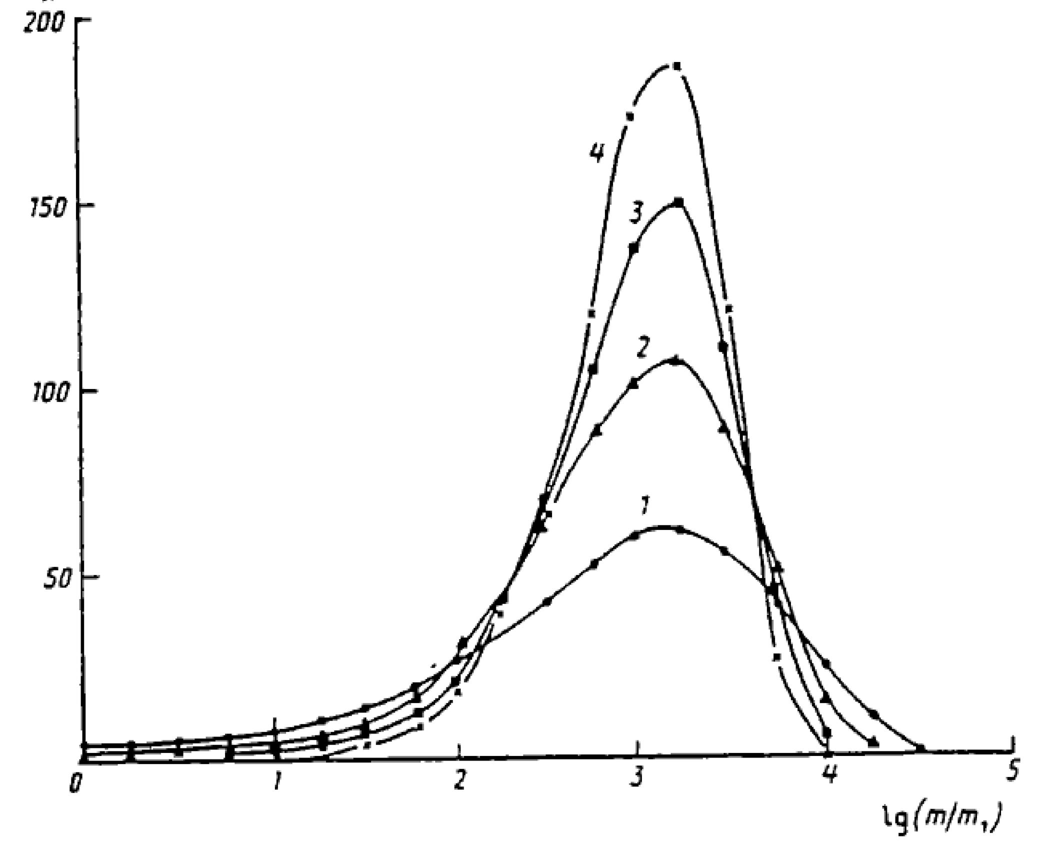

Figure 2. Number of fragments versus mass ($s_* = 1500s_1$): 1 - $\Lambda = 0.6, N_\Sigma = 475$; 2 - $\Lambda = 0.8, N_\Sigma = 635$; 3 - $\Lambda = 1, N_\Sigma = 719$; 4 - $\Lambda = 1.2, N_\Sigma = 768$.

As we can see from Fig. 2, under the same characteristic square s_* and then the parameter of fragmentation quality Λ increased then the total number of fragments $N_\Sigma = \sum_{j=1}^{20} N_j$ increased too. Moreover, the maximum of n attained for the group $m_{14} = 1.63g$ for all this cases. If the parameter of fragmentation quality Λ fixed and the characteristic square s_* changed then if s_* growing then total number of fragments N_Σ decreased and maximum of n displaced to side of big fragments (Fig. 3).

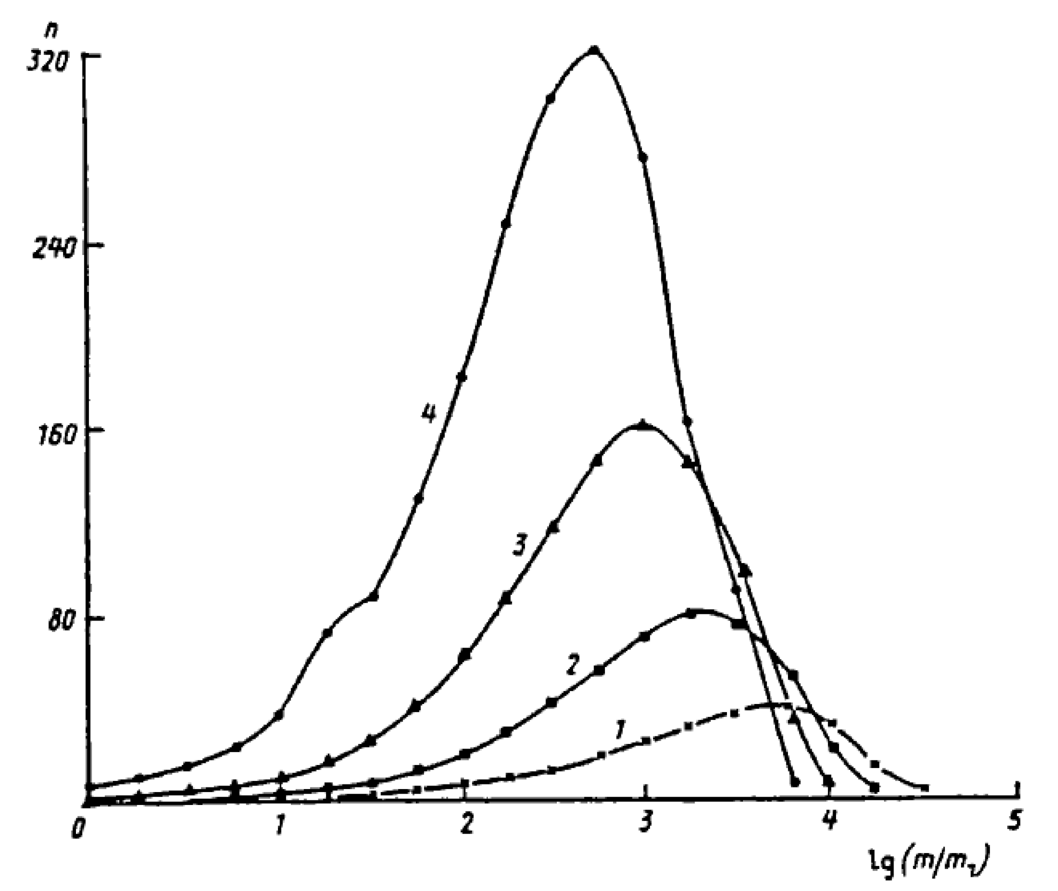

Figure 3. Number of fragments versus mass ($\Lambda = 0.8$): 1 - $s_* = 4000s_1, N_\Sigma = 231$; 2 - $s_* = 2000s_1, N_\Sigma = 473$; 3 - $s_* = 1000, N_\Sigma = 955$; 4 - $s_* = 500, N_\Sigma = 1921$.

On the whole, results of numerical investigations satisfactory agree with experimental data [1] (Fig. 4).

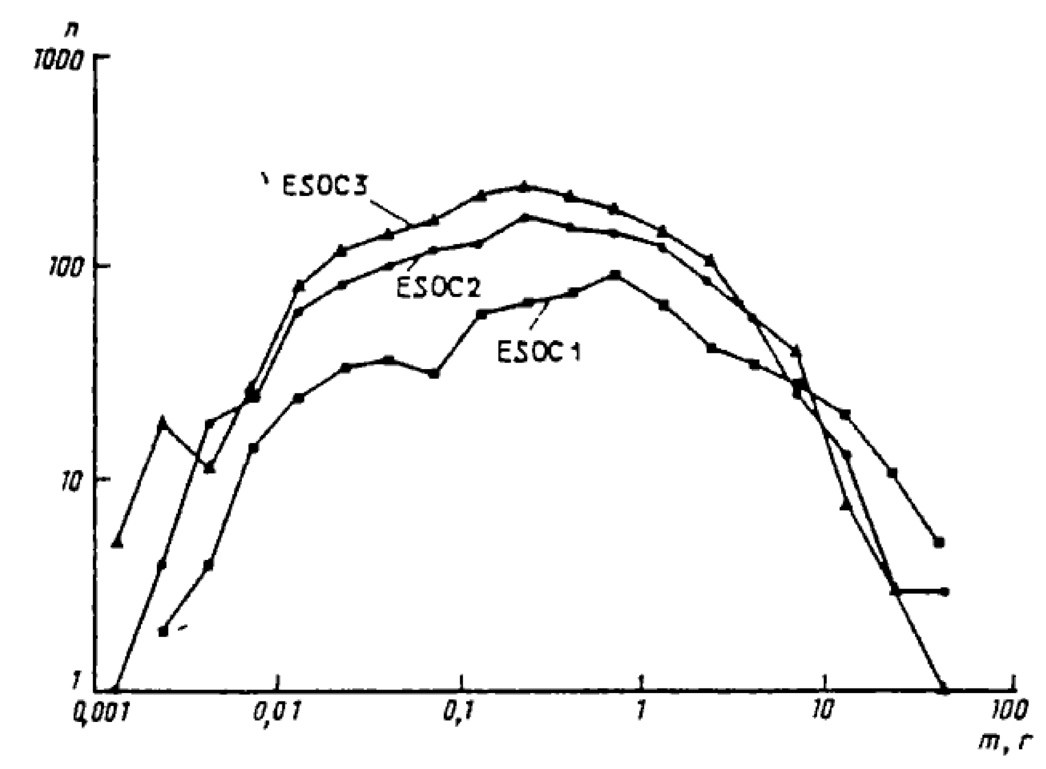

Figure 4. Number of fragments versus mass: ESOC experiments [1].

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