A DETERMINIST APPROACH FOR COLLISION RISK ASSESSMENT AND DETERMINATION OF AN OPTIMAL AVOIDANCE MANEUVER*

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ABSTRACT

This paper describes some applications of NORAD's two-lines elements sets (TLE) catalogue of orbiting objects regarding the collision hazard with space debris. A simple method permits us to eliminate from the catalogue the TLEs of the objects which will certainly not be dangerous for a certain satellite. Working with the remaining TLEs, we can study the satellite-debris distances, and get the minimum distance points. Simulations conducted with LEO, MEO and GEO satellites provide figures about the risk of close encounters within ranges under 200 km. In case of a too close encounter, we compute the optimal avoiding maneuver (an impulsive ΔV) that will respect a certain security distance. The study of a close encounter case shows that the cost of the maneuver increases linearly with the security distance imposed, and that the characteristics of the optimal ΔV maneuver are differents whether the maneuver is early or late.

1. INTRODUCTION

The main data source of orbital parameters for orbiting objects are the two-lines elements sets (TLE), which are maintained by U.S. Space Command. TLEs represent an up-to-date catalogue of more than 8000 objects (active satellites, rocket bodies, debris...), which finds new applications with the surge of the space debris threat. The TLE data format is associated with specific propagation models, in particular the SGP4 and SDP4 models (for periods greater and lower than 225 *mn* respectively). If we are interested in the collision risk for a specific, maneuverable satellite, two major utilizations can be made of the TLE database and predictions:

1) looking for close approaches and comparing miss-distances with a security distance.
2) using predictions for computing an optimal avoiding maneuver, when a unacceptable close encounter is predicted.

These two applications will be detailed in this paper.

2. SORTING POTENTIALLY DANGEROUS DEBRIS

For a particular satellite, not all of the orbiting objects are worth studying for a collision risk assessment. Therefore, it would be useful to separate potentially dangerous objects from undoubtedly harmless ones, with the purpose of saving computation time for a collision risk analysis. There is a simple method of making sure that an object will never pass closer than a specified security distance (d_{min}) from the satellite of concern. It consists in comparing the radii of apogee (R_a) and perigee (R_p) of the satellite and the choosen debris according to the following rule.
For two satellites (n°1 & n°2), if we have:

$$R_{a1} + d_{min} < R_{p2}$$
$$\text{or} \quad R_{a2} + d_{min} < R_{p1}$$

then the volumes in which the two orbits can be will not overlap and the range between the two objects will always be greater than d_{min} (Fig. 1). So, if d_{min} is greater than the precision with which the distance between the 2 objects is known, then the debris will represent no danger for the satellite.

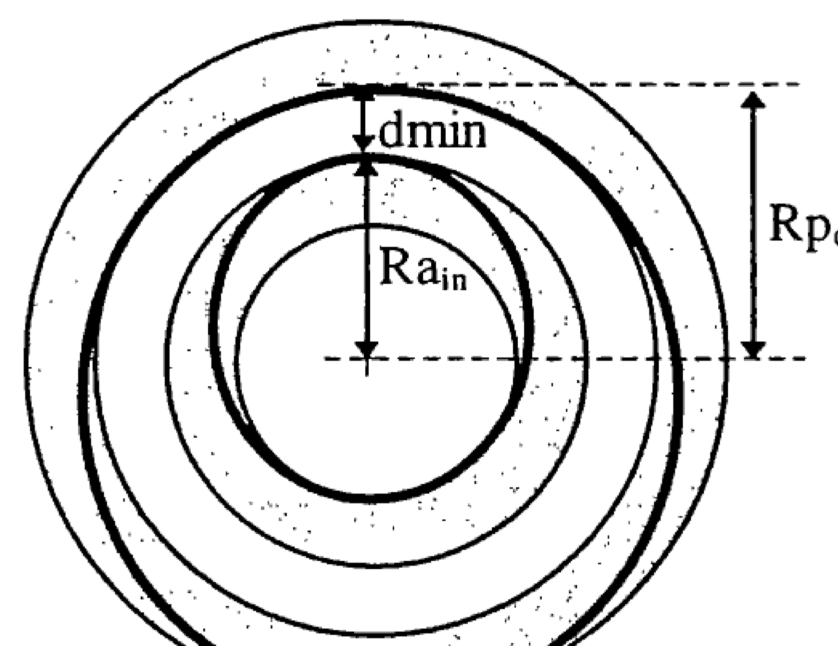

Figure 1. Principle of the sorting method.

This is a coarse but sure method to eliminate useless TLEs for a collision risk assessment method. It is easy to predict that the TLE elimination rate shall not be very good if the satellite of interest is very eccentric, and that it will depends very much on the security range we choose and on the density of population at the satellite's

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altitude. We assessed the efficiency of this algorithm for 4 satellites (see table 1 below) belonging to 4 interesting classes of orbit: 2 LEOs (space station MIR & SPOT 3), a MEO (GPS-satellite NAVSTAR 36) and a GEO (TELECOM-2B).

name	NORAD number	mean altitude (km)
MIR	16609	~ 400 (LEO)
SPOT 3	22823	~ 825 (LEO)
NAVSTAR 36	23027	~ 20150 (MEO)
TELECOM 2B	21939	~ 35800 (GEO)

Table 1. Satellites chosen for the study.

Figure 2 gives the results for the 4 satellites, with security distances ranging from 0 to 40000 *km* (beyond this value, the security distance defines a volume that encompasses virtually every debris).
The steps which can be observed on the graphics correspond to the security distances that reach the densely populated altitudes (essentially LEO & GEO).
The graphics show great differences between the results of the 4 satellites, but for all of them, this sorting method yields very interesting elimination rates with security distances that fulfill the needs of a collision assessment method, i.e small ones (Table 2).
This rate does not grow very much when the security distance increases from 0 to 50 *km*, except in the case of SPOT 3, which belongs to a very densely populated class of altitude. Even in this case, the TLE processing is useful since it shows that only less than 40 % of the initial TLE database represent potentially dangerous objects with a security distance of 50 km. The TLE elimination rate is even higher with the three other satellites (up to 90 % with MIR, 50 *km*). This pre-processing will be very useful for the miss-distance calculations explained below since the computation volume will be reduced in the same proportions.

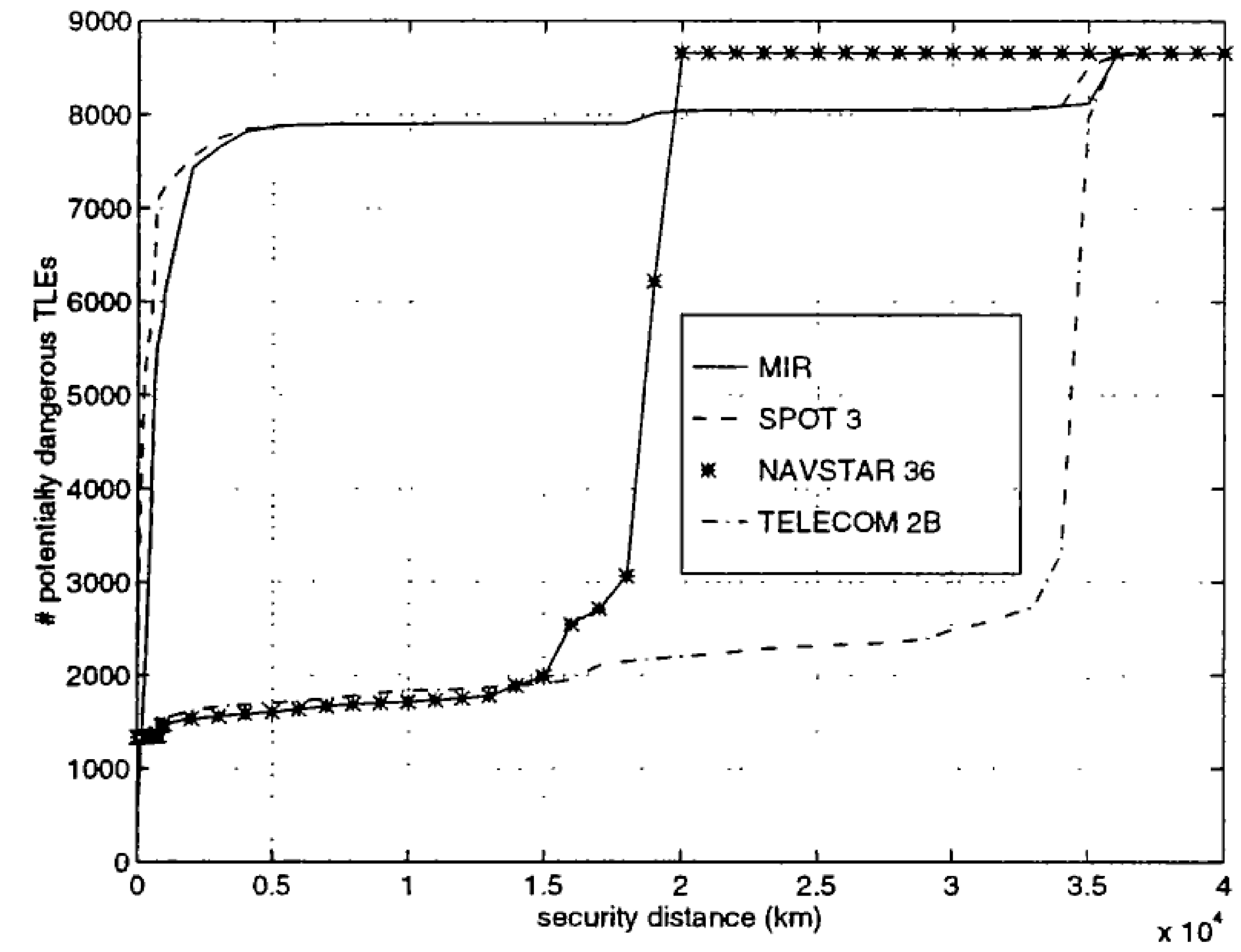

Figure 2. results of the TLE elimination method.

	# potentially dangerous TLEs (+ remaining TLEs rate)					
security distance (km)	10 km		20 km		50 km	
MIR	668	(8.4 %)	687	(8.7 %)	788	(9.9 %)
SPOT 3	2360	(29.8 %)	2638	(33.3 %)	3089	(39.0 %)
NAVSTAR 36	1332	(16.8 %)	1332	(16.8 %)	1332	(16.8 %)
TELECOM 2B	1251	(15.8 %)	1253	(15.8 %)	1279	(16.2 %)

Table 2. Results of the TLE elimination method for 10, 20 and 50 km.

3 - EXAMPLES OF COLLISION RISK ASSESSMENT USING MISS-DISTANCES COMPUTATIONS

The most important information for a determinist collision risk assessment is the evolution of the satellite-debris distance. With TLEs and the associated propagation model SGP4 / SDP4, we can easily predict it, and get the dates where it is minimum (the values at these dates are the miss-distances). With them, we can look for close encounters by studying the evolution of the distance between the satellite of interest and the debris. Naturally, the confidence we shall have in the predictions will depend much on the level of precision obtained with the propagation model. Regarding the propagation model associated with TLEs, we know that it is not adapted to long-term prediction, so a collision risk assessement should focus essentially on the close encounters whose date are not too far from the TLEs epoch. Nevertheless, simulations on a long duration can still provide valuable information on the collision risk because they often shows a synchronisation phenomenon between the orbits of the satellite and the debris. For instance, a debris that gets very close at many consecutive orbits should be watched over.
We have made such long-duration simulations with the same 4 satellites as in the former paragraph, in order to get an overview of the collision threat. We made simulations over two one-month period with the potentially dangerous debris, taken among a TLE database of nearly 8000 objects. For each satellite, we

compute the intervals in which a debris comes closer than 200 km, as well as the miss distances. Tables 3 and 4 show the result for both periods of simulation. We give:
- the number of potentially dangerous objects, sorted with the method explained above, and the corresponding percentage relatively to the total number of TLEs we started with.
- the number of objects that do not respect the 200 km limit we set over the entire one month period, and the corresponding percentage.
- the total number of less-than-200 km approaches over the one month period (there can be many for a single object).
- the record for the smallest miss-distance (with the designation and NORAD number for the debris).

satellite	**# potentially dangerous objects**	**# dangerous objects (d < 200 *km*)**	**# approaches < 200 *km***	**smallest miss distance**
MIR	1388 (17.5 %)	671 (8.5 %)	3152	0.163 *km* Mir debris (# 23695)
SPOT 3	4511 (57 %)	3576 (45.2 %)	39122	1.428 *km* Delta 1 deb. (# 8533)
NAVSTAR 36	957 (12.1 %)	4 (0.05 %)	4	116.063 *km* SL-6 R/B (# 8018)
TELECOM 2B	1067 (13.5 %)	10 (0.13 %)	14	33.380 *km* SATCOM-3R (# 12967)

Table 3. First period: 31 days; starting 12/19/1995 (7916 TLEs database).

satellite	**# potentially dangerous objects**	**# dangerous objects (d < 200 *km*)**	**# approaches < 200 *km***	**smallest miss distance**
MIR	1331 (16.9 %)	672 (8.6 %)	3026	1.245 *km* Mir debris (# 23690)
SPOT 3	4468 (56.9 %)	3562 (45.3 %)	44985	1.327 *km* [N/A] (# 87943)
NAVSTAR 36	919 (11.7 %)	1 (0.01 %)	1	184.022 *km* NAVSTAR-6 (# 11783)
TELECOM 2B	1052 (13.4 %)	6 (0.08 %)	32	9.434 *km* TELECOM 1B (# 15678)

Table 4. Second period: 31 days; starting 3/4/1995 (7859 TLEs database).

The comparison between tables 3 and 4 shows that results are close on average from one period to another.
We can see that such a study is useful whatever the type of orbit: for MIR, nearly 9 % (670+) of the 8000 orbiting object tested in this simulation pass within a 200 km range over a one-month period. For higher LEO orbits, the risk seems even more important: for SPOT 3, this percentage goes up to 45 % ! (i.e. more than 3500 debris passing within 200 *km* over one month). This percentage is much lower at higher altitudes, since a great part of orbiting debris and satellites is in LEO, but simulations usually yield less-than-200 *km*-close encounters in MEO and GEO as well.
Unlike LEO, the main cause for the common occurrence of close encounters in MEO and GEO is not an important density of objects, but the proximity of the orbital parameters of active satellites and debris. For instance, in our second simulation, we found that for both NAVSTAR 36 and TELECOM 1B, the debris which is responsible for the closest miss-distance is a same-class desactivated satellite which has been moved to a nearby graveyard orbit.

On the histograms of Figure 3, we can read directly, for our four satellites, the total number of close encounters that occured during a one-month period within a certain minimum range in the interval 0-200 *km*.

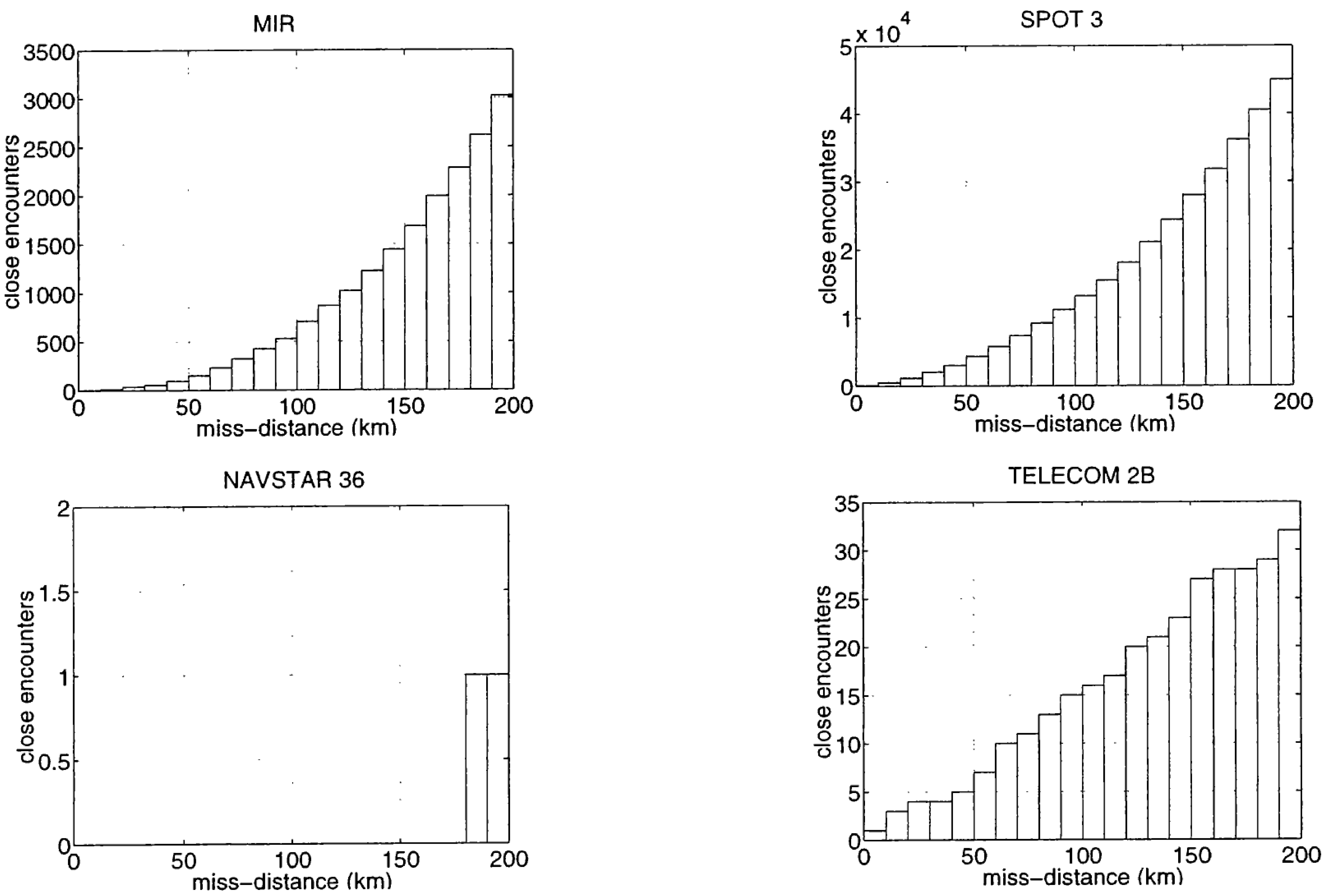

Figure 3. Histograms of close encounters (distance ≤ 200 km).

For collision risk assessment, close encounters within a range of a few tens kilometers are the most interesting.
On Figure 4 are the same histograms with a higher scale and miss-distances up to 50 km for MIR and SPOT 3.
The figures of the histograms, which represent a 31-days period, permit us to obtain interesting information under the form of a mean time between approaches within a range less than a given minimum value (Table 5). We see that *predicted* close encounters within a 5 or 10 km range from MIR are not rare, and only a few of the objects involved are catalogued as «MIR debris» (of course, we have not taken into account MIR modules and Progress/Soyuz docking spacecrafts). The close approach frequency is much higher with SPOT 3 (about 100 approaches a day within 50 km).
The method described here, and the corresponding software, have been used to investigate the case of the collision between the french micro-satellite Cerise and an Ariane's debris that occurred on July 24th, 1996.

satellite	closest range	mean time between approaches
MIR	≤ 5 km	~ 10 days
	≤ 10 km	~ 6 days
	≤ 50 km	~ 8 hours (3.1 / day)
SPOT 3	≤ 5 km	~ 21 hours (1.1 / day)
	≤ 10 km	~ 6 hours (4.3 / day)
	≤ 50 km	~ 15 minutes (99 / day)

Table 5. mean time between less than 5, 10 & 50 km close approaches.

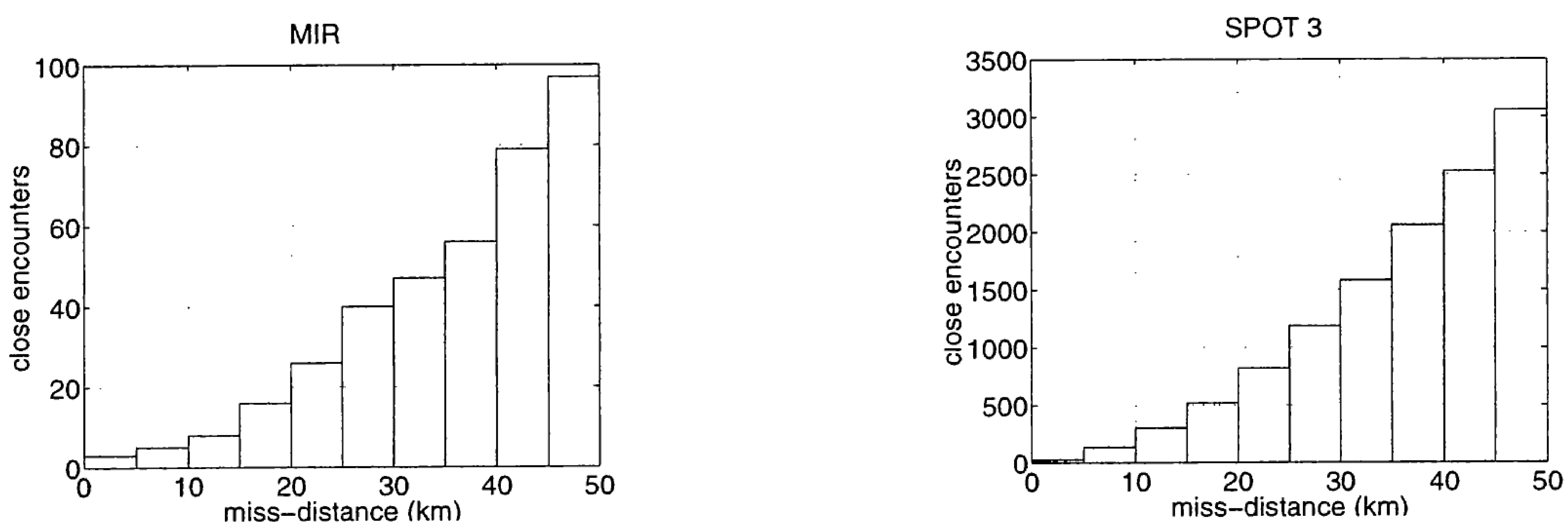

Figure 4. Histograms of MIR and SPOT 3 close encounters (distance ≤ 50 km).

4. EXAMPLE OF STUDY OF AN OPTIMAL AVOIDANCE MANEUVER

If, with a certain debris, predictions yield a minimum distance thought unacceptable, a maneuver has to be carried out by the satellite so that the miss-distance with the debris increases to an acceptable value. The satellite maneuver is assumed to be an impulsive speed variation (or ΔV) and it should be as small as possible so that it will not jeopardize the satellite mission nor have too much effect on the satellite life-span (maneuver are fuel-consuming). So, the problem of the optimal avoidance maneuver is a classic optimization problem with the following characteristics:

- **3 parameters**: the 3 coordinates of ΔV.
- **one constraint**: the minimum distance with the debris over a specified time interval should be greater than a certain acceptable distance (security distance).
- **a cost function**: the norm of ΔV should be minimized.

The date of the maneuver could be taken as a fourth parameter with a certain domain of variation, but then the nature of the problem is different whether this domain is small or large relatively to the satellite's period. We have not included this fourth parameter in the optimization problem in order to study its effects on the cost of the maneuver when we make it vary over a 15-days time span. Moreover, fixing the date of the maneuver leads to a better evaluation of the influence of the security distance alone on the characteristics of the optimal ΔV. We used the following procedure (see Figure 5):
1) our simulation program spots a close encounter with an unacceptable miss-distance. It computes the dates of the corresponding time interval (called **the danger interval**) where the distance between the satellite and the debris is lower than a given security distance.
2) a second program computes the optimal avoidance maneuver, according to the following specifications:
- **the optimization interval** = time interval where the constraint (i.e. the minimum distance) for the optimization problem will be calculated. This interval should include the initial danger interval (as it has been defined in (1)) and it should be large enough so that the minimum distance date we are interested in will not exceed its limits when the algorithm tries different maneuvers. Besides these considerations, our interest is to have this interval short for time computation reasons.
- **the maneuver delay** = the duration between the maneuver and the beginning of the former interval.
- **the security distance** = the value the minimum distance in the optimization interval should reach.

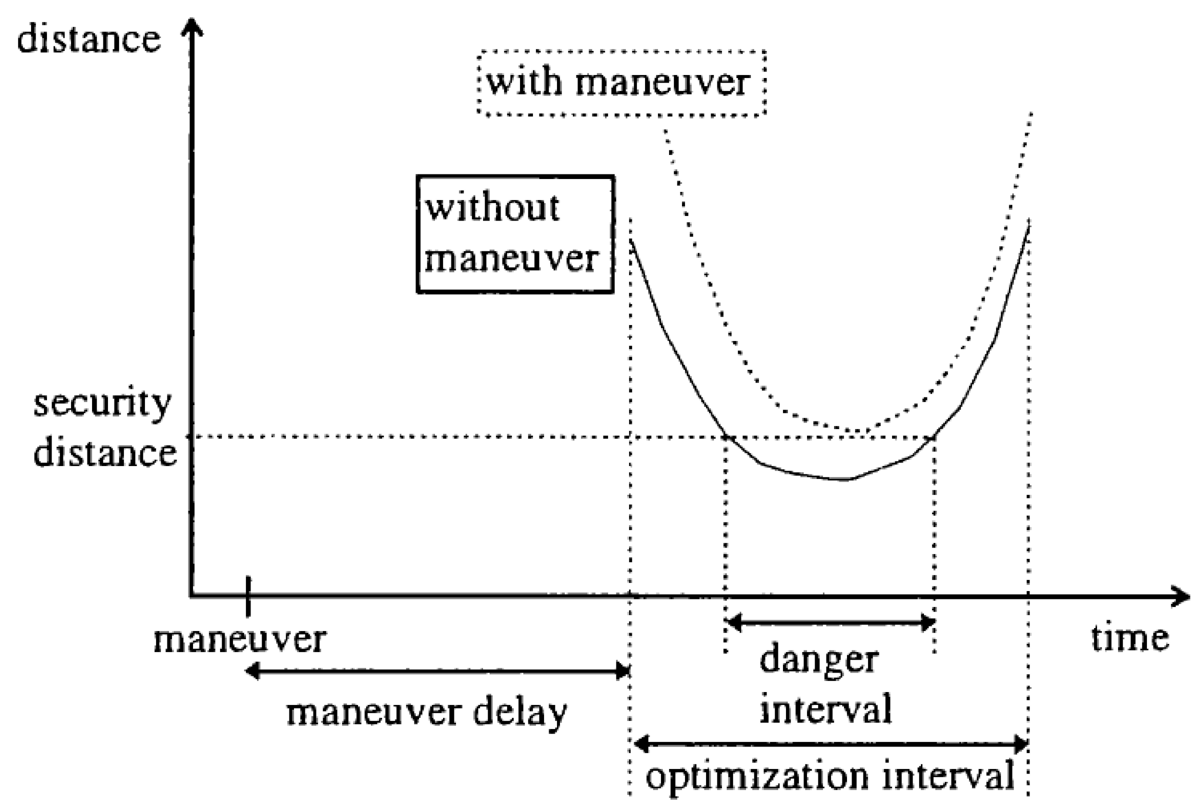

Figure 5. definitions for the optimal maneuver problem.

We used the optimization method known as «generalized projected gradient method».
Since we want to study the optimal avoiding maneuver corresponding to a *single* close encounter, we chose the case of a close encounter involving two objects with very different orbits (so that the collision risk is concentrated at a single orbit crossing rather than a series of successive ones). The benefit for our study is that it is easier to find an optimization interval that will always include the close encounter we are interested in (and only this one) whatever the ΔV.
Our case is a 1.2 km close encounter between SPOT 3 (LEO), which will be our maneuvering satellite, and a KYOKKO 1 debris (near GTO, high eccentricity, NORAD number 12133). We took a 100 minutes optimization interval centred on the 200 km-danger interval, which is a few seconds long.
Figure 6 is a 3D representation giving the norm of the optimal maneuver ΔV, for security distances ranging from 5 to 100 km and maneuver delays in the following list:

10 mn	30 mn	1 hour	2 hours	4 h	8 h
16 h	1 day	2 days	5 days	10 days	15 days

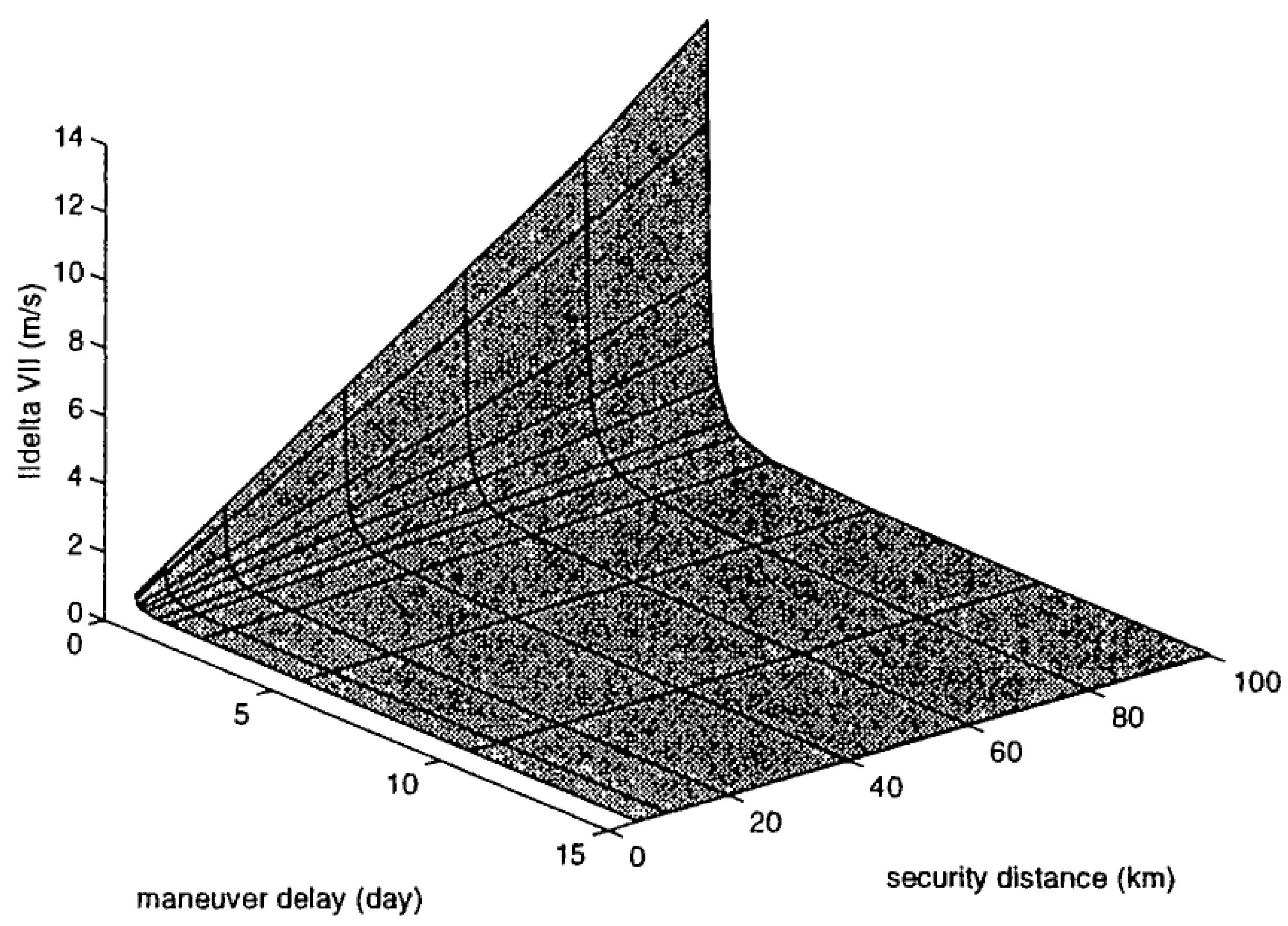

Figure 6. 3D representation of the ΔV of maneuver versus maneuver delay and security distance.

We can see that the results can be separated in two-classes in relation to the ΔV level:
- for a maneuver delay in the range between 15 days and 16 hours: $\Delta V \leq 1$ m /s
- for a maneuver delay in the range between 8 hours and 10 minutes: 1 m/s $\leq \Delta V \leq 12.5$ m /s

On Figure 7 are the ΔV versus security distance representations for these two classes of results.

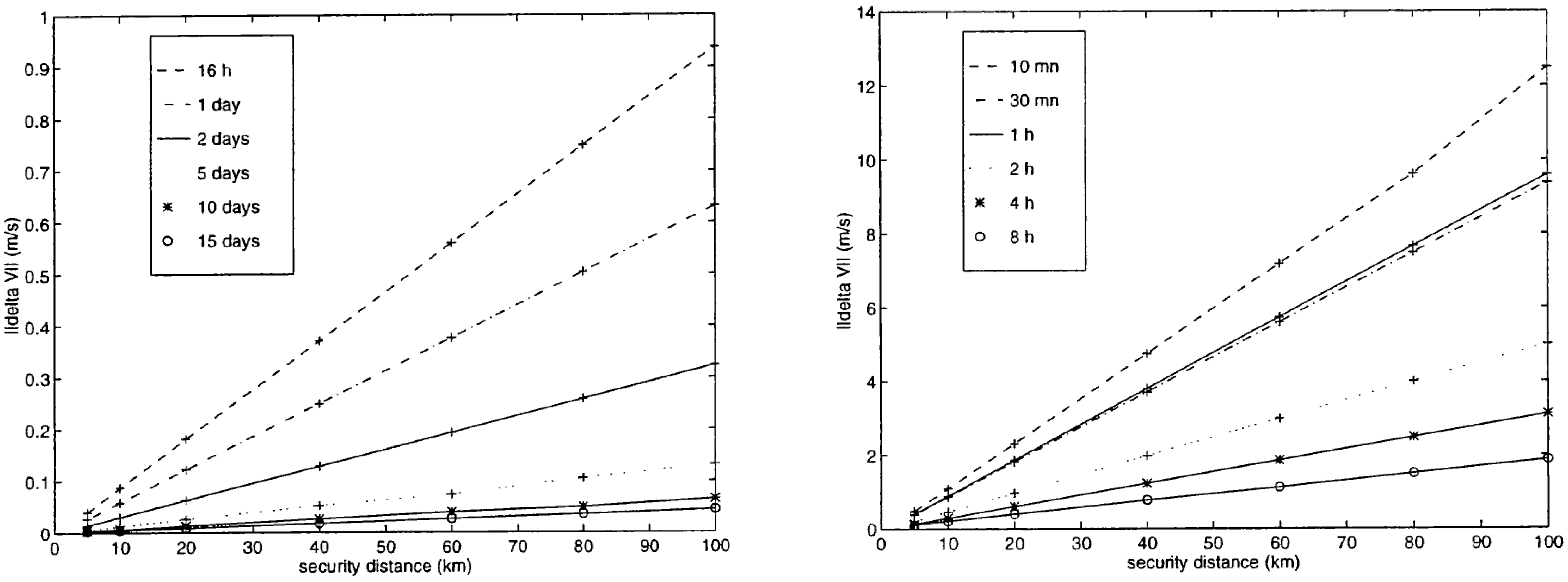

Figure 7. Cost of the maneuver versus security distance for different maneuver delays.

With any (available) value for the maneuver delay, the evolution of the cost of maneuver when security distance changes is remarkably linear, and of course, the ΔV maneuver is all the more important as the security distance is high. As for the influence of the maneuver delay on the cost of maneuver, the graphs show that the rule is different whether is it a late or an early maneuver:

- early maneuver: with maneuver delay values from 15 days to 1 hour (and with any security distance inferior to 100 km), ΔV increases exponentially as the maneuver decreases. We can infer from this result that, with such hypothesis, the less costly maneuver with a given security distance will always be the earliest one, so the maneuver should be performed as soon as possible. However, this rule will certainly not be true anymore if the length of the interval where we can choose the maneuver delay is about the satellite's orbit period: in such a case, the optimal maneuver is likely to be performed at the apogee (where the speed is minimum, and the orbit more sensitive to speed variations), which has not necessarily the earliest date.

In addition, this rule should be tempered on the basis of operational considerations: one should remember that the long-term predictions, which are conducted during the computation of an early maneuver, may not be very precise, so we should compensate this by choosing a larger security distance. This results in a more costly maneuver than it would be with a perfect propagation model. Another consequence of the imprecision of long-term predictions is that a maneuver should not be triggered too early, since it may turn out with later TLEs that it is useless.

- late maneuver: with a maneuver delay shorter than 1 hour (which is about the satellite's orbit period), the cost of the maneuver does not always increase when the maneuver delay is shortened: we see on Figure 7 that the 30 mn curve is *below* the 1 h curve, then ΔVs are higher again on the 10 mn curve. This phenomenon meets the fact that the ΔV required for the maneuver may have non-uniform variations when we move the maneuver point along the orbit. These variations depend on the characteristics of the close encounter problem, in particular the position (or anomaly) of the close encounter point on the orbit.

So, for a late maneuver, the minimum ΔV maneuver is not always the earliest one. Of course, the 1 hour limit between «late» and «early» maneuvers may be different with others satellite-debris pairs. Another difference between the two maneuver classes in our example is the direction of the optimal ΔV: for an early maneuver, it is always nearly colinear to the speed vector (angle < 1°), while the angle can be as high as 12° (with a 10 mn maneuver delay and a 100 km security distance) for a late maneuver.

Naturally, if we work under operational conditions, we have to make sure that the maneuver will not induce other dangerous close encounters with different debris or with the same one at different times.