PARTICLE FLUXES ON ORBITING STRUCTURES

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ABSTRACT

Commonly used meteoroid flux models (e.g. the Grün flux model [1]) consider the particle flux with respect to a non-moving (stationary w.r.t. the Earth surface) virtual randomly oriented plate. For the design of spacecraft, the design engineers have to consider the meteoroid fluxes with respect to orbiting oriented surfaces.

In this paper, an analytical expression is derived for the flux on an oriented surface, disregarding the Earth shielding effects using the omnidirectional flux model. The analytical expression can be used with either fixed particle velocities or with a velocity distribution function like the NASA 90 or Cour-Palais models. The analytical results are cross-checked with a simulation model using ray tracing.

1. INTRODUCTION

Two factors interfere with the particle impact flux computation on an orbiting spacecraft, namely :

1) The influence of the spacecraft velocity on the impacting flux, referred to as the „k" or "f_t" factor in several papers, and
2) The Earth shielding of the omnidirectional particle flux.

The first topic was investigated by various authors (e.g. [2]) and several flux scaling factors have been used in the flux prediction on oriented orbiting surfaces. Tools like the ESABASE/DEBRIS software consider these flux increase factors to predict the risk posed on spacecraft by the space debris and meteoroid environment. Provided that the analysis software use a mesh of the spacecraft for the analysis, the derivation of particle fluxes on oriented orbiting surfaces from the omnidirectional flux models have to be derived.

In this paper, a theoretical review and a numerical investigation of the spacecraft velocity effect are presented. The second topic, Earth shielding, is not explicitly treated here.

It is assumed that the particle flux is omnidirectional. This is the approach used in the derivation of the Grün model of meteoroid fluxes (see [1]).

The influence of the spacecraft velocity will first be investigated for two factors :

1) The "f_t" factor describing the relation between the flux on a moving oriented surface element and the flux on a virtual stationary surface element.
2) The "k_f" factor describing the relation between the flux on the forward side of a moving, oriented surface (or plate) to the average flux on the surface (average = 0.5·[forward + lee fluxes]).

As will be seen, these two factors are linked and depend on the orientation of the surface element. Drawn from the above two factors, two additional factors can be defined :

3) The "f_t^t" factor, which is obtained by evaluating the f_t factor on three perpendicular planes, which corresponds to the ratio between the flux on a moving "random tumbling" surface element and the flux on a virtual stationary surface element. This factor can also be used for the total flux increase on a symmetric spacecraft.
4) The "k" factor, which is the ratio between the flux on the forward side of a surface element (or plate) to the flux on a virtual stationary surface element.

2. THEORETICAL FORMULATION OF THE PARTICLE FLUX ON A MOVING PLATE

For the general case, we need to consider an omnidirectional particle flux and an arbitrary direction of motion of the plate.

As a starting point, the situation depicted in fig. 1 of a particle hitting the moving plate under an angle α_i is considered. The plate is moving with a velocity $\vec{v}_s$ which makes an angle β with the surface normal vector $\vec{s}$.

The impacting flux from the particles impinging with the velocity $\vec{v}_i$ on the plate is obtained by the product between the probability of a particle arriving from this direction, the scalar impact velocity and the cosine of the angle between impact velocity and surface normal.

The latter can be described by the scalar product between the impact velocity $\vec{v}_i$ and the surface element normal vector $\vec{s}$: $i = n(v_m) \cdot \vec{s} \cdot \vec{v}_i$ where $n(v_m)$ is the probability of a particle arriving with a velocity v_m, taken from the velocity distribution.

Proceedings of the Second European Conference on Space Debris, ESOC, Darmstadt, Germany, 17-19 March 1997, (ESA SP-393, May 1997)

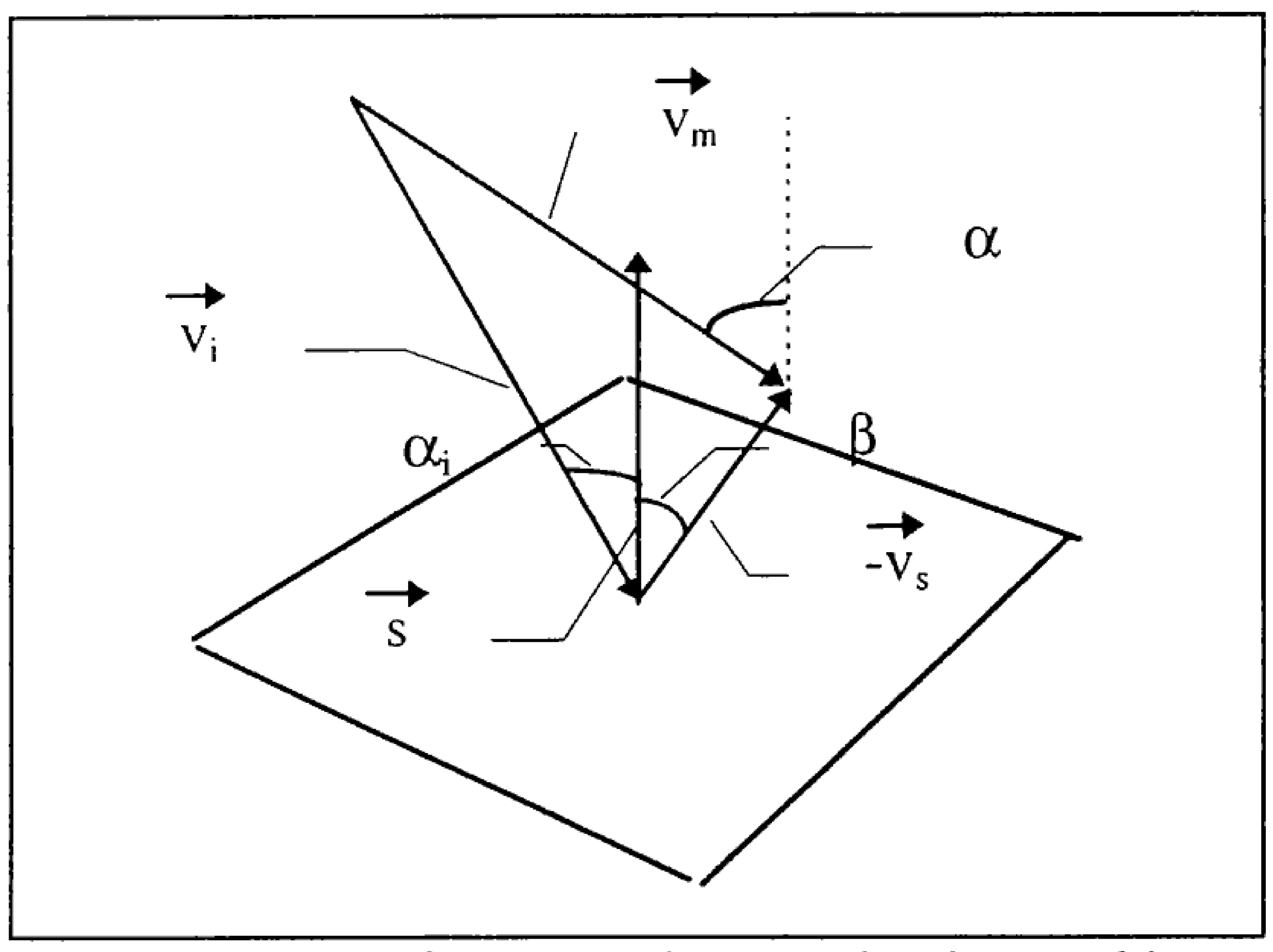

Figure 1 Oblique flux on a plate moving in an arbitrary direction

The impact velocity can be expressed as : $\vec{v}_i = \vec{v}_m + \vec{v}_s$, where $\vec{v}_s$ is chosen positive in the direction towards the plate.

Using the distributivity of the vector sum with respect to the scalar product, we obtain :

$$\begin{aligned} i &= n(v_m)\cdot\vec{s}\cdot(\vec{v}_m+\vec{v}_s) = n(v_m)\cdot(\vec{s}\cdot\vec{v}_m+\vec{s}\cdot\vec{v}_s) \\ &= n(v_m)\cdot s\cdot(v_m\cos\alpha + v_s\cos\beta) \end{aligned} \quad (1)$$

In order to assess the complete situation, we must evaluate the integration limits of the particle angle α of the whole „captured" spherical portion seen by the plate. The situation is illustrated in 2 dimensions in fig. 2.

As can be seen in fig. 2 above, the grazing impact directions allow to compute the limit particle directions knowing that the scalar product in this case is zero :

$$\vec{v}_i\cdot\vec{s} = \vec{v}_m\cdot\vec{s}+\vec{v}_s\cdot\vec{s} = 0$$

The numerical expression of the limit angle, which is constant around the whole captured sphere, is thus :

$$\cos\alpha_{lim} = \frac{-v_s\cdot\cos\beta}{v_m} \quad (2)$$

In order to obtain the total impact flux I on the plate, we must integrate over the sphere captured by the plate and the velocity distribution : $I = \int_0^{\infty}\int_0^{\alpha_{lim}(v_m)} di$

$$di = i(v_m,\alpha)\cdot ds = n(v_m)\cdot(\vec{v}_i\cdot\vec{s})\cdot 2\pi\sin\alpha\cdot d\alpha\cdot dv_m$$

$$di = 2\pi\cdot n(v_m)(v_m\cos\alpha + v_s\cos\beta)\sin\alpha\cdot d\alpha\cdot dv_m$$

2πsinα•dα is the surface integration argument computed from the integration over φ of the spherical surface differential sinα•dφ•dα .

We thus obtain the following integral :

$$I = \int_0^{\infty}\int_0^{\alpha_{lim}} 2\pi\cdot n(v_m)\cdot(v_m\cos\alpha+v_s\cos\beta)\cdot\sin\alpha\cdot d\alpha\cdot dv_m \quad (3)$$

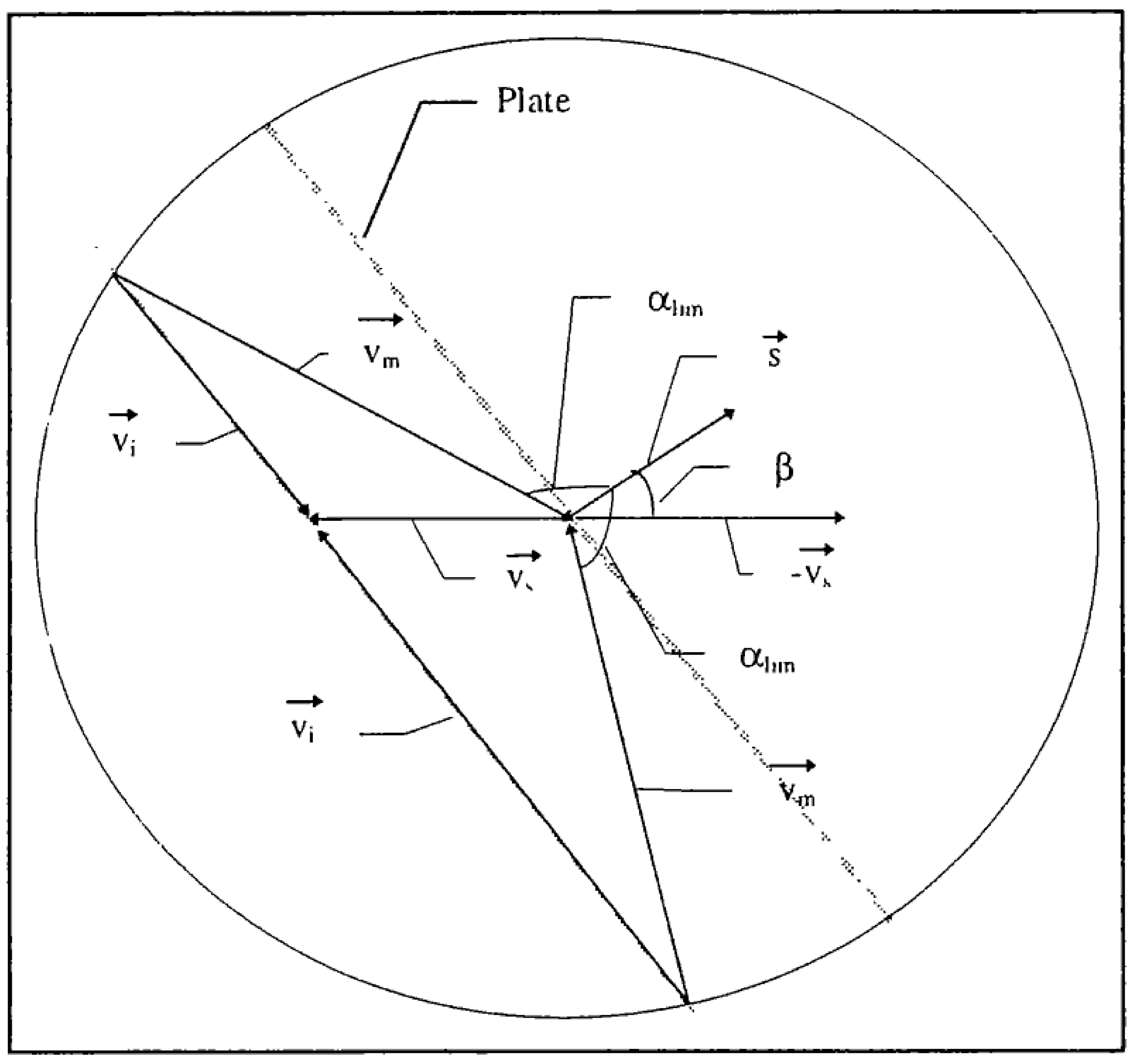

Figure 2 Integration limits for an arbitrary direction of motion

α_{lim} is obtained from equation (2).

2.1 Particle flux on a plate with unique velocity

We shall first consider the situation where the particles have a unique velocity. This case corresponds to the numerical analysis described in chapter 3 and to the investigations with average particle velocities in [2].

With a unique particle velocity, equation (3) boils down to a single integral :

$$I = 2\pi\cdot n_0\int_0^{\alpha_{lim}}(v_m\cos\alpha+v_s\cos\beta)\cdot\sin\alpha\cdot d\alpha$$

with n_0 being the probability of a particle arriving from a random direction. This integral can easily be solved analytically :

$$I = 2\pi\cdot n_0\int_0^{\alpha_{lim}}(v_m\cos\alpha+v_s\cos\beta)\cdot\sin\alpha\cdot d\alpha$$

$$I = 2\pi\cdot n_0\left[\frac{-v_m}{2}\cos^2\alpha - v_s\cos\beta\cdot\cos\alpha\right]_0^{\alpha_{lim}}$$

Using equation (2) for α_{lim}, we obtain :

$$I = 2\pi\cdot n_0\left[\frac{-v_m}{2}\left(\frac{v_s^2\cos^2\beta}{v_m^2}-1\right)-v_s\cos\beta\left(\frac{-v_s\cos\beta}{v_m}-1\right)\right]$$

$$I = 2\pi\cdot n_0\left[\frac{v_m}{2}-\frac{v_s^2\cos^2\beta}{2v_m}+\frac{v_s^2\cos^2\beta}{v_m}+v_s\cos\beta\right]$$

$$I = 2\pi\cdot n_0\frac{v_m^2+v_s^2\cos^2\beta+2v_m v_s\cos\beta}{2v_m}$$

$$I = \frac{\pi\cdot n_0}{v_m}(v_m+v_s\cos\beta)^2$$

The above expression for I permits the analytical computation of the k and f_t factors. The f_t factor needs the total flux on an orbiting structure :

For an oriented plate:

$$I_{tot} = \frac{\pi \cdot n_0}{v_m}\left[(v_m + v_s\cos\beta)^2 + (v_m - v_s\cos\beta)^2\right]$$

For the case with $v_s = 0$: $I_{tot} = 2\pi n_0 v_m$.

We can now derive the analytical expressions of the two factors :

$$k_t = \frac{2 \cdot I(+v_s)}{I(+v_s) + I(-v_s)} \quad (4)$$

$$f_t = \frac{I_{tot}(v_s)}{I_{tot}(v_s = 0)} \quad (5)$$

A spacecraft can be symbolised by a regular box, moving with one side normal to the flight direction, i.e. two sides with $\beta = 0$ and four sides with $\beta = 90°$. This of course implies a symmetrical spacecraft structure. We can now derive the f_t^t factor for the whole spacecraft :

$$f_t{}' = \frac{1}{6v_m^2}\left[4v_m^2 + (v_m + v_s)^2 + (v_m - v_s)^2\right] = \frac{6v_m^2 + 2v_s^2}{6v_m^2}$$

We finally obtain :

$$f_t{}' = 1 + \frac{v_s^2}{3v_m^2} \quad (6)$$

Equation (6) is identical to the formula of f_t derived by Don Kessler for the case $v_s \le v_m$ in [2].

In the same way one can compute the k factor:

$$k = \frac{I(+v_s)}{I(v_s = 0)} = \frac{2 \cdot I(+v_s)}{I(+v_s) + I(-v_s)} \cdot \frac{I(+v_s) + I(-v_s)}{2 \cdot I(v_s = 0)} = f_t \cdot k_t \quad (7)$$

Tables 1 and 2 show the f_t, k_f and k factors for two velocity values and a set of values of β.

$$I^{*+} = \frac{I(+v_s)}{\pi n_0 v_m} \qquad I^*_{tot} = \frac{I_{tot}(+v_s)}{\pi n_0 v_m}$$

Case 1: $v_s = v_m$

β [deg]	0	15	30	45	60	75	90
I^{*+}	4	3.86	3.48	2.91	2.25	1.58	1
I^*_{tot}	4	3.87	3.5	3.0	2.5	2.13	2
k_f	2.0	1.99	1.99	1.94	1.8	1.49	1.0
k	4.0	3.84	3.48	2.91	2.25	1.59	1.0
f_t	2.0	1.93	1.75	1.5	1.25	1.07	1.0

Table 1 Analytical values for k and f_t with $v_s = v_m$

$f_t^t = 1.333$

Case 2: $v_s = 7.6$ km/s; $v_m = 16.8$ km/s

β [deg]	0	15	30	45	60	75	90
I^{*+}	2.11	2.06	1.94	1.74	1.5	1.25	1
I^*_{tot}	2.41	2.38	2.31	2.2	2.1	2.03	2
k_f	1.75	1.73	1.68	1.58	1.43	1.23	1.0
k	2.1	2.06	1.93	1.74	1.5	1.24	1.0
f_t	1.2	1.19	1.15	1.1	1.05	1.01	1.0

Table 2 Analytical values for k and f_t with $v_s = 7.6$ km/s; $v_m = 16.8$ km/s

$f_t^t = 1.068$

This corresponds to the value given in [1] (page 18).

2.2 Flux on a plate with a given velocity distribution

The general expression of I is given by equation (3) in paragraph 2.1. Proceeding as in the previous section, we can solve the first integration step (over α). We now have the following equation :

$$I = \int_0^\infty \frac{\pi}{v_m} n(v_m) \cdot (v_m + v_s\cos\beta)^2 dv_m \quad (8)$$

In order to proceed further, we need to define the form of the velocity distribution $n(v_m)$.

An easy way to approach the velocity distribution is to define a function composed of a series of straight curves as depicted in figure 3. Actually most velocity distributions can be approximated with such a function.

To derive the general formulation for I, let's integrate the portion of the velocity function between v_1 and v_2 :

$$v_1 < v_m < v_2 \qquad n(v_m) = n_2 + (n_1 - n_2)\frac{v_2 - v_m}{v_2 - v_1}$$

In fig. 3, one can split the integration into four steps.

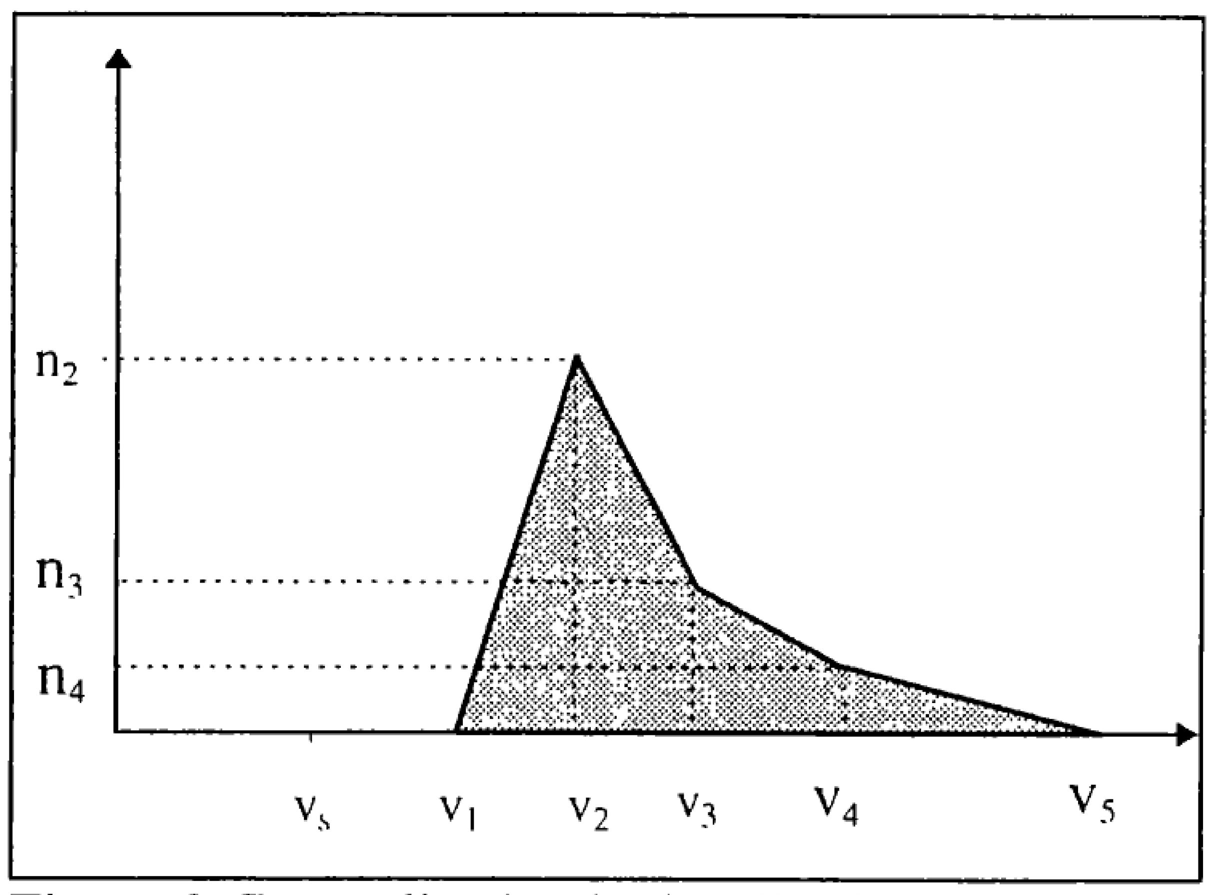

Figure 3 Generalised velocity distribution

We'll first compute the first step I_2 between v_1 and v_2.

$$I_2 = \pi \int_{v_1}^{v_2} \left(n_2 + \frac{(n_1 - n_2)(v_2 - v_m)}{v_2 - v_1}\right) \frac{(v_m + v_s \cos\beta)^2}{v_m} \cdot dv_m$$

The above expression can be analytically solved :

$$I_2 = \frac{\pi}{v_2 - v_1} \int_{v_1}^{v_2} \{-(n_1 - n_2)v_m^2$$
$$+[n_2(v_2 - v_1) + (n_1 - n_2)(v_2 - 2v_s\cos\beta)]v_m$$
$$+[2n_2(v_2 - v_1) + (n_1 - n_2)(2v_2 - v_s\cos\beta)]v_s\cos\beta$$
$$+[n_2(v_2 - v_1) + (n_1 - n_2)v_2]\frac{v_s^2\cos^2\beta}{v_m}\}dv_m$$

$$I_2 = \frac{\pi}{v_2 - v_1}\{-(n_1 - n_2)\frac{v_m^3}{3}$$
$$+[n_2(v_2 - v_1) + (n_1 - n_2)(v_2 - 2v_s\cos\beta)]\frac{v_m^2}{2}$$
$$+[2n_2(v_2 - v_1) + (n_1 - n_2)(2v_2 - v_s\cos\beta)]v_s\cos\beta \cdot v_m$$
$$+[n_2(v_2 - v_1) + (n_1 - n_2)v_2]v_s^2\cos^2\beta \cdot \ln(v_m)\}_{v_1}^{v_2}$$

$$I_2 = \frac{\pi}{6(v_2 - v_1)}\{-2(n_1 - n_2)(v_2^3 - v_1^3)$$
$$+3[n_2(v_2 - v_1) + (n_1 - n_2)(v_2 - 2v_s\cos\beta)](v_2^2 - v_1^2)$$
$$+6[2n_2(v_2 - v_1) + (n_1 - n_2)(2v_2 - v_s\cos\beta)]v_s\cos\beta \cdot (v_2 - v_1)$$
$$+6[n_2(v_2 - v_1) + (n_1 - n_2)v_2]v_s^2\cos^2\beta \cdot \ln\left(\frac{v_2}{v_1}\right)\}$$

The above expression can be rearranged:

$$I_2 = \pi\{\frac{2n_2 + n_1}{6}v_2^2 - \frac{2n_1 + n_2}{6}v_1^2 + \frac{n_1 - n_2}{6}v_2 v_1$$
$$+(n_1 + n_2)v_s\cos\beta(v_2 - v_1) + (n_2 - n_1)v_s^2\cos^2\beta$$
$$+\frac{n_1 v_2 - n_2 v_1}{v_2 - v_1}v_s^2\cos^2\beta \cdot \ln\left(\frac{v_2}{v_1}\right)\}$$

Using the expression above, one can formulate the general expression as the sum over all the $(n;v_m)$ points of the velocity distribution :

$$I = \pi\sum_{i=2}^{m} \{\frac{2n_i + n_{i-1}}{6}v_i^2 - \frac{2n_{i-1} + n_i}{6}v_{i-1}^2 + \frac{n_{i-1} - n_i}{6}v_i v_{i-1}$$
$$+(n_{i-1} + n_i)v_s\cos\beta(v_i - v_{i-1})$$
$$+(n_i - n_{i-1})v_s^2\cos^2\beta + \frac{n_{i-1}v_i - n_i v_{i-1}}{v_i - v_{i-1}}v_s^2\cos^2\beta \cdot \ln\left(\frac{v_i}{v_{i-1}}\right)\} \quad (9)$$

Equation (9) above is easily programmable. The two standard meteoroid velocity distributions, the NASA 90 model and the Cour-Palais model were approximated with 5 (n;vm) points and computed with equation (9). The results are presented in tables 3 and 4.

For comparison with the data in chapters 2.2 and 3, the plate velocity was set at 7.6 km/s.

Analytical flux computation of the NASA 90 model

The NASA 90 velocity distribution can be approximated as follows :

Velocity	Normalised Probability
11.0	1.0
15.2	1.0
20.0	0.25
28.0	0.03
50.0	0.0

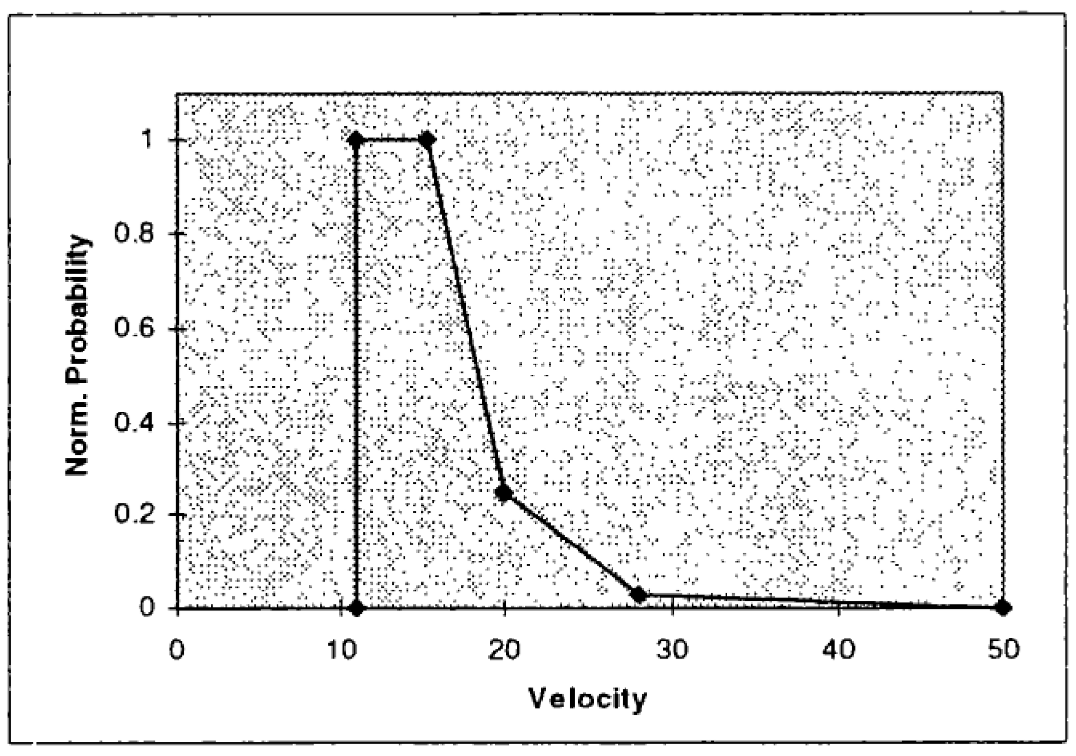

Figure 4 NASA velocity distribution

The analysis over the above velocity distribution with equation (9) gives the following results:

	$v_s = 0$	$v_s = 7.6$			
β [deg]	N/A	0	30	60	90
Flux positive	451.6	966	885.3	683.5	451.6
Flux total	903.2	1106	1055	953.8	903.2
k_f factor	1.0	1.75	1.68	1.43	1.0
k factor	N/A	2.14	1.97	1.52	1.0
f_t factor	N/A	1.22	1.17	1.06	1.0

Table 3 NASA 90 Velocity distribution $v_s = 7.6$,

The f_t^t factor amounts to 1.075. This figure compares closely to the 1.068 evaluated from table 2.

Analytical flux computation of the Cour-Palais model

The Cour-Palais velocity distribution can be approximated as follows :

Velocity	Normalised Probability
11.0	0.0
15.5	1.0
27.5	0.24
35.0	0.05
50.0	0.0

The analysis over the above velocity distribution with equation (9) gives the results in table 4.

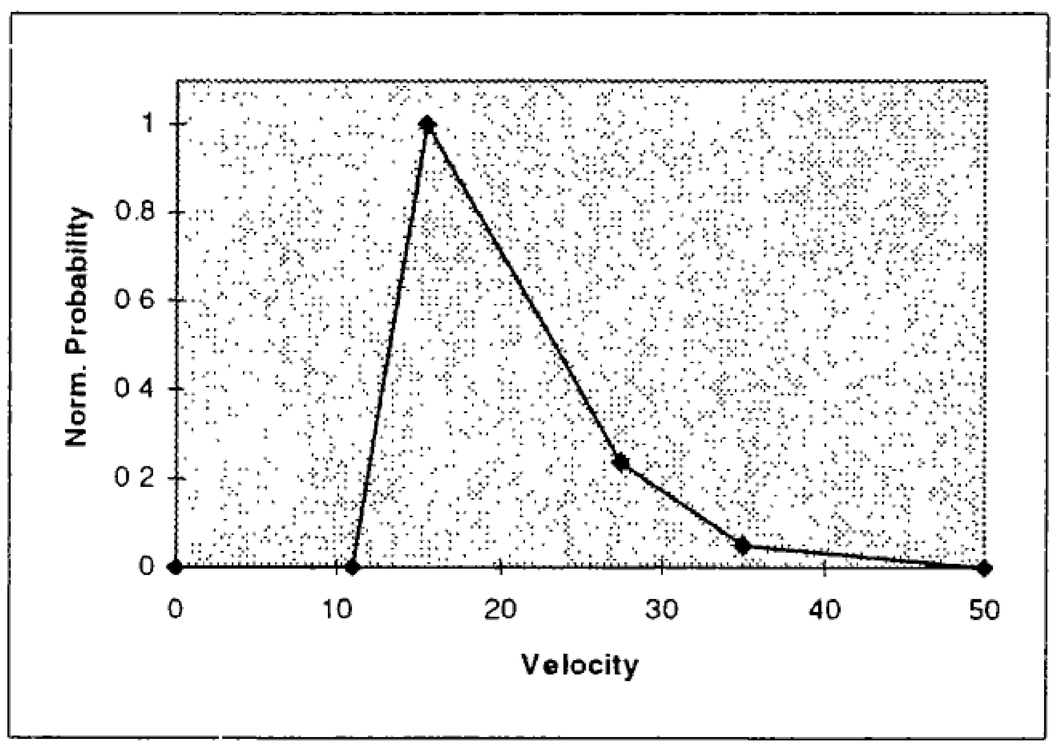

Figure 5 Cour-Palais velocity distribution

	$v_s = 0$	$v_s = 7.6$			
β [deg]	N/A	0	30	60	90
Flux positive	724.0	1362	1265	1017	724
Flux total	1448	1660	1607	1501	1448
k_f factor	1.0	1.64	1.57	1.36	1.0
k factor	N/A	1.89	1.74	1.41	1.0
f_t factor	N/A	1.15	1.11	1.04	1.0

Table 4 Cour-Palais Velocity distribution $v_s = 7.6$,

The $f_t^{'}$ factor amounts to 1.049.

3. NUMERICAL ANALYSIS OF PARTICLE FLUXES ON A MOVING PLATE

As a cross-check of the theoretical expressions derived in chapter 2, a numerical simulation of particle fluxes on a moving target plate using ray-tracing was performed

The two programs simulated this situation by :

1) Firing rays from a unit sphere with random particle direction and fixed particle and plate velocities and to assess the impacts and impact fluxes on a plate centred on the origin of the sphere.
   This simulates the true environment.
2) Firing rays from the plate centre with random particle direction and fixed particle and plate velocities and to assess the impact flux on the plate.

The second method is typically used for engineering simulation tools like ESABASE/DEBRIS.

3.1 Rays fired from a unit sphere

The program K_SPHERE simulates the "true" environment by firing particles from a unit sphere centred on the plate centre in random directions. It is checked whether the impact velocity trajectory hits the plate; and if so, the impact flux is computed.

The program input is read from a free format file :

- Number of rays to fire
- Particle velocity
- Plate dimension
- Plate velocity (scalar)
- Plate velocity β angle (angle between velocity and Z-axis)
- Plate velocity φ angle (angle between the projection of the velocity on the XY-plane and the X-axis)

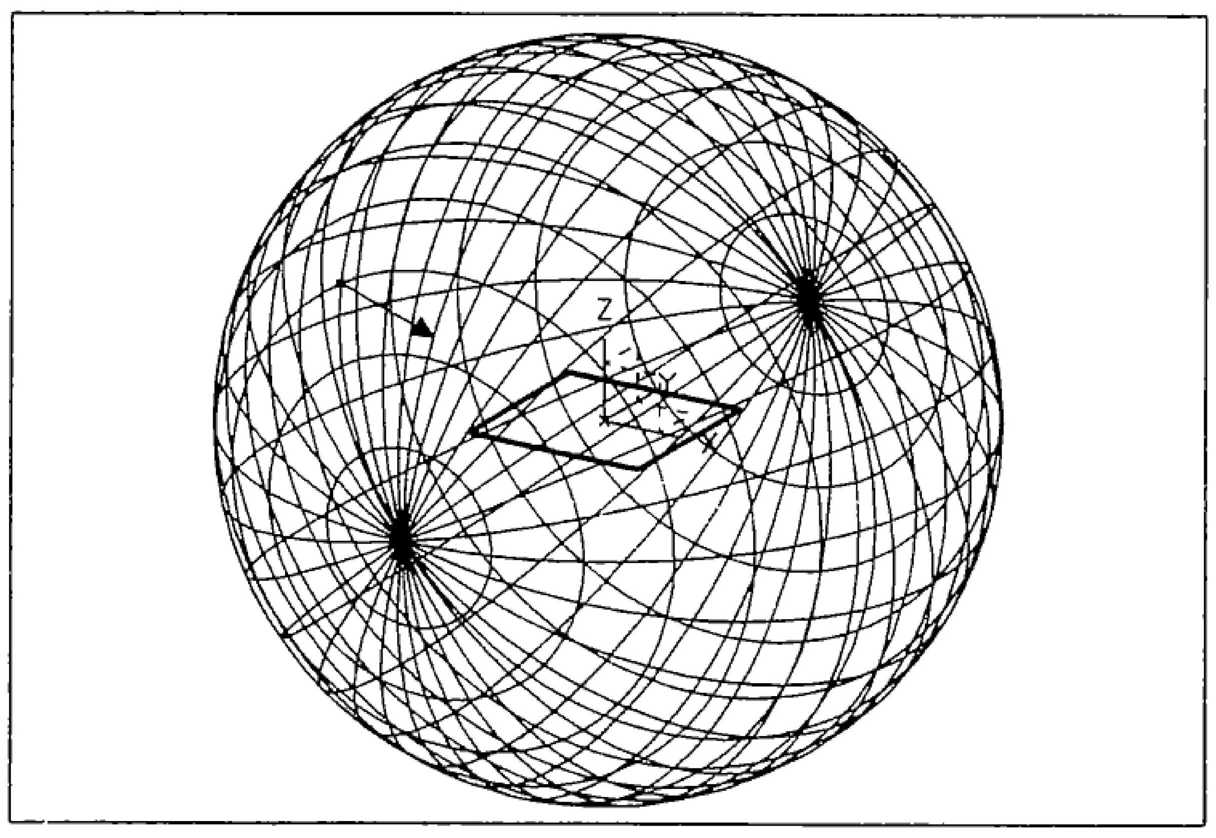

Figure 6 Geometrical set-up of program K_SPHERE

Program flow

1) Initialise random number generator with the system time.
2) Zero all counters.

Loop over rays :

3) Get random firing point on the unit sphere.
4) Get random firing direction of the particle.
5) Compute the impact velocity
   $\vec{v}_i = \vec{v}_{particle} - \vec{v}_{plate}$.

For each axis plane (XY, YZ, ZX) :

6) Check whether impact vector direction passes through the plate. If YES, then:
   - If the impact direction is opposite the plate positive axis (+Z, +X or +Y), a hit on the positive side is recorded.
   - If the impact direction is the same as the plate positive axis (+Z, +X or +Y), a hit on the negative side is recorded.
7) If a hit is recorded: compute the flux as the product between the scalar impact velocity and a scaling factor.
8) Summarise results :
   - Total number of rays hitting the plate (positive side, negative side, both)
   - Total impact flux hitting the plate (positive side, negative side, both)
   - k factor

Results

The program K_SPHERE was run with 2.0E6 rays. The velocities equal the case of [2], i.e. with 16.8 particle velocity and 7.6 plate velocity. The flux scaling factor is 0.001.

	$v_s = 0$	$v_s = 7.6$			
β [deg]	N/A	0	30	60	90
Nhit positive	12893	21278	20071	16835	12510
Nhit negative	12787	5855	6567	8890	12567
Nhit total	25680	27133	16638	25725	25077
Flux positive	216.6	455.6	420.0	328.5	215.8
Flux negative	214.8	65.5	80.2	129.1	216.6
Flux total	431.4	521.1	500.2	457.6	432.4
Flux on 3 plates	1301	1386	1390	1390	1386
k_f factor	1.004	1.75	1.68	1.44	1.0
f_t factor	N/A	1.21	1.16	1.061	1.0
k factor	N/A	2.12	1.95	1.53	1.0
f_t' factor	N/A	1.065	1.068	1.068	1.065

Table 5 Results from K_SPHERE

	$v_s = 0$	$v_s = 7.6$			
β [deg]	N/A	0	30	60	90
Nhit positive	12913	18894	18054	16027	12979
Nhit negative	13087	7106	7946	9973	13021
Nhit total	26000	26000	26000	26000	26000
Flux positive	108.1	230.2	211.0	165.1	109.0
Flux negative	109.6	32.7	40.5	65.3	109.2
Flux total	217.7	292.8	251.5	230.4	218.2
Flux on 3 plates	654.9	700.3	700.0	700.0	700.3
k_f factor	0.99	1.75	1.68	1.43	1.0
f_t factor	N/A	1.21	1.16	1.058	1.0
k factor	N/A	2.12	1.95	1.51	1.0
f_t' factor	N/A	1.069	1.069	1.069	1.069

Table 6 Results from K_PLATE

3.2 Rays fired from the plate

The program K_PLATE corresponds to the engineering simulation approach firing particles from the plate in random directions. The impact flux is computed. The program logic is similar to K_SPHERE.

Program flow

1) Initialise random number generator with the system time.
2) Zero all counters.

Loop over rays :

3 Get random firing direction of the particle.
4) Compute the impact velocity

$$\vec{v}_i = \vec{v}_{particle} - \vec{v}_{plate}.$$

For each axis plane (XY, YZ, ZX) :

5) Compute the flux as the scalar product between the impact velocity vector and the surface normal, multiplied by a scaling factor. The scalar product corresponds to the $\cos\alpha_i$ term of the analytical expressions in chapter 2, and is necessary here since we fire the rays *from* the plate.
6) Summarise results :
   - Total number of rays hitting the plate (positive side, negative side, both)
   - Total impact flux hitting the plate (positive side, negative side, both)
   - k factor

Results

The program K_PLATE was run with 26000 rays. The plate and particle velocities are 7.6 and 16.8 km/s respectively. The flux scaling factor is 0.001.

4. CONCLUSIONS

Full agreement between analytical and numerical methods are achieved for the computation of the *k* and f_t factors. Agreement is also achieved with [2].

A few important observations can further be stated :

- For a plate, the flux is strongly dependent on the orientation of the plate with respect to the flight direction. Also the total flux (i.e. the f_t factor) depends on this. For a plate in LEO, the flux increase when the plate is normal to the flight direction compared to the flux when parallel to the flight direction amounts to *20%*.
- The f_t factor as it is stated in [2] is only valid for perfectly symmetrical structures (or randomly oriented structures). For non-symmetric structures, as are most spacecraft, the f_t factor will depend on the space-craft orientation.
- The ray tracing technique (as used in the enhanced ESABASE/DEBRIS software) perfectly captures the true particle flux situation, provided that the $\cos\alpha$ term (angle between impact direction and surface normal) is introduced in the flux computation.

Finally, this papers proves the suitability of the ray tracing method for the simulation of particle fluxes on orbiting space structures.

REFERENCES

[1] E. Grün, H. Zook, H. Fechtig, R. Giese, 1985: *Collisional Balance of the Meteoritic Complex*

[2] Don Kessler et al, manned space center, 1971: *A guide to using meteoroid-environmental models for experiment and spacecraft design applications*