THE NEW FAMILY OF METHODS FOR REVEALING LATENT PERIODICITIES IN SIGNALS

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ABSTRACT

The new approach to solving the problem of revealing real harmonic structure of physical processes rendered by their signatures, is presented and analysed. The advantages of the proposed methods are as follows: 1) the presence of a non-periodical component of a rather complex form is assumed; 2) the signatures to be analysed can be given at fairly short time intervals; 3) the number of periodical components is allowed to be unknown in advance; 4) no need to know beforehand the frequency range; 5) the influence of measurement errors to the analysis quality is essentially softened; 6) periodical components may differ from harmonic form. The methods can be applied to the analysis of radiometric, photometric, spectrometric, polarimetric and other signals, of solar radiation variation process, processes related with space equipment and spacecraft vibrotests and so on.

Revealing latent periodicities is an important stage of structure analysis of some processes related with space surveillance or different space technologies. For example, this problem arises when investigating the el-set evolution process fine structure, the solar radiation intensity variations and other helio-geophysical processes, when analysing signatures of observed radar, photometric, spectrometric, polarimetric and other space related signals, when investigating mechanical processes generated during vibrotests of space techniques, supervelocity impact tests, resonance phenomena and so on.

Due to immensity of application area of the methods for revealing latent periodicities, as well as a considerable age of the problem itself, presently there exists a lot of such methods (Ref. 1) however suffering from grave shortcomings. The last forced us to look for new approaches to construct this kind of methods.

The most common shortcoming of the known methods is that they were mostly developed for polyharmonic processes, that is, containing no non-periodical component. Many methods (Refs. 2, 3) require processing very long signatures for fine and clear separating all the harmonics. The other methods (Refs. 4, 5) require to indicate in advance the number of periodical components. All the selective methods need beforehand determination of the frequency range of harmonics sought for. Most methods are very sensitive to the measurement errors.

The general idea of the approach proposed here consists in creating special mapping D transferring the initial analysed process into a special function minima of which are in a crisp relationship with periods of periodical components of the process. Namely, this function has characteristic "sharp" minima at points divisible by the period of the highest harmonic. The envelope of these minima in its turn turns into minima at points divisible by the period of the second highest harmonic and so on. So the D-image of the initial process renders the information on its harmonic composition easily accessible for decoding.

Before proceeding to describe this approach in details let us make two notes:

- Here, the problem consists in phenomenological revealing the real harmonic structure of processes presented by their observed signatures rather than looking for theoretically anticipated harmonics in signals.
- For solving this problem, it is generally incorrect to apply the harmonic analysis methods like Fourier expansions, since such approach imposes to the analytical rendering of the process an artificial occasional parameter - the dimension of the signature interval domain, which is alien to physical essence of the process analysed, though the skillful handling with this technique can give useful results (Ref. 6).

So, let the process have a form

$$X(t)=\sum_{k=0}^{\bar{k}} a_k t^k + \sum_{r=1}^{\bar{r}} A_r \sin(\frac{2\pi}{T_r} t + \varphi_r), \qquad (1)$$

its signature $\overline{X}(t)$ being determined over the time interval $[0, \bar{t}\,]$. Parameters $\bar{k}$, $\bar{r}$, A_r, T_r, φ_r are unknown. All T_r and $\bar{r}$ should be determined.

Introduce a new independent variable τ having non-negative real meanings and define the next function

$$\zeta^n(t,\tau) = X^n(t+\tau) - X^n(t) \ ,$$

where X^n is the n-th derivative of X(t). The function ζ^n is determined for all t and τ not bringing the process out of its signature domain $[0, \bar{t}\,]$.

Next, by definition,

$$D_X^n(\tau) = \frac{1}{\bar{t}-\tau} \int_0^{\bar{t}-\tau} |\zeta^n(t,\tau)| dt. \qquad (2)$$

By that, D-mapping is constructed over the set of processes of form given by Eq. 1.

The differentiation carries two functions here: damping the influence of a non-periodical polynomial component and increasing the ratios of amplitudes of higher harmonics to lower ones. The modulus operation imparts clear sense to the integration here and, besides, causes a characteristic "sharp" form of the function $D_X^n(\tau)$'s minima.

After discovering and filtration of the highest harmonic, the D-image of the remainder of the process should be built, which could be used then for determination of the second highest harmonic's period and so on. The following simple "sliding" filter can be used here for eliminating the highest harmonic found:

$$\widetilde{X}(t) = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} X(\lambda) d\lambda,$$

where T is the period of a harmonic to be suppressed.

In those cases when periods of harmonics differ significantly (by several times), the filtration procedure can be avoided, since the envelopes of the minima can be used for revealing harmonics.

So, the main part of the decision procedure is the search for minima (several first ones for more accuracy of period determination) of D-image of the initial process and of its remainders of the 1st, 2nd, 3d,.... orders.

The function $D_X^n(\tau)$ has a set of important properties the use of which allows to simply construct a reliable procedure for searching its minima. This properties are as follows:

- $D_X^n(\tau)$ is concave between each two consecutive minima.
- It turns into minima at points kT, k = 0, 1, 2, ..., where T is the highest harmonic's period.
- The first derivative of $D_X^n(\tau)$ exists everywhere except its minima.
- The second derivative of $D_X^n(\tau)$ converges to zero when τ approaches any minimum from any side.

These properties by themselves prompt the way of organizing the economy minimization procedure.

Now some words on diagnostics of capabilities of one or another method of D-family as to revealing harmonic structure of one or another process. It can be shown that the absolute and relative errors ΔT_1 of the highest harmonic's period determination due to influence of the rest harmonics can be estimated "from above" respectively as

$$|\Delta T_1| < \frac{2}{\pi(\bar{s}_1+1)} \frac{A_2 T_1^{n+1}}{A_1 T_2^n}, \quad \frac{|\Delta T_1|}{T_1} < \frac{2}{\pi(\bar{s}_1+1)} \frac{A_2 T_1^n}{A_1 T_2^n}, \qquad (3)$$

where T_2 - period of the second highest harmonic (as the most influential), A_2 - its amplitude, $\bar{s}_1$ - the number of accounted minima of function $D_X^n(\tau)$ for calculating the estimate of T_1.

If the feasible relative errors $\sigma_i = \frac{|\Delta T_i|}{T_i}$ are given, then by that the class of processes X(t) is determined for which the problem of revealing their harmonic structures is solvable.

To conceive this, introduce designations $A = \frac{A_2}{A_1}$ and $T = \frac{T_2}{T_1}$ and neglect the influence of harmonics lower than the second highest one. Then in accordance with Eq. 3

$$\sigma > \frac{2}{\pi(\bar{s}_1+1)} \cdot \frac{A}{T^n}.$$

Consider the mapping M which for each process (1) puts into correspondence a point in AT-plane with coordinates A and T. M-mapping transfers all the processes (1) into the area Ω of the first quadrant consistent with inequality T > 1. Of interest are only the processes being mapped into the area Σ (σ_1, n, $\bar{s}_1$) which is cut out of Ω by the curve

$$A(T) = \frac{\pi\sigma_1(\bar{s}_1 + 1)}{2} T^n$$

and located under the curve. So the inverse M-image of the Σ area (that is $M^{-1}\Sigma$) represents a set of processes for which D-mapping solves the structure revealing problem with the given accuracy. The simplicity of constructing the Σ-area should be acknowledged.

Now consider some examples of application of the 1st order D-mapping (i.e. D_1-mapping) as the more investigated one (Ref. 7).

With the help of D_1-mapping, the harmonic structures of real processes of orbital elements variations were studied. As a result, in particular, the presence of periodical components like

$$A_n \sin[n\omega(t) + \varphi_n]$$

predicted by D.Brouwer (Ref. 8) and M.Kislick (Ref. 9) was confirmed. Here, n is a natural number, $\omega(t)$ - argument of perigee, φ_n - phase of the harmonic. Their parameters were determined and the harmonics were isolated in pure form. And also there were found the fact of "upswinging" of these components towards the end of a satellite orbital existence.

Structure of solar radiation intensity variations was analysed. By means of D_1-mapping predominantly 3 periodical components were revealed in some signatures over the time intervals of 50 - 140 days. Below are their estimates:

T1: 6.2; 8.0; 6.43; 6.91; 7.21
T2: 14.1; 15.18; 16.3
T3: 26.8; 26.84; 27.8; 28.3 .

A notable dispersion of these estimates witnesses the instability of the process and the availability of significant measurement errors.

As for analysis of photometric signals, the archive data recorded in 1967 were taken. Those were signatures of photometric signals proportional to the reflected light flux from some Earth satellites:

MIDAS 5. The lowest and most powerful harmonic's period was 9.972 s. Such was perhaps the period of the satellite's own rotation. One more harmonic was revealed having the period 0.785 s (1.27 Hz), which can be accounted for by a complex configuration of the satellite surface or by its resilient vibration.

MIDAS 6 (spent). Of interest are 2 intensive periodical components with the same period 3.63 s but differently phased, the shift of phases being slow changed. These can be explained by rotation of the satellite and precession of the axes of rotation.

MIDAS 7. Three harmonics were revealed. Their periods are 28.94 s, 1.436 s (0.696 Hz) and 0.947 s (1.05 Hz).

In all above examples periodical components had forms badly different from harmonic. Nevertheless, all of them were revealed with no problems.

A process of star brightness variation was analysed. As an example observational data of HV-2063 from the Small Magellanic Cloud was taken (Ref. 10). The two steady periodicities were revealed with periods 3.73 and 11.2 days, whereas in Ref. 10 only the last one was mentioned.

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