CALCULATION OF CHARACTERISTICS OF ELECTROMAGNETIC WAVE PROPAGATION IN THE NEAR SPACE AT A PRESENCE OF ORBITAL DEBRIS WITH ARBITRARY ELECTRIC AND PHYSICAL CHARACTERISTICS IN THE PROPAGATION CHANNEL

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ABSTRACT

Signals of the short wave part of centimetre, millimetre and optic wave length ranges are being broadly used in the communication, location and remote probing systems with space channels. In this case the presence of orbital debris, space dust and other discrete formations in the propagation channel may have substantial influence upon the characteristics of wave processes, and thus upon the data system quality. Mathematical models, procedures and software complex realizing them for studying the orbital debris effect on the characteristics of electromagnetic wave propagation in the near space are analysed in this paper.

1. INTRODUCTION

It is known (Ref. 1) that the task of studying the electromagnetic wave propagation and scattering in the random discrete non-uniform media, including the formations consisting of space debris, is in general statement solved by two steps: first of all it is necessary to analyse the characteristics of scattering and absorption for a single particle and then to study the characteristics of the wave process at a presence of a large number of particles, distributed arbitrary in space. The complexity of solving this task even in particular cases leads to a necessity to use the numerical methods for obtaining the final quantitative result. Besides, it is rather interesting to make studies at the most general assumptions concerning the characteristics of discrete medium and electromagnetic wave propagating through it.

That is why an aim was set to develop the multi-purpose mathematical models, procedures and software complex realizing them, which would allow to calculate the characteristics of propagation in the near space for both optic waves and radiowaves at a presence of formations consisting of the orbital debris and having arbitrary density and arbitrary dimensions and electrophysical characteristics for separate particles.

2. ELECTROMAGNETIC WAVE SCATTERING AND ABSORPTION BY A SINGLE PIECE OF ORBITAL DEBRIS

The back scattering cross-section σ_b, total scattering cross-section σ_s, absorption cross-section σ_a and attenuation cross-section $\sigma_t = \sigma_s + \sigma_a$ are usually used for describing the scattering and absorbing properties of a single particle. These characteristics may be calculated using the mathematical methods, known from the diffraction theory (Refs. 1-3): Rayleigh approximation, Born (Rayleigh-Debye) approximation, Ventzel-Kramers-Brillouin approximation and Mie theory.

The Mie theory, which gives a general rigorous analytic solution for a uniform spherical particle by factoring the fields into elementary spherical functions, is the universal one. Application of Mie theory for a particle with complex relative dielectric permeability $\dot{\varepsilon} = \dot{\varepsilon}' - i\dot{\varepsilon}''$ and complex relative magnetic permeability $\dot{\mu} = \dot{\mu}' - i\dot{\mu}''$ allows to obtain characteristics in the form of the following infinite sums:

$$\sigma_b = \frac{\pi R^2}{\alpha^2}\left|\sum_{n=1}^{\infty}(2n+1)(-1)^n\left(\dot{a}_n - \dot{b}_n\right)\right|^2, \qquad (1)$$

$$\sigma_s = \frac{2\pi R^2}{\alpha^2}\sum_{n=1}^{\infty}(2n+1)\left(\left|\dot{a}_n\right|^2 + \left|\dot{b}_n\right|^2\right), \qquad (2)$$

$$\sigma_t = \frac{2\pi R^2}{\alpha^2}\sum_{n=1}^{\infty}(2n+1)\operatorname{Re}\left(\dot{a}_n + \dot{b}_n\right), \qquad (3)$$

where R - particle radius, $\alpha = 2\pi R/\lambda$, λ - wave length in vacuum,

$$\dot{a}_n = \frac{\dot{\mu}\psi_n(\alpha)\psi'_n(\beta) - \sqrt{\dot{\mu}\dot{\varepsilon}}\,\psi_n(\beta)\psi'_n(\alpha)}{\dot{\mu}\xi_n(\alpha)\psi'_n(\beta) - \sqrt{\dot{\mu}\dot{\varepsilon}}\,\psi_n(\beta)\xi'_n(\alpha)}, \quad (4)$$

$$\dot{b}_n = \frac{\sqrt{\dot{\mu}\dot{\varepsilon}}\,\psi_n(\alpha)\psi'_n(\beta) - \dot{\mu}\psi_n(\beta)\psi'_n(\alpha)}{\sqrt{\dot{\mu}\dot{\varepsilon}}\,\xi_n(\alpha)\psi'_n(\beta) - \dot{\mu}\psi_n(\beta)\xi'_n(\alpha)}, \quad (5)$$

$$\psi_n(x) = \sqrt{\pi x/2}J_{n+1/2}(x), \quad \xi_n(x) = \sqrt{\pi x/2}H^{(1)}_{n+1/2}(x), \quad (6)$$
$$\beta = \sqrt{\dot{\mu}\dot{\varepsilon}}\alpha,$$

$J_{n+1/2}(x)$, $H^{(1)}_{n+1/2}(x)$ - Bessel and Hankel functions of the first kind of half-integer indexes.

Relations (1) - (3) require substantial time for computation at numerical summation. That is why for small ($\alpha \ll 1$) and large ($\alpha \gg 1$) particles it is advisable to use asymptotic approximations.

In the first case ($\alpha \ll 1, \left|\sqrt{\dot{\mu}\dot{\varepsilon}}\right|\alpha \ll 1$) it is sufficient to use two first members of the Mieserieses, which finally results in the following relations:

$$\sigma_b = 4\pi R^2\alpha^4 \left|\frac{\dot{\varepsilon}-1}{\dot{\varepsilon}+2} - \frac{\dot{\mu}-1}{\dot{\mu}+2}\right|^2, \quad (7)$$

$$\sigma_s = \frac{8}{3}\pi R^2\alpha^4 \left(\left|\frac{\dot{\varepsilon}-1}{\dot{\varepsilon}+2}\right|^2 + \left|\frac{\dot{\mu}-1}{\dot{\mu}+2}\right|^2\right), \quad (8)$$

$$\sigma_t = 12\pi R^2\alpha \left(\frac{\varepsilon''}{(\varepsilon'+2)^2 + (\varepsilon'')^2} + \frac{\mu''}{(\mu'+2)^2 + (\mu'')^2}\right), \quad (9)$$

which at $\dot{\mu} = 1$ correspond to the classical formulae of Rayleigh approximation (Ref.1).

In the second case ($\alpha \gg 1, \left|\sqrt{\dot{\mu}\dot{\varepsilon}}\right|\alpha \gg 1$) the computations may be simplified by using the asymptotic approximations for functions (6):

$$\psi_n(x) = \cos(x + (n+1)\pi/2), \quad \xi_n(x) = (-1)^{n+1}e^{ix},$$

which gives the following expressions for the coefficients (4) and (5), used in (1) - (3):

$$\dot{a}_n = i^{n+1}e^{-i\alpha}\frac{\dot{\mu}\cos x tg y - \sqrt{\dot{\mu}\dot{\varepsilon}}\sin x}{\dot{\mu} tg y + i\sqrt{\dot{\mu}\dot{\varepsilon}}}, \quad (10)$$

$$\dot{b}_n = i^{n+1}e^{-i\alpha}\frac{\sqrt{\dot{\mu}\dot{\varepsilon}}\cos x tg y - \dot{\mu}\sin x}{\sqrt{\dot{\mu}\dot{\varepsilon}} tg y + i\dot{\mu}}, \quad (11)$$

where

$$x = \alpha - (n+1)\pi/2, \quad y = \beta - (n+1)\pi/2. \quad (12)$$

Thus, the relations (1) - (12) comprise a mathematical model for calculating the scattering and absorption characteristics for a spherical particle of arbitrary radius with arbitrary electrophysical characteristics and generalize the classical results (Refs. 1-3) for the case $\dot{\mu} \neq 1$. The software for calculating the characteristics of electromagnetic wave interaction with a single orbital debris is developed on the basis of the model presented.

Scattering and absorption characteristics for a single orbital debris as functions of its relative dimension α, obtained using the developed software, are presented in Fig.1 as an example.

3. ELECTROMAGNETIC WAVE PROPAGATION AND SCATTERING IN THE LAYER OF ORBITAL DEBRIS

A rigorous statistic-wave description of signal propagation through the random discrete media is given by a theory of multiple scattering, one of the variants of which is developed by Twersky (Ref. 1). This theory is based on a general integral equation for a double-frequency coherence function, allowing to study the propagation of arbitrary wide-band signals through the discrete media. Besides, a number of particular relations might be obtained, allowing to calculated the averaged energy characteristics of the signals.

But the integral equation of the theory of multiple scattering does not have a general analytic solution and numerical procedures a rather bulky, even for the modern computing devices. That is why the first approximation of the theory of multiple scattering (Ref. 1) is used as a mathematical instrument in the present paper. From one hand, this approximation allows to take the scattered component of the signal into account, and from the other hand - to obtain the relations acceptable for practical calculations with sufficient accuracy of results.

A model for the formation of orbital debris in the form of linear layer of thickness L, located in the far zone relative to the emission source, was chosen for studies. Under these conditions the intensity of direct signal, having passed through the layer of constant density ρ is defined by the expression:

$$I_f = e^{-\gamma} + 2\pi\int_0^{\pi/2}\frac{|f(\theta)|^2}{\sigma_t}\frac{e^{-\gamma} - e^{-\gamma/\cos\theta}}{1-\cos\gamma}\sin\theta d\theta, \quad (13)$$

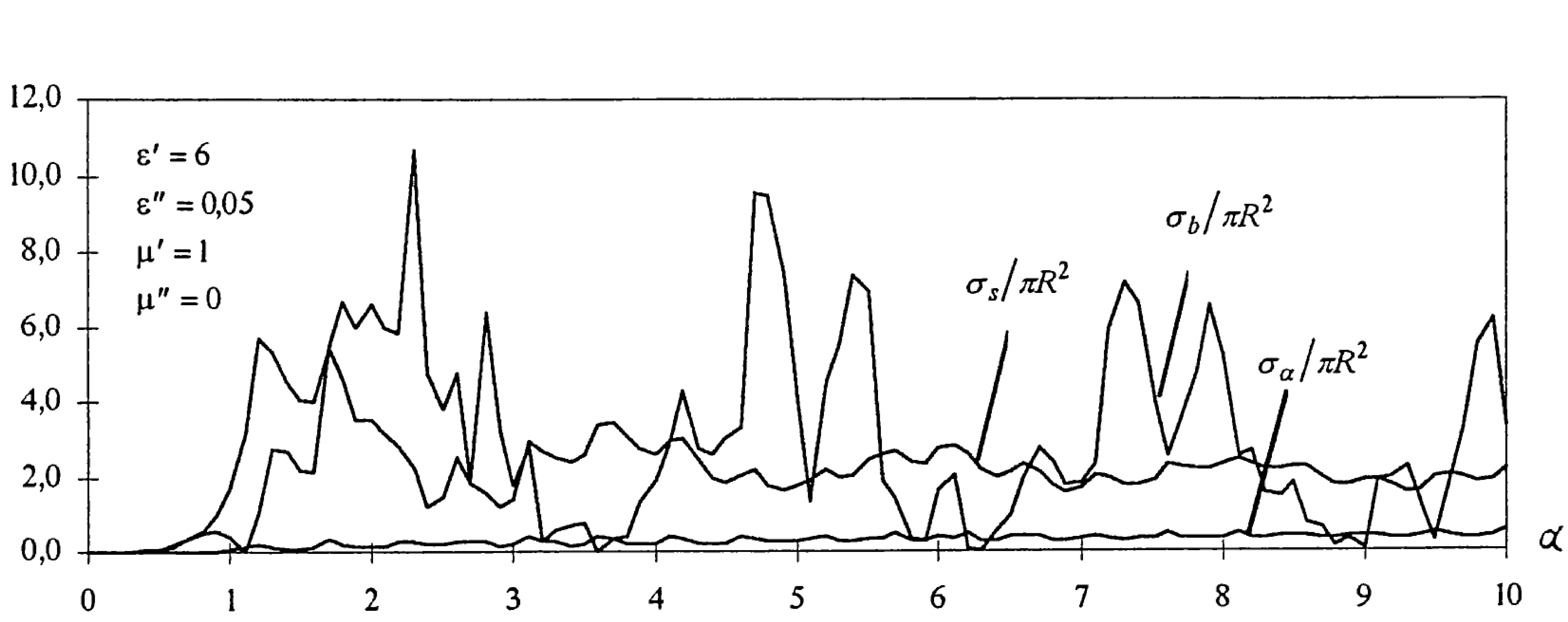

Figure 1. Scattering and absorption characteristics for a single piece of orbital debris

where $\gamma = \rho\sigma_t L$ - attenuation coefficient, θ - angle of scattering measured from the normal line to the layer boundary, $f(\theta)$ - complex amplitude of a signal scattered by a single particle.

In general case the integration in (13) is made numerically, and asymptotic approximations are used for small and large particles. So, at $\alpha \ll 1$ the scattering on a particle is isotropic practically and the intensity of the passed signal may be transformed to the following form:

$$I_f = (1 + \rho\sigma_s L/2)e^{-\gamma}, \quad \alpha \ll 1. \qquad (14)$$

At $\alpha \gg 1$ the use of Gaussian approximation for the square of the scattering amplitude modulus $|f(\theta)|^2$ gives the following:

$$I_f = (1 + \rho\sigma_s L)e^{-\gamma}, \quad \alpha \gg 1. \qquad (15)$$

The signal intensity for the back scattering from the layer under the studied conditions is defined by the following formula:

$$I_b = (1 - e^{-2\gamma})\sigma_b/(8\pi\sigma_t). \qquad (16)$$

The problem of the orbital debris influence on the "visibility" of objects is of interest for the location tasks. In view of this the "visibility" coefficient was introduced into the study. This coefficient is a relation of powers for the back scattered signal, received by the location system in the case of absence and presence of the orbital debris layer:

$$K_v = I_f^2 + I_b S/\sigma_0, \qquad (17)$$

where S is the layer surface area and σ_0 is the back scattering cross-section for the object being located.

Thus, the relations (13) - (17) are a mathematical model for calculating the energy characteristics of a signal at an orbital debris layer presence in the channel of electromagnetic wave propagation. Fig.2 shows an example for the dependences of the energy characteristics of signals on the wave length, calculated by the software realizing the model (13) - (17).

Calculation of the pulsed characteristic for the location channel containing orbital debris is realized also in the developed software complex on the basis of the first approximation of the theory of multiple scattering. For a layer with constant density ρ the pulsed characteristic is described by the following relation:

$$g(t) = \begin{cases} 0, \quad t < 2R/c, \\ \dfrac{G_a S_a \sigma_b S \rho c k}{2(4\pi)^2 R^4} e^{-\rho\sigma_t c(t-2R/c)}, \quad 2R/c \le t < 2(R+L)/c, \\ \dfrac{G_a S_a \sigma_0}{(4\pi)^2 R^4} e^{-2\gamma}\delta(t - 2(R+L)/c), \quad t = 2(R+L)/c, \\ \dfrac{G_a S_a \sigma_b S \rho c k}{2(4\pi)^2 R^4} e^{-\rho\sigma_t c(t-2R/c)} \times \\ \times \ln\left(\dfrac{ct - (2R+L)}{ct - 2(R+L)}\right), \quad t > 2(R+L)/c, \end{cases}$$

where G_a - gain coefficient of the receiving-transmitting antenna; S_a - antenna effective area, $c = 3 \cdot 10^8$ m/sec - light velocity, $k = 1$ sec - dimensional factor, R - distance between the antenna and a layer of discrete scatterers.

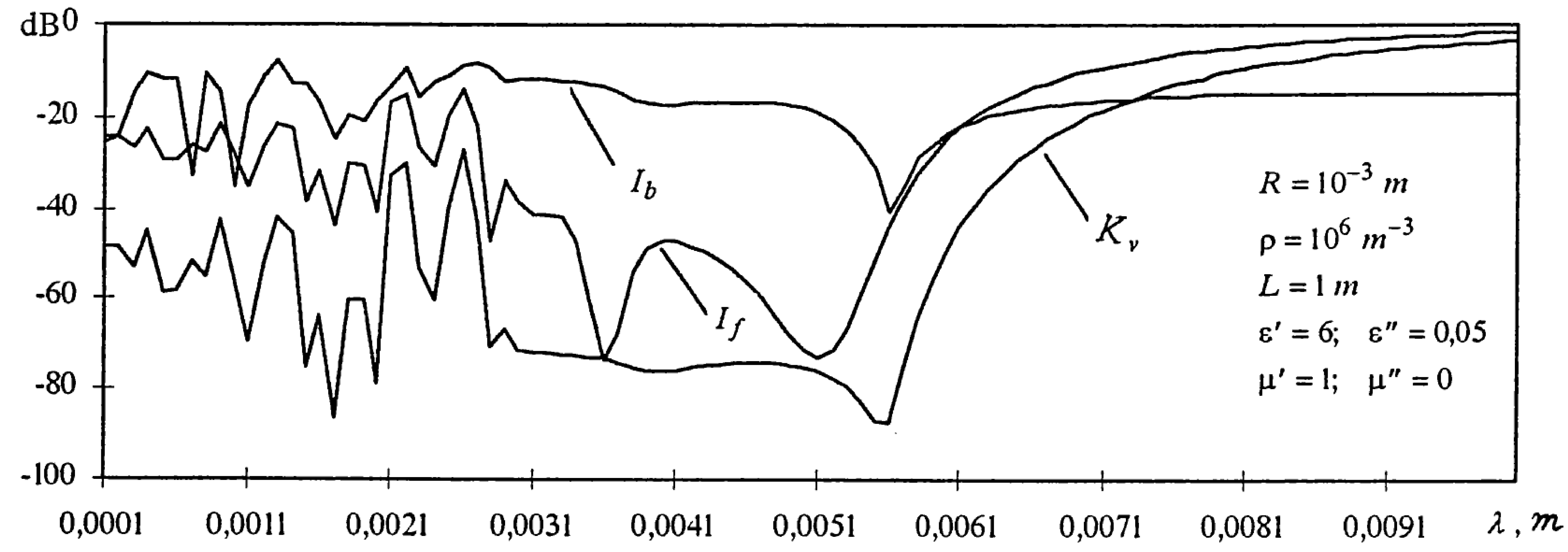

Figure 2. Energy characteristics of signals

Besides, calculation of the pulsed characteristic of the channel with orbital debris for the linear and exponential laws of particle density distribution within the layer is realized also in the developed software complex. The average power envelopes for the signals at the channel outlet at different laws of modulation for the radiated signal are calculated on the basis of pulsed characteristics by calculating the convolution for the power envelope of the radiated signal and the channel pulsed characteristic.

The procedure, based on the substitution of a dense layer of debris by a continuous medium with equivalent electrophysical characteristics is used for studying the characteristics of wave processes inside the dense formations, in which the distance between separate particles is less that a wave length.

4. CONCLUSION

Thus, the mathematical models and procedures realized in the form of a software complex are presented in this paper, which allow to study:
- scattering and absorption cross-sections for the orbital debris modeled by spherical particles with arbitrary dimensions, dielectric and magnetic permeabilities;
- energy characteristics for the signals propagating and being scattered in the layer of orbital debris with different laws of particle density distribution;
- pulsed characteristics of the location channels containing a layer of orbital debris;
- average power envelopes for the signals at the channel outlet at arbitrary laws of the radiated signal modulation.

The developed mathematical models, procedures and software may be used for studying the influence of space debris on the quality indexes for the location and communication systems and for designing the systems for the near space remote probing.

Analysis of the results obtained using the presented procedures shows that they differ from the known ones (Refs. 1-3) by no more than 2-10%.

5. REFERENCES

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Chapter 3

Dust and Debris Experiments and Analysis of Material Returned from Space