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IMPLICATIONS OF SPACE DEBRIS FOR CONSTELLATION SURVIVABILITY AND DESIGN

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ABSTRACT

This paper considers the probability of a cascade fragmentation occurring within a constellation of satellites. Using analytic descriptors, equations are constructed for the short and long term collision probability of a satellite passing through a debris cloud produced by the breakup of another constellation satellite. Constellation design parameters such as the number of satellites and orbital planes, satellite mass and dimensions, and orbital altitude and inclination are all found to influence the survivability of a system following such an event.

1. INTRODUCTION

Recent studies (Refs. 1 and 2) suggest that the spatial densities of operational satellites and space debris within the more populated orbital regimes may have reached critical values. Future satellite breakups will then lead to self sustaining *cascade* fragmentations resulting in the development of debris belts about the Earth. The probability that the cascade fragmentation process will occur is dependent upon the number and spatial distribution of the possible collision partners. The complex interaction between the sources (launch rate / fragmentations) and sinks (orbital decay / deorbit / reorbit) of the Earth's satellite population is found to be the dominant factor determining whether the conditions for cascade fragmentation exist.

Past research has focussed upon the conditions required within the general satellite population for cascade fragmentation to occur. The current vogue for constellations of large numbers of satellites within low altitude circular orbits presents an alternative problem to be analysed. It may be possible that the fragmentation of one satellite element could lead to a cascade fragmentation within the constellation as a whole, resulting in the eventual loss of all the satellites. Such an incident would have a severe impact on the orbital environment raising the collision probability of the local satellite population with the resulting debris. If this were to occur in low Earth orbit the probability of collision between a fragment of debris and a *high risk* object such as a Space Station or nuclear power source (NPS) would be greatly increased. This paper presents results of a study to determine the probability of the cascade phenomenon occurring within a constellation system. Design parameters such as constellation altitude, orbital inclination, mass and number of the satellite elements are varied to investigate their relative influence on the probability of secondary or cascade fragmentations.

The analyses use theory developed for the AUDIT (Ref. 3) suite of software which is based on analytic fragmentation and orbital theory. This analytic approach lends itself easily to the definition of criteria related to the potential pollutive impact of space systems on the orbital environment.

2. COLLISION PROBABILITY WITHIN CLOUD

The probability of collision with a fragment as a satellite passes through a cloud of debris is given in its simplest form by the expression (Ref. 4):

$$P_c = \frac{C_N \, \sigma \, L}{V} \tag{1}$$

where C_N is the number of fragments within the cloud
σ is the collision section of the satellite (km^2)
L is the path length through the cloud (km)
V is the volume of the cloud (km^3).

In this paper our aim is to model the specific conditions relating to the formation of a debris cloud resulting from the fragmentation of a constellation satellite and the subsequent passage through that cloud of the surviving satellites. Our first objective is to find analytic descriptors which are able to represent the above parameters.

3. FRAGMENT DISTRIBUTIONS

The mechanisms of satellite breakup are not well understood. There is a sparse amount of empirical data derived from observations of in-orbit and laboratory-based fragmentation events. The exponential and power laws that are applied in an attempt to characterise the observed distributions of mass, dimensions and velocities of fragments can produce misleading and unrepresentative results. McKnight (Ref. 5) has reviewed, and attached caveats to the application of, the data and laws that are currently available. These guidelines have been observed in our analyses.

3.1 Fragment Mass Distribution

It is necessary to determine the number of fragments in the cloud which are large enough to cause catastrophic fragmentation upon collision with one of the constellation satellites. The mass distribution of fragments resulting from the breakup of a satellite is dependent upon the nature of the fragmentation event. The fragmentation may be induced either by a collision or an explosion. In both cases we assume that the breakup has been catastrophic (i.e. total breakup of satellite).

Su and Kessler (Ref. 6) have developed an empirical relationship for the mass distribution of fragments for the explosive breakup of a body of mass m_{sat} (g), which is of the form[5] :

$$C_N = j \, m_{sat} \exp\left(-k \sqrt{m_f}\right) \tag{2}$$

C_N is the cumulative number of fragments with mass greater than a limiting mass m_f(g).

Kessler and Cour-Palais (Ref. 7) derived a similar power law for a collision-induced fragmentation. For a catastrophic collision, McKnight recommends an expression of the form:

$$C_N = b\,(m_f\,/m_{sat})^{-c} \qquad (3)$$

For the analyses of short term collision probability we will only be concerned with the number of fragments with a mass greater than a threshold value m_f. This is used as a crude measure of the shielding capability of the vehicle. In considering the long term collision probability, when it will be necessary to improve the resolution of the cloud structure by mass partitioning (Ref. 3), we will need to ensure that conservation of mass is observed when deriving the mass distributions.

3.2 Fragment Velocity Distribution

The acquisition of data on velocity distributions of fragments is very difficult because of the very high velocities associated with the breakup event. The most reliable velocity information currently available is derived indirectly from observations of the trajectory changes of fragments following a satellite breakup in orbit. Given the change in the observed orbital elements relative to those of the *parent satellite locus* (the position in inertial space that the parent satellite would have occupied had it remained intact) it is possible to determine the velocity impulse that caused this trajectory change (Ref. 8). Such an approach, empirical in nature and similar to that above to derive mass distributions of fragments, was adopted to determine the velocity impulse imparted to trackable-size debris when fragmentation of the Ariane V16 3rd stage was observed in November 1986. If we follow the approach of Jehn (Ref. 9) then the maximum imparted velocity (km/s) is related to the fragment mass by an expression of the form:

$$\Delta v_{max} = p\; m_f^{-q} \qquad (4)$$

This expression will be used to represent the velocity distribution derived from both collision and explosion induced breakups.

3.3 Fragment Ballistic Coefficient Distribution

McKnight proposes a similar power law expression for the ballistic coefficient which will dictate the rate of orbital decay of a fragment. Again in terms of the mass of a fragment m_f, the ballistic coefficient is given by the expression:

$$\delta = 1728 \sqrt[g]{\frac{m_f^{2-g}}{f^2}} \qquad (5)$$

The nominal values of b, c, f, g, j and k recommended by McKnight are 1.0, 0.65, 45000, 2.26, 0.0005, and 0.04 respectively. Jehn recommends values of 0.65 and 0.159 for p and q.

The modular and analytic nature of the AUDIT approach permits all of these expressions and parameter values to be replaced or updated as more detailed information becomes available from laboratory-based experiments.

4. CONSTELLATION GEOMETRY

For the purposes of this analysis we restrict ourselves to the family of constellations which use circular orbits (common semimajor axis a). A constellation is designed, and the trajectories of the satellites phased such that there is no possibility of two satellites arriving at one of the intersection points between the trajectories at the same time and a direct collision between the satellites occurring (see figure 1).

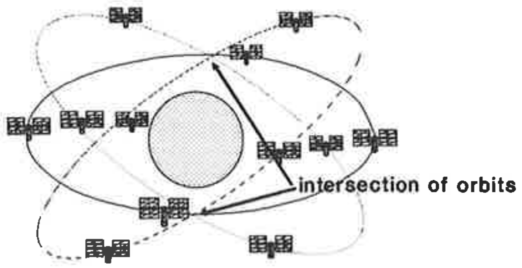

Figure 1. Satellite constellation and intersection nodes

Let us define a parameter, the *intra-plane separation window* ϕ (see figure 2), which represents the angular separation between satellites within an orbital plane. If the constellation comprises of more than one plane of satellites, we need to define a further parameter, the *inter-plane separation window* χ, which represents the angular separation between satellites within the constellation. If we assume that each of the M orbital planes contains the same number of N satellites, then these parameters (in radians) are given simply by:

$$\phi = \frac{2\,\pi}{N}\ , \quad \chi = \frac{\phi}{M} \qquad (6)$$

It is immediately apparent that as the number of orbital planes and satellites within those planes is increased, then the intra- and inter-plane separation windows decrease.

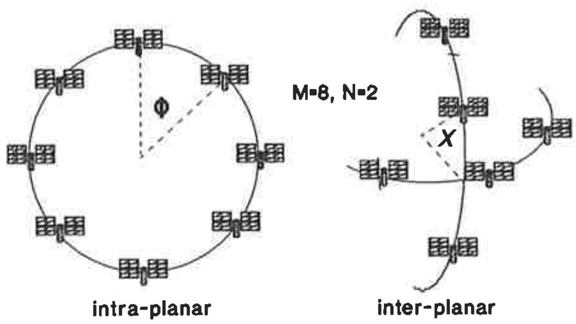

Figure 2. Satellite separation windows

In order to avoid collisions between the constellation satellites active *formation keeping* is necessary to maintain this separation under the influence of perturbing forces.

5. SHORT TERM EVOLUTION OF CLOUD

When a satellite fragments in orbit, the resulting debris cloud migrates around the parent orbit until it evolves into a torus. In the absence of perturbations such as atmospheric drag and gravitational anomalies, the cloud takes the form of a torus after several days. This torus has a pinch point at $\theta = 2m\pi$ ($m = 0,1,2,3,\ldots$) as the fragments pass through the point of breakup. In addition the torus has a pinch wedge which is extended in the radial direction at $\theta = (2m+1)\pi$ ($m = 0,1,2,3,\ldots$) where the crosstrack dimension collapses to zero. This torus defines the maximum extent of the cloud in inertial space (see figure 3). The complete torus is formed when the limbs of the cloud, which extend with time both ahead of, and behind the parent satellite locus each extend for 2π around the orbit. The limb that advances in front of the parent satellite locus contains fragments with semimajor axis values below the initial circular orbit value. The limb which retreats behind the parent satellite locus contains fragments which have semimajor axis values greater than the circular orbit value. The rate at which these limbs advance and retreat in relation to the parent satellite locus are dictated by their relative orbital periods. This perturbation-free model for the cloud evolution is valid for several orbit revolutions after the fragmentation at all but the lowest altitudes.

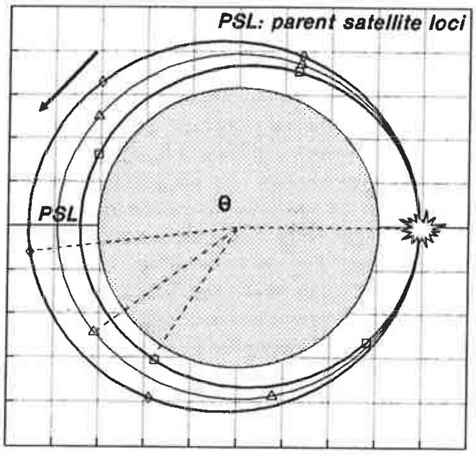

Figure 3. Torus defining extent of debris cloud

5.1 Characterisation Of Cloud Evolution

The short term volume of a debris cloud resulting from the fragmentation of a satellite in a circular orbit can be expressed analytically in terms of the initial semimajor axis a(km) and the maximum velocity Δv(km/s) imparted to a fragment (Ref.10).

The volume of the limb which advances ahead of the parent satellite locus (denoted by the subscript λ) is given by:

$$V_\lambda(t) = \frac{4}{\pi} a^3 \left(\frac{1-\lambda}{2-\lambda}\right) \cos^{-1}\beta \left[k_0\,(\theta_\lambda - \alpha t) + \sum_{n=1}^{\infty} k_n \left(\sin n\theta_\lambda - \sin n\alpha t\right)\right] \tag{7}$$

The volume of the limb that retreats behind (denoted by the subscript γ) is given by:

$$V_\gamma(t) = \frac{4}{\pi} a^3 \left(\frac{\gamma-1}{2-\gamma}\right) \cos^{-1}\beta \left[k_0\,(\alpha t - \theta_\gamma) + \sum_{n=1}^{\infty} k_n \left(\sin n\alpha t - \sin n\theta_\gamma\right)\right] \tag{8}$$

$$\gamma = \left[1+\frac{\Delta v}{a\alpha}\right]^2, \quad \lambda = \left[1-\frac{\Delta v}{a\alpha}\right]^2, \quad \beta = 1-\frac{a}{2\mu}(\Delta v)^2, \quad \alpha = \sqrt{\frac{\mu}{a^3}} \tag{9}$$

$$\theta_\lambda = \alpha t(2-\lambda)^{3/2} - \left(2e_\lambda - \frac{1}{4}e_\lambda^3\right)\sin \alpha t(2-\lambda)^{3/2} + \frac{5}{4}e_\lambda^2 \sin 2\alpha t(2-\lambda)^{3/2} \tag{10}$$

$$\theta_\gamma = \alpha t(2-\gamma)^{3/2} + \left(2e_\gamma - \frac{1}{4}e_\gamma^3\right)\sin \alpha t(2-\gamma)^{3/2} + \frac{5}{4}e_\gamma^2 \sin 2\alpha t(2-\gamma)^{3/2} \tag{11}$$

$$e_\lambda = \sqrt{1-\lambda(2-\lambda)}\,, \quad e_\gamma = \sqrt{1-\gamma(2-\gamma)} \tag{12}$$

k_n are numerical constants whose values do not vary with a and Δv. In practice only the first six terms are needed to accurately represent the cloud and are given by: $k_0=1.044$, $k_1=-0.422$, $k_2=-0.396$, $k_3=0.070$, $k_4=-0.021$, $k_5=0.017$.

The volumes of the cloud limbs following a fragmentation from an initial circular orbit of altitude 1500km and a limiting velocity impulse of 250m/s are shown in figure 4. Also shown are the predicted volumes for the same Δv and a using the classical Chobotov approach (Ref. 4).

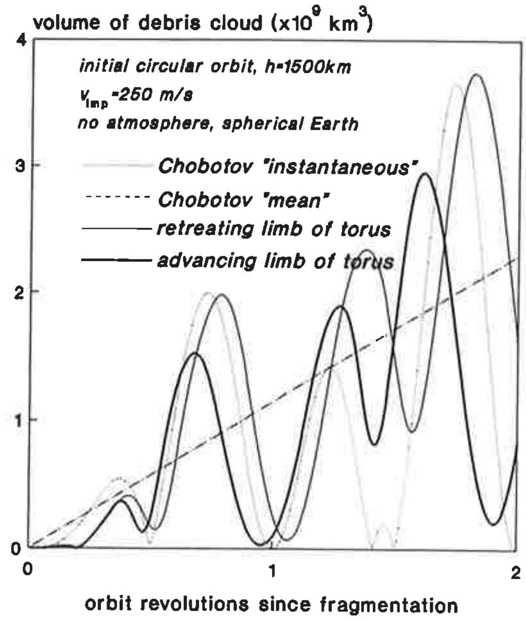

Figure 4. Comparison of predicted volumes of debris cloud

We can see that the torus model appears to offer the combined benefits of both versions of the Chobotov model, namely a periodic variation in volume that does not return to zero at the pinch points/wedges. In addition it is easy to determine the growth, with time, of the limbs of the cloud both ahead of, and behind the parent satellite locus. This characteristic of the model is very useful as it will allow us to determine when the cloud is able to interact with, and present a hazard to, the remaining satellites within the constellation.

5.2 Encounter Criterion

As we have seen, as the cloud evolves the limbs extend ahead of, and behind the parent satellite locus. The cloud will therefore be able to interact with other satellites at the intersection of the trajectories when either of the limbs extends away from the parent satellite locus by an angle greater than χ.

This *encounter criterion* is represented by:

$$\begin{aligned} \theta_\lambda - \alpha t &\geq \chi \\ \alpha t - \theta_\gamma &\geq \chi \end{aligned} \qquad (13)$$

5.3 Short Term Collision Probability

The path length of another constellation satellite through the resulting debris cloud is a function of the intensity of the fragmentation, the altitude of the constellation, the time elapsed since fragmentation and the relative orientation of the orbital planes, and is given by:

$$L(t) = \frac{2\ a\ |\sin\ \alpha t|\ \cos^{-1}\beta}{\sin\dfrac{m\pi}{M}} \qquad (14)$$

($m = 1 \rightarrow M-1$)
Substitution of equation (4) into equations (7), (8), (9) and (10) will give the volume, equations (10) and (11) give the limb growth and equation (14) the path length through the cloud of debris. Combining these equations into equation (1), with either equation (2) for an explosion-induced breakup or equation (3) for a collision-induced breakup, we can construct an expression for the short term collision probability for a constellation satellite passing through the cloud. This is in terms of the threshold minimum fragment mass that could prove lethal (a measure of shielding capability), the initial altitude, and the area and mass of the constellation satellite.

From equation (6) the number of satellites N and orbital planes M employed within the constellation provide the inter-plane separation window and permit us to determine when the encounter criterion of equation (13) is met.

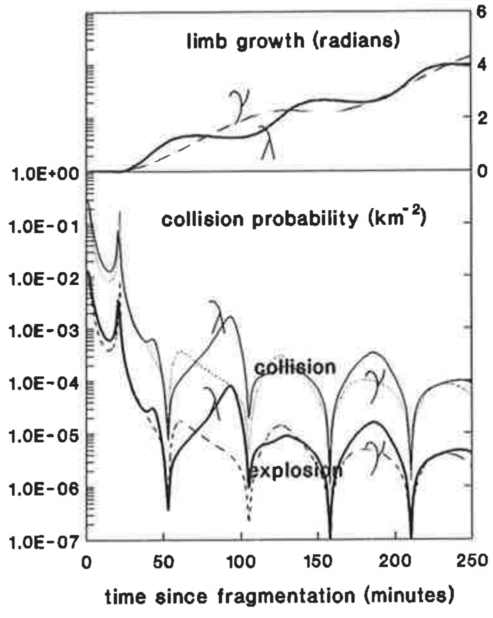

Figure 5. Predicted collision probability and limb growth

The *area specific* collision probability assuming a satellite mass of 500kg, an initial altitude of 1000km and a limiting shieldable fragment mass of 1g is plotted in figure 5 for both an initial collision-induced and explosion-induced breakup. Also shown is the limb growth. Assuming that there are 5 orbital planes and 8 satellites in each plane, the inter-plane separation window for the constellation is 0.157 radians ($=9^0$). We can then identify the time in figure 5 when the encounter criterion of equation (13) is met and the cloud can interact with the other constellation satellites.

The *critical collision probabilities* (the maximum possible once the encounter criterion has been met) for the test cases are 0.002(km^{-2}) for the collision-induced breakup and 0.0001(km^{-2}) for the explosion-induced breakup. We find that the interaction between the number of satellites and their initial mass, altitude and shielding capability is very complex even when using the simple analytic descriptors introduced above. In general, as the number and mass of the constellation satellites increase, then so the critical collision probability increases. Conversely an increase in constellation altitude will reduce the short term critical collision probability. An improvement in shielding capability reduces the number of fragments that are lethal (i.e. can produce a catastrophic fragmentation upon collision) and therefore reduces the corresponding critical collision probability. It is clear that appropriate selection of the constellation design parameters can optimise the short term system survivability following the fragmentation of one of the satellites. Conversely, inappropriate choice of constellation parameters can drastically increase the vulnerability of the system (and the general satellite population) to such an event.

6. LONG TERM COLLISION PROBABILITY

6.1 Dispersion Of Cloud Around Central Body

Due to the asphericity of the Earth, we find that secular changes in the argument of perigee and right ascension of ascending node of the fragment orbits result in a dispersion of the cloud into a band around the Earth which is confined within a latitude band related to the initial orbital inclination of the parent satellite (see figure 6).

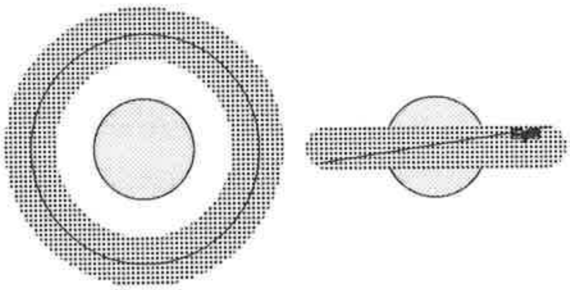

Figure 6. Dispersion of cloud around central body

The volume of the cloud when it has dispersed and (we will assume) spread uniformly around the Earth is given by:

$$V_{\gamma+\lambda} = \frac{4\pi}{3}\, a^3 \left[(2-\gamma)^{-3} - (2-\lambda)^{-3}\right] \sin\ i \qquad (15)$$

Assuming a common orbital inclination for all constellation satellites, then the path length through the cloud each orbital revolution is given by:

$$L = 2\ \pi\ a \qquad (16)$$

6.2 Orbital Decay Of Fragments

In determining the long term collision probability between a constellation satellite and the cloud it is also necessary to take account of the orbital decay of the fragments if the initial constellation altitude is below 1000km. Due to the dissipative influence of the atmosphere the orbital energy of the fragments will be reduced and their altitudes will decrease. We find that those fragments whose specific orbital energy is less than that of the parent satellite decay in their orbit much more quickly that those whose relative orbital energy is increased following breakup. This suggests that we should adopt the same energy partitioning approach as section 5.

We define the time when the apogee of a debris fragment trajectory has decayed a safe distance h (km) below the constellation altitude as t_h, i.e. when it is no longer possible for a constellation element to collide with the fragment.

King-Hele (Ref. 11) has derived an expression for the reduction in eccentricity $\Delta e = e_\lambda - e_h$ required to produce a decrease in apogee altitude Δr_a.

The expression for the time to decay this distance is given by:

$$t_h = \frac{e_\lambda^2}{2B}\left[1 + w + e_\lambda x + e_\lambda^2 y + \frac{H}{a_\lambda} z\right] \qquad (17)$$

where w, x, y, and z are functions of e_λ and e_h, H is the density scale height at the constellation altitude, and B is a function of the atmospheric density, e_λ and the ballistic coefficient δ.

If we are considering those fragments with specific orbital energies greater that the parent satellite and their associated decay period t_γ, then $\Delta r_a = (a_\gamma - a) + h$. If we are considering those fragments with specific orbital energies below that of the parent satellite then t_λ is given by $\Delta r_a = h$.

In order to improve the resolution of the cloud further, in addition to partitioning the fragments in terms of their specific orbital energy, we also partition the ensemble of fragments by mass and use the concept of the *macro-fragment* (Ref. 3) which is representative of a particular zone of the cloud. The properties of a macro-fragment are:

1) a characteristic mass m_f which is representative of that particular mass partition,

2) a number of representative fragments within that particular mass partition,

3) a representative volume of the zone.

We can then derive the long term collision probability between a constellation satellite and a fragment within a particular zone of the cloud by again substituting equations (15) and (16) and either (2) or (3) into equation (1). This will give us the collision probability for a particular constellation satellite. This value must be multiplied by an enhancement factor *(N.M)-1* if we wish to determine the probability that any constellation satellite will collide with a cloud fragment.

In addition we can calculate the time for the orbits of the fragments to decay below the constellation altitude.

We have calculated the area specific collision probability for similar fragmentation scenarios to those in section 5. This time we vary the altitude and inclination to observe their influence on the long term collision probability. The results are shown in figures 7 and 8. Comparing the results with those for the short term predictions we see that there is a complex tradeoff between the increased volume of the cloud and the increased path length of a satellite passing through it. This is further complicated by the enhancement factor *(N.M)-1* as more satellites are able to encounter the cloud.

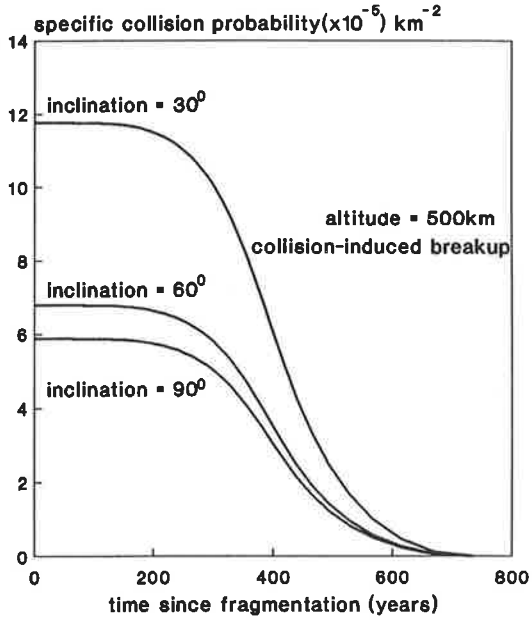

Figure 7. Long term collision probability (*i* dependence)

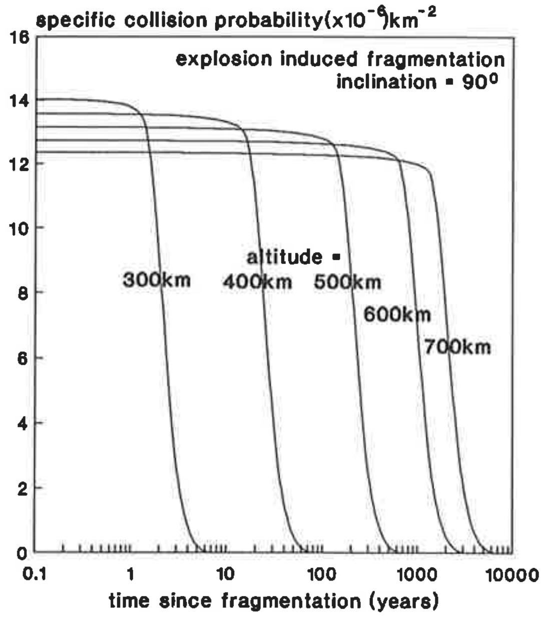

Figure 8. Long term collision probability (*a* dependence)

It is also very clear that the orbital inclination and constellation altitude have a dominant influence on the long term collision probability. In general as the orbital inclination approaches 90^0 the collision probability decreases. As the constellation altitude decreases the collision probability increases but there is a tradeoff as fragment orbits will decay quicker within the atmosphere and the more hazardous conditions will not last as long. We find that the majority of the low energy fragments decay very quickly and that it is the fragments with specific orbital energies greater than the parent satellite which contribute the majority of collision partners.

7. DISCUSSION

This attempt to construct a simple model of the breakup of a constellation element and the subsequent collision probability of a surviving satellite passing through the resulting debris cloud has illustrated that the short term problem is very complex and multi-dimensional. The trade-off between the collision probability and the number of satellites, their dimensions, the inclinations and number of orbital planes, and the altitude of the constellation must be considered for each specific constellation design. The general conclusions that we are able to draw from the long term analysis may simply reflect the relatively simplistic approach that was adopted to model the structure of the debris cloud during this phase. Further work is necessary to accommodate the influence of relative velocity between the cloud fragments and the target satellites, and to account for possible *hot spots* in the cloud density within the band.

Further development also is necessary to consider the modelling of the transition from the short term evolution (torus) to long term evolution (band) of the debris cloud. The resolution of the short term evolution of the cloud could be improved by adopting the macro-fragment approach used in the long term predictions. In all cases the data and empirical laws representing the breakup characteristics should to be extended to include more representative satellite structures as this information becomes available.

8. CONCLUSIONS

It has been shown that the design of a satellite constellation can be varied in order to decrease the vulnerability of the system to the breakup of one of the constellation satellites. The vulnerability of any space system to debris (and therefore its potential pollutive impact on the space environment) should become a primary consideration in the tradeoff analysis which is carried out during the design phase of any project.

9. ACKNOWLEDGEMENT

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