CHARACTERIZING THE MATERIAL IN A DEBRIS CLOUD CREATED IN A HYPERVELOCITY IMPACT

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ABSTRACT

All earth-orbiting spacecraft are susceptible to damaging impacts by pieces of orbital debris. The damage caused by these hypervelocity impacts can lead to catastrophic failure and possible loss of life. In the design of dual-wall protective systems, the inner wall material must be chosen so that the loading of the debris cloud created by the initial impact of the debris particle on the outer wall does not compromise the integrity of the inner wall. To calculate the response of the inner wall to the debris cloud loading, the state of the material within the debris cloud and the amount of material in each of the three states of matter must be known. This paper presents a method that can be used to calculate to first-order accuracy the amount of material in a debris cloud created by a hypervelocity impact that is solid, molten, and/or vaporized.

INTRODUCTION

All earth-orbiting spacecraft are susceptible to impacts by pieces of orbital debris which can strike the spacecraft at any time during its mission at speeds as high as 16 km/sec. The damage caused by these impacts can lead to catastrophic failure and possible loss of life. Traditional damage-resistant spacecraft wall design for earth-orbiting vehicles consists of a thin shield placed at a small distance in front of the inner wall of the spacecraft [1]. This concept has been studied extensively over the last thirty years as a means of reducing the perforation threat of hypervelocity particles. Dual-wall configurations have been shown repeatedly to provide significant increases in protection against perforation by high-speed particles over equivalent single-wall structures.

An important component in the design of such dual-wall systems is the selection of inner wall parameters so that the integrity of the spacecraft is not compromised. From a perforation-resistance point of view, the inner wall must be able to withstand the impact loading of the debris cloud created by the impact of the debris particle on the shield. To calculate the response of the inner wall to the debris cloud loading, the amount of debris cloud material in each of the three states of matter must be known.

This paper presents a method to calculate to first-order accuracy the amount of material in a debris cloud created by a hypervelocity impact that is solid, molten, and/or vaporized. The method is applied to aluminum particles impacting aluminum shields at velocities of up to 16 km/sec. Limitations of the method are presented and discussed as are recommendations for improving the method.

SHOCK LOADING AND RELEASE ANALYSIS

Consider the normal impact of a cylindrical projectile on a flat target plate (see Figure 1). Upon impact, shock waves are set up in the projectile and target materials. As the shock waves propagate, the projectile and target materials are heated adiabatically and non-isentropically. The release of the shock pressures occurs isentropically through the action of rarefaction waves that are generated as the shock waves interact with the free surfaces of the projectile and target. This process leaves the projectile and target materials in high energy states and can cause either or both to fragment, melt or vaporize.

![Figure 1. Impact of a Cylindrical Projectile on a Flat Target](image)

At very early times during the impact event, only the area in the immediate vicinity of the impact site is affected by the impact. For the projectile and target geometries considered in this study, the shock waves can be considered to be initially planar. This allows one-dimensional relationships to be used for analyzing the creation and release of shock pressures. The pressures, velocities, energies, and densities in the projectile and target materials after the passage of a shock wave are calculated using the three 1-D shock-jump conditions, a linear relationship between the shock wave velocity and particle velocity in each material, and continuity of pressure and velocity at the projectile/target interface.

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While the shock loading of a material is an irreversible process, the release of a shocked material occurs isentropically along an 'isentrope'. The difference between the area under the isentrope and the energy of the shocked state is the amount of residual energy in the material. A sketch of a generic Hugoniot and a generic release isentrope with initial, shocked, and final material states highlighted is shown in Figure 2. To calculate the release of the projectile and target materials from their respective shocked states, an equation-of-state (EOS) is needed. To keep the analysis relatively simple, the Tillotson EOS [2] is used in this study.

![Figure 2. Generic Hugoniot and Release Isentrope](image)

In its original form, the Tillotson EOS has two parts [2]. The choice of which part to use depends on the location of the release isentrope within P-V-E space. The first part applies when the material is in compression regardless of the internal energy (i.e., for \( V<V_H \) and for all \( E>0 \)) and in the small region of expansion in which \( V<V_H \) provided that \( E \times E_{-H} \), where \( E \) is the 'total' heat needed to produce incipient vaporization and \( E_H \) is the latent heat of vaporization. The quantity \( V=1/\rho \) corresponds to the specific volume (or density) of a material that completes its release process with an internal energy \( E \). In these two regions, the Tillotson EOS has the form

\[
P_2 = \left[ a + b f(E,\rho) \right] E + Q \quad \text{(1)}
\]

where \( \mu = \sqrt{\gamma - 1} \rho \) and

\[
Q = A_0 + B_0^2 \quad \text{(2a)}
\]

\[
f(E,\rho) = \left[ (E/E_H)(\rho/\rho_H)^2 \right] + 1 \quad \text{(2b)}
\]

Equation (1) applies in particular to shock loadings in which the material remains a solid after it returns to ambient pressure. In equation (1), the subscript 'o' refers to ambient conditions, \( a = c_0^2 \) and \( b = b_0 \), where \( c_0 = \sqrt{(K/\rho)} \) is the adiabatic sound speed, \( K \) is the bulk modulus, \( c_0 \) is the volumetric coefficient of thermal expansion, and \( \rho_0 \) is the specific heat at constant pressure. For most metals, a value of \( a = 0.5 \) will yield satisfactory results.

Tillotson states that the constants \( E_0 \) and \( B_0 \) should be adjusted to give the best fit for the EOS surface [2]. Recent efforts by Mullin, et al. [3] show that the constant \( B \) can be approximated reasonably well in terms of various thermal/mechanical material properties by expanding the Tillotson EOS in Taylor series form; however, \( E_0 \) still has to be treated as a curve-fitting parameter. One of the dangers of improperly guessing a value for \( E_0 \) is that the isentrope could actually curve up from its starting point instead of curving down. If this were to occur, the release process would have to be terminated, another value of \( E_0 \) would have to be selected.

In a highly expanded state (i.e., for \( V<V_H \) regardless of internal energy) or if the internal energy is high enough to cause complete vaporization even in a moderately expanded state (i.e., for \( V<V_H \) and if \( E>E_H \)), the Tillotson EOS has the form

\[
P_2 = a E + \left[ b E_0 f(E,\rho) + A_0 e^{[-\beta(V/V_H - 1)]} \right] e^{[-c(V/V_H - 1)^{2/3}]} \quad \text{(3)}
\]

where the constants \( \alpha \) and \( \beta \) are adjusted to control the rate of convergence of the EOS to that of an ideal gas. The exponential factors force the second term in equation (3) to approach zero at large expansion volumes. The remaining first term is then equivalent to the ideal gas term with \( \gamma = 1.5 \), which is reasonable for real gases [2].

In this two-part form, the Tillotson EOS is asymptotically correct in the compression and expansion regimes. It should be noted that the release process as described by the Tillotson EOS does not always terminate in a simple manner as it does with, for example, the Mie-Gruneisen EOS [4]. When the material remains in a solid state upon release, the isentrope generated with the Tillotson EOS will cross the V-axis in P-V space in a manner analogous to that which is observed when using the Mie-Gruneisen EOS. However, for impact conditions that lead to material melt and vaporization, the isentrope created with the Tillotson EOS approaches the V-axis asymptotically. Therefore, an additional user-supplied parameter must be a cut-off point for the release process in the event of extreme gaseous expansion.

Closed-form expressions for \( P_2 \) along the isentrope described by equations (1) and (3) can be obtained using the procedure described in [4]. In this procedure, equation (1) is manipulated so that the unknown pressure \( P_1 \) at the current increment in the release process is written in terms of quantities at the previous increment. The manipulations involved in deriving a closed-form expression for \( P_1 \), can be reduced significantly if equation (3) is rewritten in the following form:

\[
P_2 = a E + b E_0 f(E,\rho) + C \quad \text{(4)}
\]

where \( f(E,\rho) \) is still given by equation (2b), \( b = b_0 \) and \( C = UC_0 \) where

\[
S = A_0 e^{[-\beta(V/V_H - 1)]} \quad \text{(5)}
\]

\[
U = A_0 e^{[-\beta(V/V_H - 1)]} \quad \text{(6)}
\]

Thus, the expression for \( P_1 \) can be written in exactly the same form as the expression for \( P_2 \). As a result, the expressions that are derived for \( P_1 \) can be used to give us \( P_2 \), as well provided that in every instance \( b \) is replaced with \( b U \) and \( C_0 \) is replaced with \( U C_1 \).

If we examine equations (1) and (3) in more detail, we note that they are continuous across \( V=V_H \), which implies that the Tillotson EOS is continuous across \( V=V_H \) for very high impact energies.
However, at \( V > V_s \), there is a discontinuous, abrupt jump in the release isentrope for moderate impact energies, that is, when \( E_s < C E_s ' \) at \( V = V_s \). This is the case illustrated in Figure 3. The jump in the EOS occurs because according to the original formulation proposed by Tillotson, whenever \( E_s < C E_s ' \) equation (1) is used, even in the \( V < V_s \) region of the curve. However, once we move across \( V = V_s \), equation (3) is invoked regardless of the impact energy. Since these two equations are not continuous at \( V = V_s \), neither is the isentrope.

![Figure 3. Tillotson Equation-of-State, \( E_s < C E_s ' \) (MPF - Mixed Phase Formulation)](image)

A modification in the form of a 'Mixed Phase Formulation' of the Tillotson EOS was proposed in an attempt to lessen the effects of the discontinuity at \( V = V_s \) [5]. The Mixed Phase Formulation proposes that if \( E_s < C E_s ' \) as the release isentrope crosses \( V = V_s \), then for \( V < V_s \) the pressure is to be calculated using the equation

\[
P_3 = \left[ P_2 (E_s - E_s) + P_1 (E_s - E_s') \right] / (E_s - E_s')
\]

(7)

This ensures that the EOS and the release isentrope are continuous if \( E_s \) or if \( E_s > E_s' \) at \( V = V_s \). This modification was motivated by the fact that if \( E_s \) as the isentrope crossed \( V = V_s \), then enough energy would be present to cause 'partial vaporization.' Hence, the region \( V < V_s \) is referred to as a 'mixed-phase region' in which some gas is present in addition to the original solid material. Thus, rather than continue to use equation (1) when \( E_s < C E_s ' \) in the regime \( V < V_s \), equation (7) is to be implemented to account for some additional expansion of the material. This implies that equation (1) is valid in \( V > V_s \) only if \( E_s \) instead of \( E_s ' \) as proposed by Tillotson.

A point worthy of note is the if \( E_s < C E_s ' \) in the regime \( V < V_s \), then the isentrope must terminate at some value of specific volume greater than \( V_s \). Thus, in this case, the isentrope crosses \( V = V_s \), equation (3) is invoked, and the jump in the isentrope at \( V = V_s \) is created. While the Mixed Phase Formulation does allow for some gaseous expansion in moderately high energy impacts not possible with the original Tillotson EOS, it still does not address the discontinuity at \( V = V_s \). In this case, the isentrope continues along a path that becomes asymptotic to the \( V \)-axis. Thus, the original Tillotson EOS and the Mixed Phase Formulation both tend to overpredict the amount of expansion that occurs in the release of a material from a moderately energetic state.

To overcome this difficulty, it is proposed that when \( V > V_s \) and \( E_s < C E_s ' \) (i.e., in moderately high energy impacts), the jump in Tillotson EOS can be eliminated by uniformly subtracting the magnitude of the jump at \( V = V_s \) from the pressure values calculated when \( V > V_s \) using equation (3), that is, the original Tillotson EOS equation applicable when \( V < V_s \). Thus, if \( E_s < C E_s ' \) as the isentrope crosses \( V = V_s \), then for \( V > V_s \), the pressure is to be calculated using the equation

\[
P_4 = P_2 - \left[ P_2 \left( \frac{V_s}{V} \right) - P_3 \left( \frac{V_s}{V} \right) \right]
\]

(8)

in which \( P_4 \) is calculated using equation (3) and \( P_3 \) is calculated using equation (7). As can be seen from equation (8), this correction is not intended to replace the Mixed Phase Formulation of the Tillotson EOS, but rather to complement its use.

The quantity within the square brackets of equation (8) is the amount of the jump, in the release isentrope; it is largest if \( E_s (E_s') \) at \( V = V_s \), and decreases as \( E_s ' \). In the event that \( E_s > E_s ' \), \( V = V_s \), the proposed modification in the Tillotson EOS disappears, the EOS reverts back to its original form (i.e., \( P = \frac{V}{r} \)), and continuity at \( V = V_s \) is maintained. If \( E_s < E_s ' \), \( V = V_s \), then the isentrope never reaches \( V = V_s \), so that in such cases, the correction is never invoked. Thus, the proposed correction is only invoked when needed, that is, if \( E_s < E_s ' \) as the isentrope crosses \( V = V_s \). Use of the proposed jump correction forces the release isentrope to cross the \( V \)-axis at a value much less than that which would be obtained with the Mixed Phase Formulation (or with the original Tillotson EOS). The following is a summary of which equation to use in which regime of \( P \)-\( V \)-\( E \) space to generate a release isentrope with the Tillotson EOS.

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Figure 4 shows the result of implementing the jump correction given by equation (8) for a 10 km/sec aluminum-on-aluminum impact. In such a scenario, a fair amount of melting and expansion would be expected to occur. The Tillotson EOS release isentrope shown in Figure 4 after implementing the correction is more reasonable because it terminates at a specific volume that is greater than that predicted by the Mie-Grüneisen EOS which cannot account for greatly expanded states, yet is substantially less than that which would be obtained following the path of complete vaporization. The Tillotson EOS in which the jump correction is performed using equation (8) in conjunction with the Mixed Phase Formulation is hereafter referred to as the Modified Tillotson EOS.

The differences in the final specific volumes obtained in aluminum-on-aluminum impacts using the Mie-Grüneisen, Tillotson, and Modified Tillotson equations-of-state are shown in Figure 5. For low energy impacts (below approx. 9 km/sec), the results are, as expected, nearly identical. For very
high energy impacts (above approx. 18 km/sec), the final values predicted by the Tillotson EOS and the Modified Tillotson EOS (upper curve) overlap and exceed those predicted by the Mie-Gruneisen EOS (lower curve) due to the gaseous expansion of the released material at those impact velocities. The odd behavior in the final values of specific volume due to the jump in the Tillotson EOS begins for aluminum at approximately 9 km/sec (upper curve in Figure 5). However, the Modified Tillotson EOS (middle curve) produces a smooth transition as the material changes from a solid state (below approx. 6 km/sec) to a liquid state (between approx. 6 and 11 km/sec) to a gaseous state (above approx. 11 km/sec). It is the Modified Tillotson EOS that was used throughout the remainder of this study.

Figure 4. Close-up of Moderate Energy Impact Shock Loading and Release Curves (H - Hugoniot; MG,T,MT - Mie-Gruneisen, Tillotson, Modified Tillotson EOS Release Isentropes)

Figure 5. Final Specific Volume vs. Impact Velocity (MG,T,MT - Mie-Gruneisen, Tillotson, Modified Tillotson EOS Values)

DEBRIS CLOUD MATERIAL CHARACTERIZATION

Computing the Percentages of Solid, Liquid, and Gaseous Debris Cloud Material

Once the residual internal energies in the shocked and released portions of the projectile and target materials are obtained, the percentages of the various states of matter in the resulting debris cloud are estimated using the following procedure. This procedure requires the knowledge of the materials’ solid and liquid specific heats (Cp, Cg), their melting and boiling points (Tm, Tb), and their heats of fusion and vaporization (Hf, Hv) in addition to the residual energy (E).

If E < Cg T, then all of the shocked and released material is considered to remain in a solid matter state. If Cg T < E < Cg T + Hv, then the quantity (E - Cg T) / Hv represents the fraction of the shocked and released material that was melted, while the remaining shocked and released material is assumed to be in solid form. If Cg T + Hv < E < Cg T + Hv + (T - Tm), then all of the shocked and released material is considered to be in liquid form. If Cg T + Hv + (T - Tm) > Hv, then all of the shocked and released material is vaporized.

Computing the Masses of the Solid, Liquid, and Gaseous Debris Cloud Material

The material in the debris cloud created by the initial impact consists of the target material removed by the impact and the impacting projectile mass. While the mass of the projectile material in the debris cloud is known a priori, the mass of the target material in the debris cloud is found once the diameter of the hole created in the target plate by the initial impact is calculated using an empirical equation for hole diameter [6].

To calculate the masses of the various states of the projectile and target materials in the debris cloud, the amounts of shocked and released target and projectile material need to be determined. These quantities are obtained by determining the locations in the target plate and in the projectile where the rarefaction waves overtake the corresponding shock waves [7]. It is the material through which both the shock wave and the release wave travel that is shocked and released and which is therefore either melted or vaporized, depending on the particulars of the impact event. Any material beyond the point at which the rarefaction wave overtakes the shock wave is assumed, for the purposes of this study, not to be shocked and to remain in a solid matter state. If the point at which the release wave overtakes the shock wave is beyond the thickness of the target plate or the length of the projectile, then all of the target and/or projectile material is shocked and released.

Once the projectile and target mass contributions to the debris cloud and the fractions of these masses that are shocked and released are obtained, the masses of the target and projectile materials in each of the three states of matter are computed by multiplying each shocked and released percentage by the appropriate total shocked and released mass. The mass of the solid shocked and released material (if any) is then added to the mass of the unshocked material (if any) to obtain the total mass of the solid material component.

A significant limitation of this procedure is the assumption that the impact pressure acts uniformly on an area equal to the target plate hole area. In fact, if shear and viscous forces are neglected, enforcing pressure continuity at the projectile/target interface, and noting that force is the product of pressure and area implies that the effective area on which the impact pressure acts must, to an first-order approximation, equal the presented area of the projectile. This in turn implies that the shocked target material comes from an area of the target approximately equal to the presented area of the projectile (see also [8,9]).

Another limitation of this procedure is the assumption that no further projectile ejection or transfer of loading and unloading occurs beyond the point where the release waves overtake the corresponding shock wave. This is not completely correct since the
shock wave does not simply cease to exist once it is overtaken by a rarefaction wave. Rather, its magnitude decreases over a finite amount of time and a finite extent of material. Some additional projectile and target material will be heated and possibly melted until the strength of the shock wave diminishes to a point below which melt no longer occurs.

RESULTS AND DISCUSSION

Figures 6-9 present the results obtained by applying the procedure developed herein to aluminum-on-aluminum impacts at velocities between 4 and 25 km/sec. Figures 6-8 compare the effects of using the Mie-Cruneisen, Tillotson, and Modified Tillotson EOS to determine the percentages of the various matter states in aluminum-on-aluminum impacts. Figure 9 shows the distribution of the projectile material among the three matter states for some of the impact velocities considered.

As can be seen in Figure 6, the Mie-Cruneisen EOS predicted only a small amount of vaporized material at an impact velocity as high as 25 km/sec. However, both the Tillotson and the Modified Tillotson EOS predicted that the aluminum was completely vaporized at an impact velocity between 20 and 25 km/sec. This difference is due to the fact that the Mie-Cruneisen EOS did not account for the expansion of the material as it nears vaporization and completed the release process with the material in a much lower energy state than the Tillotson EOS.

Comparing Figures 7 and 8 reveals that the Tillotson and the Modified Tillotson equations-of-state agreed in the percentages of the various states of matter at speeds below approx. 9 km/sec and above approx. 18 km/sec. However, within the moderate impact energy regime, the Modified Tillotson EOS predicted vaporization to begin at an impact velocity that was lower than that predicted by the Tillotson EOS. Had this characterization scheme been used in an actual inner wall response assessment for an impact velocity between approx. 9 and 18 km/sec, the result would have been conservative since there would have been fewer lethal solid fragments remaining in the debris cloud.

In Figure 9, the total projectile mass remained constant because the projectile length and diameter were fixed in all of the impact scenarios considered. The solid dark region represents the mass of the projectile that was unshocked and therefore was not subjected to melting and/or vaporization. This quantity increased with impact velocity because the speed of the rarefaction wave in the projectile increased at a faster rate than did the speed of the shock wave in the projectile. As the impact velocity increased, the rarefaction wave caught up with the shock wave within a shorter period of time. This in turn increased the amount of the projectile material that was not subject to melting and/or vaporization. The remaining shaded areas in Figure 9 show the amounts of the shocked and released projectile material in each of the three matter states as the impact velocity increased from 4 to 25 km/sec.

Figure 7. Debris Cloud Material Composition Using the Tillotson EOS, Aluminum-on-Aluminum Impact

Figure 8. Debris Cloud Material Composition Using the Modified Tillotson EOS, Aluminum-on-Aluminum Impact

SUMMARY AND RECOMMENDATIONS

A first-order accurate scheme has been developed to determine the amount of material in each of the three states of matter in a debris cloud created by a hypervelocity impact on a thin target. A modified version of the Tillotson EOS was used to calculate the residual energy in the projectile and target materials upon release from their respective shocked states. Elementary thermodynamic principles were used to determine the percentages of shocked and released projectile and target materials that were melted and/or vaporized during the release process. Using assumed projectile and target geometries, these percentages were then used to calculate the mass of the projectile and target materials in solid, liquid, and gaseous form. Based on the work completed thus far, the following recommendations are offered for continuing the development of a debris cloud characterization model that would be applicable in impact scenarios where material melt and/or vaporization occur.

The next step in the first-order debris cloud characterization would be to determine the nature of the debris cloud solid fragment population. This includes calculating the number of projectile and target material fragments, as well as their sizes, speeds, and trajectories. The predictions of the various fragmentation models can be compared against one another and against available experimental data to determine which fragmentation model
is best suited for continued use. Following the solid fragment characterization, a procedure needs to be developed to determine the debris cloud’s translation and expansion velocities.

After a satisfactory first-order accurate procedure that characterizes the composition and motion of a debris cloud is completed, the accuracy of the procedure needs to be improved. This includes modifying the methods presented herein to include the impact of non-monolithic projectiles, and the impact of yawed and/or obliquely incident projectiles. Additional modifications to improve the accuracy of the debris cloud calculations are as follows.

First, the method of calculating the percentages of projectile and target material in the three states of matter should also be replaced with a more rigorous thermodynamic procedure. One method (see, e.g. [10]) would require calculating the entropy of the shocked state, that is, the entropy imparted to the material by shocking it to a given pressure. The material will retain that entropy during isentropic release to the final release pressure and specific volume. The calculation is completed by identifying the material state with that entropy at the final release pressure by consulting classical thermodynamic tables.

Second, a shock wave attenuation procedure [11] should be implemented to obtain more accurate mass values for the material that is melted and/or vaporized in a high speed impact. Such a procedure will result in a residual energy profile along the length of the projectile and through the thickness of the target. Energy levels at various positions can then be compared to energy levels necessary to begin material melt or vaporization. In addition, the assumption that the impact pressure acts on an area equal to the area of the holes created in the target plate needs to be reconsidered.

Third, in its present formulation, it is possible that the value of $E_o$ in the Tillotson EOS can be different for different impact conditions [12]. Since $E_o$ is a material property, it should be constant and not depend on impact conditions. Thus, the manner in which $E_o$ is chosen needs closer examination.

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