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ESTIMATION OF THE LOW EARTH ORBIT SPACE OBJECT POPULATION USING A NON-VERTICAL STARING RADAR

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ABSTRACT

To estimate the low Earth orbit space object population measurements have been taken using the Haystack radar in a staring mode. The radar has been used in a vertical staring mode and a non-vertical mode, at several elevation angles pointed due South. The population is estimated from the number of objects passing through the radar. In this paper an algorithm for estimating the population and distribution is developed. The algorithm depends on the range, inclination and radar cross section (RCS) of each object passing through and does not depend on any previous estimates of the population distribution, such as the Norad Data Base.

1. INTRODUCTION

Over the past few years the hazard that the increasing number of space objects pose to operational spacecraft has become evident. Currently we only track objects as small as 10 cm, but we cannot shield against objects larger than 1 cm in size. Thus, the regime from 1 - 10 cm is of major concern. Recently a measurement program(Ref. 1) has begun to estimate the number of objects in this size regime and larger. In this program the Haystack radar is used in a staring mode and orbital elements and the RCS for each object passing through are estimated. From these measurements the population and distribution in space and size are estimated. Measurements have been made with the radar in a vertical staring mode and at several elevations in a non-vertical staring mode pointed South. The non-vertical staring mode is used because the Haystack latitude is greater than the planned Space Station inclination and as such cannot view the 28 deg inclination objects in the vertical staring mode. At a 10 deg elevation angle the radar can see the 28 deg inclination orbits at Space Station altitude. In Ref. 1 the current NORAD Data Base population distribution is used as a basis for estimating the total number of objects and distribution. Basically the number of objects in an altitude range is obtained from the ratio of the measured rate of objects in this altitude range to the rate for the NORAD Data Base obtained by computer simulation. In this paper an algorithm is developed for estimating the population and its distribution which does not depend on any expected rate for the NORAD Data Base. After each object passes through the radar the estimate is updated along with an estimate of the error in the estimation. The algorithm developed in this paper is for the radar in the non-vertical staring mode; it is an extension of earlier work (Ref. 2) for the radar in the vertical staring mode. In Section 2 the algorithm is developed and results from computer simulations which validate the simulation are presented in Section 3.

2. ALGORITHM DEVELOPMENT

In this section we develop the algorithm for estimating the low Earth orbit space object population and distribution from measurements of a sample of the population which pass through a non-vertical staring radar which is pointed South. For each object which passes through the beam the range, direction of motion(inclination) and RCS are obtained. First we state the assumptions and then derive the algorithm.

<u>Assumptions</u>

1. All objects are in circular orbits at the altitude they pass through the radar. Since no accurate measurement of eccentricity and argument of perigee can be obtained from passing through a very narrow beam this assumption is necessary.

2. The right ascension and argument of latitude (true anomaly plus argument of perigee) are uniformly distributed. Analysis of the NORAD Data Base validates this assumption.

3. The problem of space objects passing through a radar beam can be modeled as a Poisson process. A Poisson process is characterized by the passage of objects whose times of passage are independent and which possess a constant average rate of passage. For a constant average rate of passage through the beam the population must be uniformly distributed in right ascension and argument of latitude(Assumption 2). Two key properties of the Poisson process are:

a) The mean and standard deviation of a random process generated from the difference in times of passage of successive objects of a Poisson process are equal, and equal to the inverse of the average rate of objects.

b) The sum of independent Poisson processes is a Poisson process with a rate equal to the sum of the rates of the independent processes.

<u>Algorithm</u>

The radar under consideration has a beamwidth β, is located at latitude ψ and pointed South with an elevation angle of γ as shown in Fig. 1. First assume there is a population of N objects of equal RCS in circular orbits at altitude h, inclination I, and uniformly distributed in right ascension and argument of latitude. The intersection of the radar beam with the spherical shell at altitude h is the oval shape shown in Fig. 2. The boresight intersects the shell at latitude θ_0. For this initial discussion assume that $I >> \theta_0$. Referring to Fig. 3 the average rate λ of objects passing through the beam is

$$\lambda = \left(\frac{2N}{T}\right)\left(\frac{\alpha_e}{2\pi}\right) \tag{1}$$

where T is the period of the orbit and α_e is the range of right ascensions of objects that will pass through the beam. The first term is a result of the fact that each object crosses the equator twice per orbit and the second term is the ratio of the right ascensions that will pass through the beam. It will be shown later that α_e is a function of the altitude, inclination and RCS of the object and the radar beamwidth, latitude and elevation angle. Now let each of the objects in the population of N have a different altitude, inclination and RCS. α_e is no longer constant, but different for each object. Using the second property of the Poisson process the average rate of objects passing through the beam becomes

$$\lambda = \left(\frac{1}{\pi}\right)\sum_{i=1}^{N}\left(\frac{\alpha_{ei}}{T_i}\right) \tag{2}$$

The problem now is that we measure λ and want to estimate N from the measurements of M objects passing through the beam. If we can obtain an average value for the summation term in Eq. (2) then we can estimate N. That is,

$$\lambda = \left(\frac{N}{\pi}\right)\left(\frac{\alpha_e}{T}\right)_{tpa} \tag{3}$$

where the notation "tpa" represents total population average. The average value for the population is computed by

$$\left(\frac{\alpha_e}{T}\right)_{tpa} = \left[\frac{1}{M}\sum_{i=1}^{M}\left(\frac{T_i}{\alpha_{ei}}\right)\right]^{-1} \tag{4}$$

Let λ_m be the measured rate and N_{est} the estimated population, then

$$N_{est} = \lambda_m \pi\left(\frac{\alpha_e}{T}\right)_{tpa}^{-1} \tag{5}$$

We will now address the method for calculating α_e. When the radar is pointing vertically the intersection of the radar with the sphere at altitude h is a circle. It was shown in Ref. 2 for the vertical pointing radar that α_e is given by

$$\alpha_e = \left(\frac{h\beta}{r}\right)\left(\cos^2\psi - \cos^2 I\right)^{-1/2} \tag{6}$$

Note that there is a singularity at $I = \psi$. In Ref. 2 we made the assumption that there were no objects whose inclination was nearly equal to the latitude of the sensor. This was reasonable since there were very few objects at inclinations near the latitude of the radars being considered. However, the non-vertical radar intersects many latitudes and we will not be able to make that assumption in this analysis.

As shown in Fig. 2 when the radar is non-vertical the intersection of the radar cone and the sphere at altitude h is no longer a circle, but an oval shape. This complicates the problem significantly. We will now show that in a latitude-longitude reference frame this intersection is an ellipse when the radar beam is a narrow beam cone. Referring to Fig. 2 let P be a point where the radar cone intersects the sphere and let $(\vec{R}_e, \vec{\rho}, \vec{r})$ be the vectors from the Earth center to the radar site, the radar sight to P, and the Earth center to P. Then

$$\vec{r} = \vec{R}_e + \vec{\rho} \tag{7}$$

The latitude, θ_0, of the intersection point of the boresight and sphere at altitude h and the range, ρ_0, to the intersection point are given by

$$\cos(\psi + \gamma - \theta_0) = \frac{\cos\gamma}{1 + h/R_e} \tag{8}$$

$$\rho_0 = \frac{R_e \sin(\psi - \theta_0)}{\cos(\psi + \gamma - \theta_0)} \tag{9}$$

The following procedure is used to determine the equation of the curve of the intersection of the cone and sphere. With the angle σ defined as in Fig. 2 and assuming the radar is located at $\phi = 0$, express the vectors $(\vec{R}_e, \vec{\rho}, \vec{r})$ in terms of the spherical coordinate unit vectors $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$ defined in Fig. 4. Let

$$\begin{aligned} \theta &= \theta_0 + \eta \\ \rho &= \rho_0 + \xi \end{aligned}, \tag{10}$$

assume η and ξ are small because the beamwidth is small, and equate components of the unit vectors $(\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi)$. The following equations are obtained

$$\xi = -0.5\rho_0\beta\sin\sigma\cot\delta \tag{11}$$

$$\phi = \frac{0.5\rho_0\beta\cos\sigma}{R_e\cos\psi + \rho_0\sin(\psi+\gamma)} \tag{12}$$

$$\eta = \frac{0.5\rho_0\beta\sin\sigma}{\left[R_e\cos(\psi-\theta_0)+\rho_0\sin\delta\right]\sin\delta} \tag{13}$$

where

$$\delta = \psi+\gamma-\theta_0 \tag{14}$$

Eliminating σ from Eqs. (12) and (13) gives

$$\frac{\phi^2}{a^2}+\frac{\eta^2}{b^2}=1 \tag{15}$$

where

$$a = \frac{0.5\rho_0\beta}{R_e\cos\psi + \rho_0\sin(\psi+\gamma)} \tag{16}$$

$$b = \frac{0.5\rho_0\beta}{[R_e\cos(\psi-\theta_0)+\rho_0\cos\delta]\sin\delta} \tag{17}$$

Thus, the intersection of the radar cone with the sphere is an ellipse in the latitude-longitude plane. We now need to determine α_e. Referring to Fig. 5 there are three cases which must be considered.

Case 1 The northernmost point and the southernmost point of the orbit are outside of the latitude band defined by the radar beam. Mathematically this is expressed as

$$I \ge |\theta_0| + b \tag{18}$$

Referring to Fig. 7 we define the flight path angle ν as the angle between the trajectory and the latitude line at latitude θ. ν is given by

$$\cos\nu = \cos I / \cos\theta \tag{19}$$

Accordingly ν_0 is the flight path angle at latitude θ_0
In the latitude-longitude frame the flight path angle will be denoted by ν' and is obtained from

$$\tan\nu' = \cos\theta\tan\nu \tag{20}$$

Referring to Fig. 7 and assuming we are in the Northern hemisphere, we are looking for the point where the trajectory is tangent to the ellipse, that is, we are looking for the point (ϕ, η) where

$$\frac{d\eta}{d\phi} = \frac{a}{b}\frac{(a^2-\eta^2)^{1/2}}{\eta} = \tan\nu' \tag{21}$$

This point is given by

$$\eta = \frac{a^2}{(a^2+b^2\tan^2\nu')^{1/2}}, \qquad \phi = -\frac{b^2\tan\nu'}{(a^2+b^2\tan^2\nu')^{1/2}} \tag{22}$$

The distance or angle s is given by

$$s = \left(a^2\cot^2\nu' + b^2\right)^{1/2}\frac{\sin\nu'}{\sin\nu_0'} \tag{23}$$

and α_e becomes

$$\alpha_e = 2s \tag{24}$$

The problem now is how do we solve for ν'. An iterative method is used. The change in the flight path angle over the radar beam is small, so assume $\theta=\theta_0$ and accordingly $\nu'=\nu_0'$. Solve for η using Eq. (22), then use $\theta=\theta_0+\eta$ (use negative sign if the radar is in the Southern hemisphere) and calculate ν' using Eqs. (19) and (20). Continue the iteration until the change in η between iterations is sufficiently small. We used 0.01 degrees. When I is not close to θ_0, which is usually the case, no iterations are necessary and

$$s = \left(a^2\cot^2\nu_0' + b^2\right)^{1/2} \tag{25}$$

Case 2 Either the northernmost point or the southernmost point of the orbit is within the latitude band defined by the radar beam. This case is defined by

$$|\theta_0| - b \le I \le |\theta_0| + b \tag{26}$$

Again assuming we are in the Northern hemisphere and referring to Fig. 8 the tangent point is

$$\eta = -\frac{a^2}{(a^2 + b^2 \tan^2 v')^{1/2}}$$
$$\phi = \frac{b^2 \tan v'}{(a^2 + b^2 \tan^2 v')^{1/2}} \tag{27}$$

An iterative solution is necessary for this case and we cannot start with θ_0. The initial point chosen for the iteration is halfway between the maximum latitude and the minimum point of the ellipse, that is,

$$\theta = I - (b + I - \theta_0)/2 \tag{28}$$

Just as in Case 1 the iteration is continued until the change in η between iterations is sufficiently small. From Figs. 5 and 8 we see that

$$\alpha_e = (\pi - 2s)/2 \tag{29}$$

$$s = \sin^{-1}(\tan I \cot \theta) - \phi \tag{30}$$

The factor two in the denominator is needed to counter the factor two in Eq. (1), which resulted from the fact that a band of ascending and descending orbits pass through the radar beam. In this case an ascending and a descending pass define the band, so we must divide by two.

Case 3 Both the northernmost and southernmost points of the orbit are within the latitude band defined by the radar beam. This is defined by

$$\theta_0 - b \le I \le \theta_0 + b \tag{31}$$

This case is unlikely in that it would only occur with an almost equatorial orbit. For this case $\alpha_e = 2\pi$ However, in Eq. (5) we must use $\alpha_e = \pi$ for the same reason as we had the factor two in the denominator in Eq. (29).

3. RESULTS

To validate the algorithm we used the NORAD Data Base and selected all objects with periods less than two hours. Period (altitude) and inclination bins were selected and the number of objects in each bin determined. It was assumed all objects were large enough to be detected anywhere in the radar beam. A radar with a 10 deg elevation at the location of Haystack was used. The Haystack beamwidth is 0.05 deg. In the simulations a beamwidth of 15 deg was used rather than the 0.05 deg Haystack beam to reduce computation time, this larger beamwidth has no discernible effect on the results. Using the ephemerides in the NORAD Data Base the objects were propagated in time and the computer program determined the time and altitude of penetration of the radar beam.. These data were input into the algorithm and the population estimated sequentially after each object had passed through the beam. Three separate simulations were performed, the simulations differ only in the start time of the simulation. Fig. 9 shows the population estimate as a function of the number of objects passed through the beam and Table 1 gives the estimated population along with the percentage errors. The total number of objects, 5385, is the total number of objects in the NORAD Data Base, and does not consider those low inclination, low altitude objects which cannot be seen by the radar. Reasonably good estimates are obtained after 10% of the population has passed through the beam and after 15% have passed through very little improvement of the estimate is obtained. Figs. 10 and 11, and Tables 2 and 3 summarize the results for the estimate of the altitude and inclination distributions after approximately 25% of the total number of objects have passed through the beam. These simulations demonstrate the validity of the algorithm

4. CONCLUSIONS

An algorithm has been developed for estimating the low Earth orbit space object population and distribution in altitude, inclination and radar cross section (RCS) from measurements taken on a sample of the population by a non-vertical staring narrow beam radar. The radar measures the range, direction of motion, from which inclination is derived, and RCS of the objects which pass through the beam. The algorithm uses only the radar parameters and measurements and does not rely on expected rates for the objects in the NORAD Data Base. Bounds on the error in the estimates are given as a function of the number of objects which have passed through the beam. Simulations showed that a good estimate of the population can be obtained when 10% of the objects have passed through the beam. From these error bounds the operational time of the radar to achieve a desired accuracy can be computed.

5. REFERENCES

1. Stansbery, E.G., Bohannon, G., Pitts, C.C., Tracy, T., Stanley, J.F., Characterization of the Orbital Debris Environment Using the Haystack Radar, NASA JSC-32213, April 24, 1992.

2. Alfriend, K.T., Lewis, D.L., "Estimation of the Low Earth Orbit Debris Population and Distribution", presented at the 1992 AIAA Astrodynamics Conference, Hilton Head, SC., Paper No 92-4444, August 1992, to appear in AIAA Journal of Spacecraft and Rockets.

Table 1. Estimated Population			
	Sim 1	Sim 2	Sim3
Est. No. of Objects	5171	5242	5444
% Error	-4.0	-2.7	1.1
Actual No. of Objects = 5385			
Standard Deviation = 2.5%			

Table 2. Estimated Period Distribution				
Period (min)	Actual	Sim 1	Sim 2	Sim 3
87 - 95	214	211	234	288
95 - 100	606	747	708	651
100 - 105	1753	1648	1679	1707
105 - 110	904	808	774	845
110 - 115	741	588	550	622
115 - 120	827	757	807	801
120 - 240	340	411	490	530

Table 3. Estimated Inclination Distribution				
Inclination (deg)	Actual	Sim 1	Sim 2	Sim 3
0 - 20	9	12	8	10
20 - 40	276	291	299	295
40 - 60	179	152	164	157
60 - 70	938	774	750	768
70 - 80	945	856	932	894
80 - 90	1233	1168	1202	1283

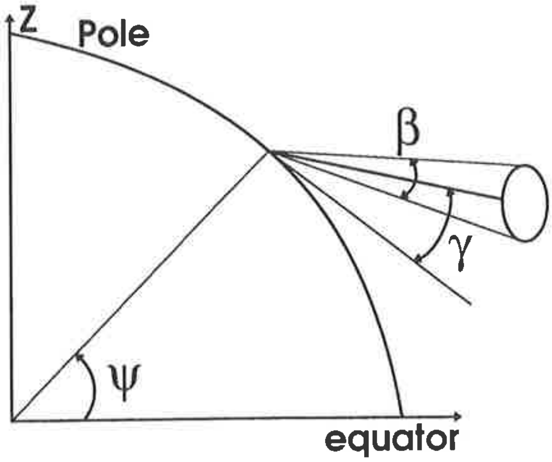

Figure 1. Radar Configuration

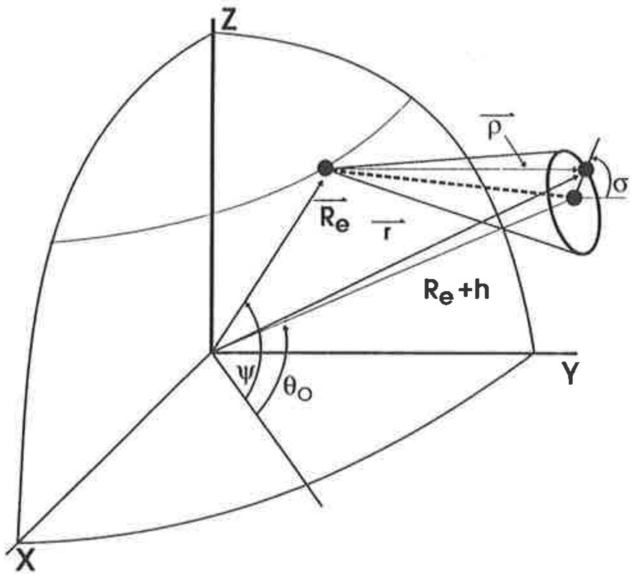

Figure 2. Variable Definitions

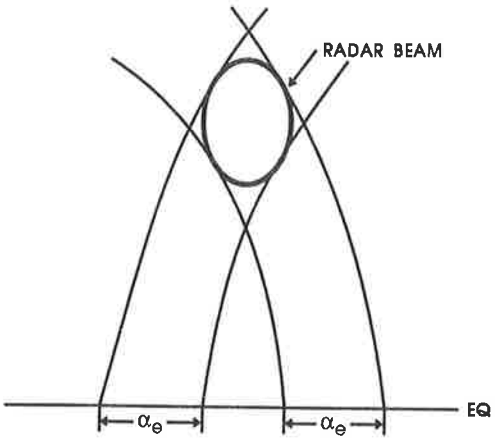

Figure 3. Right Ascension Range of Objects Which Pass Through the Beam

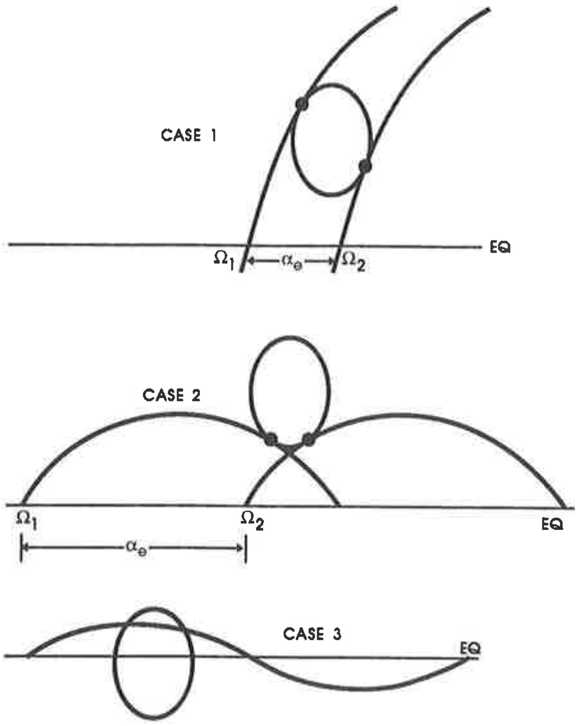

Figure 5. Classes of Objects Which Can Pass Through the Radar Beam

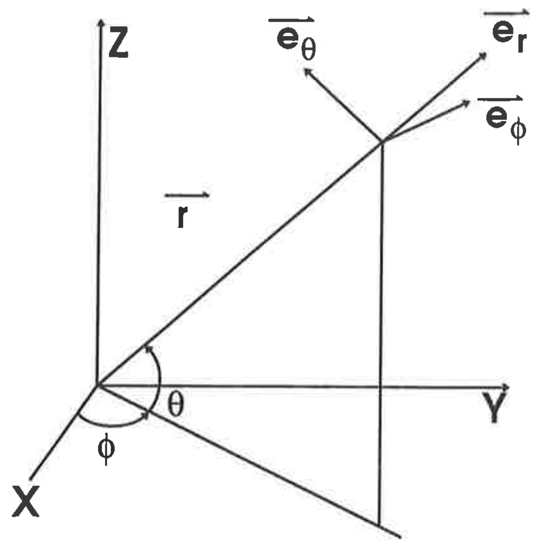

Figure 4. Spherical Coordinate Definition

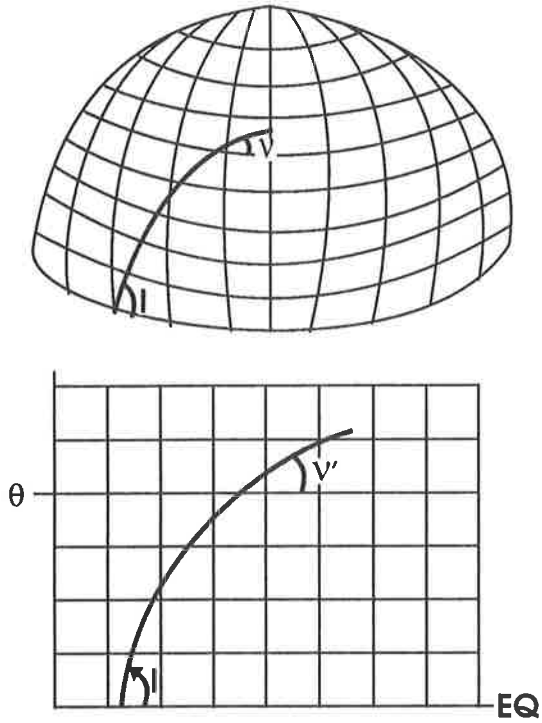

Figure 6. Angle Definition

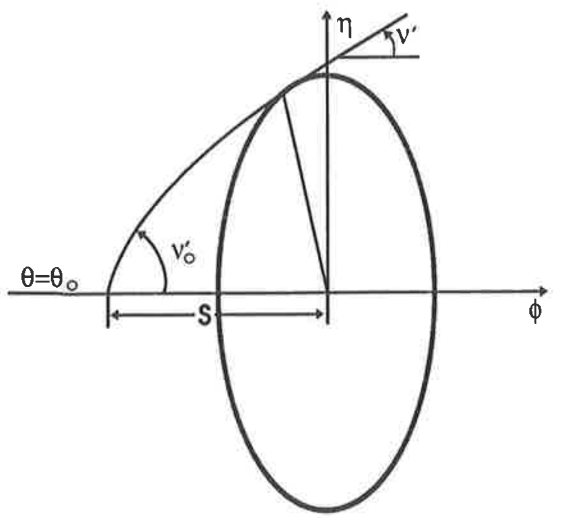

Figure 7. Case 1 Tangent Point

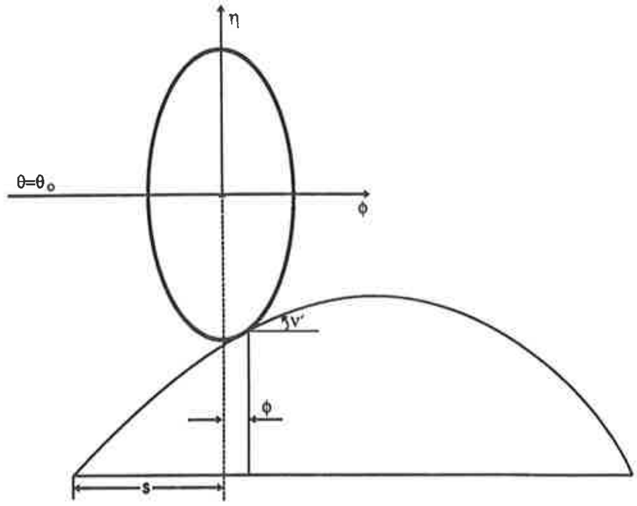

Figure 8. Case Tangent Point

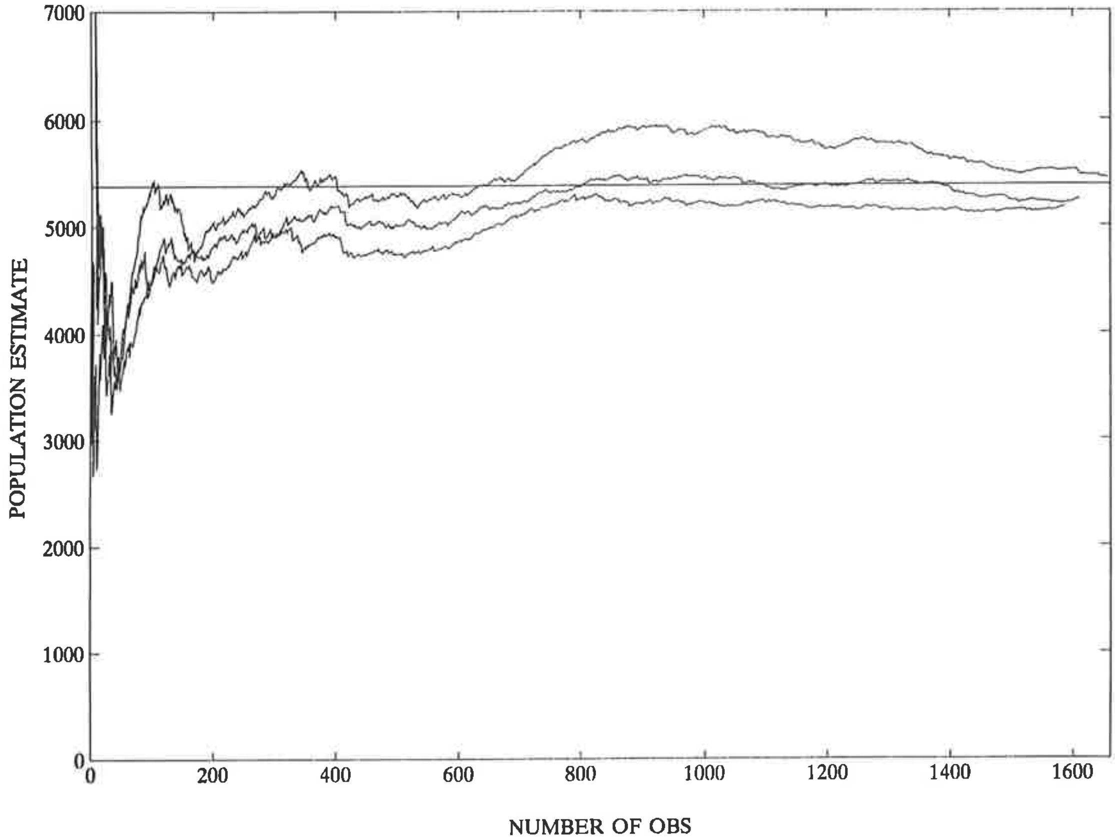

Figure 9. Total Population Estimate

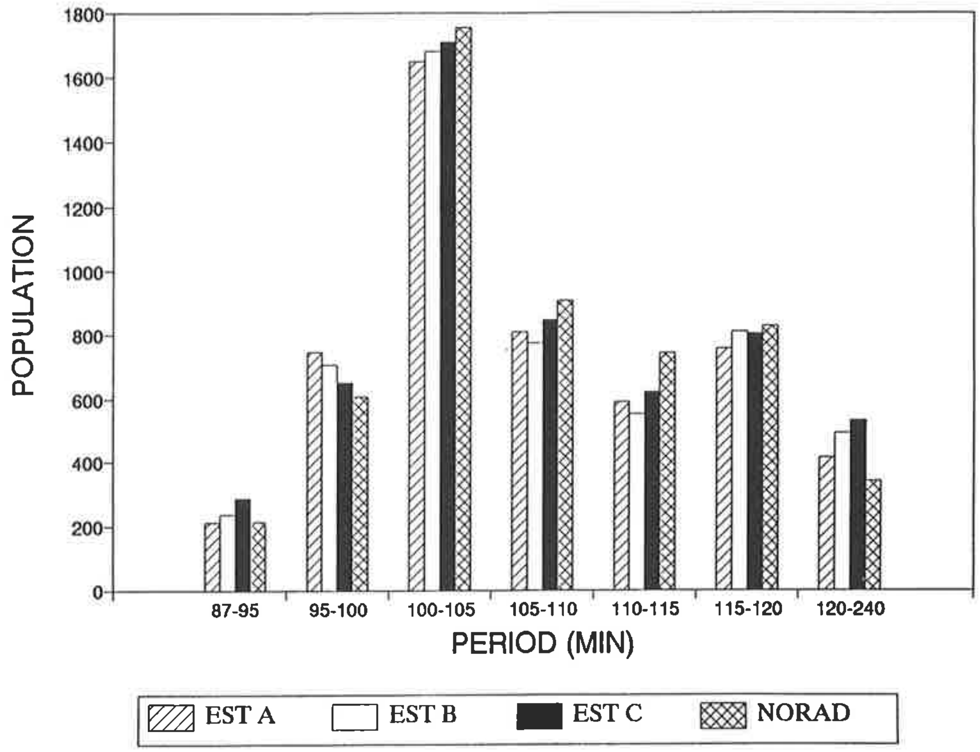

Figure 10. Estimated Period Distribution

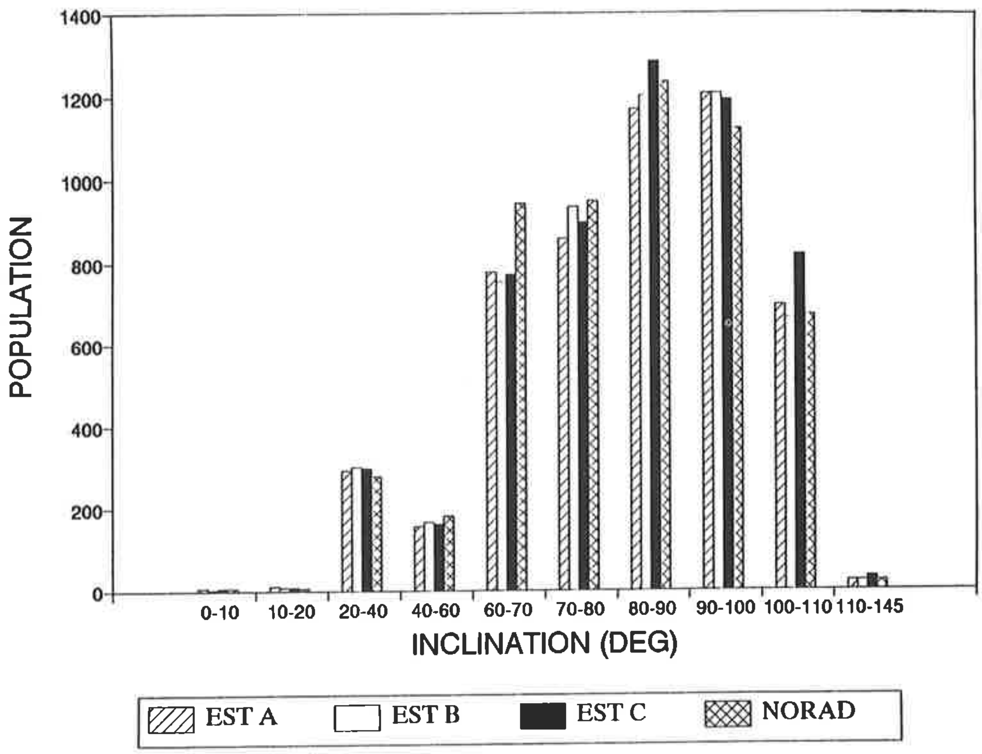

Figure 11. Estimated Inclination Distribution