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METHOD AND RESULTS OF PREDICTION OF ORBITAL DEBRIS SPATIAL DENSITY

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ABSTRACT

The problem of the prediction of spatial density of orbital debris taking into account the atmospheric drag and for given initial values of the density is considered. To solve the problem, a simple dependence of the upper atmosphere density on the altitude providing sufficient accuracy of the density calculation is suggested. It is assumed that at the initial time the debris spatial density is inversely proportional to the diameter to some power and the distribution function of debris mass density is given. Equations simple enough for the calculation of the spatial density versus time and altitude are obtained for debris at the near-circular orbits; the equations contain a definite integral which has to be calculated numerically in general case, but can be found analytically in some special cases. Results of the prediction of debris spatial density for different initial conditions are given.

1. INTRODUCTION

Orbital debris are of a growing threat to the safety of the space flights and to the normal functioning of onboard instruments in near Earth space. And the small particles are of the most danger because of their large number and nonobservability. Therefore the problem of obtaining the authentic assessment of the number and spatial distribution of small particles and prediction of future environment is very topical. The analysis of the environment evolution under the influence of different perturbations is a very important component of the prediction, and the atmospheric drag is the major among the perturbations for low Earth orbits. Due to the big uncertainty in the small debris population and in the parameters of their interaction with upper atmosphere, a simplified approach to the analysis, i.e. approximate models of the debris motion and atmosphere density etc., may be used. In this paper such an approach is considered which allowed to obtain simple equations for the prediction of the spatial density of the debris. The paper deals with the evolution of existing population, i.e. new inputs are not considered.

2. UPPER ATMOSPHERE DENSITY

The parameters of the small particles interaction with the atmosphere are not accurately known, thus there is no sense in employing an accurate and sophisticated upper atmosphere density model when the effect of the atmospheric drag on the particles spatial distribution is studied, a simplified model is sufficient. At the same time the model employed should be accurate enough for the results obtained to be reliable. In this section simple expression describing upper atmosphere density with sufficient accuracy is obtained.

Let us assume that in a vicinity of any given geocentric radius r_1 the upper atmosphere density varies exponentially:

$$\rho(r) = \rho(r_*) \exp \left[- \frac{r-r_*}{H_*} \right] , \qquad (1)$$

where H_* is the density scale height at r_*. If the dependence $H = H(r)$ is known within some range of radii as well as the density $\rho_1 = \rho(r_1)$ at a certain initial radius r_1, then at any radius r within the range the density may be calculated from

$$\rho(r) = \rho_1 \exp\left[-\int_{r_1}^{r} \frac{dr}{H(r)}\right] \qquad (2)$$

We consider the following linear dependence

$$H = \alpha + \beta r \qquad (3)$$

which corresponds to linear variation of so-called molecular temperature with height (Ref. 1). It can be easily found from (2) that in such case

$$\rho(r) = \rho_1 \left[1 + \frac{r - r_1}{E}\right]^{-c}, \qquad (4)$$

where $c = 1/\beta$, $E = \alpha/\beta + r_1$ (similar equation for the density based on linear approximation (3) were used in Refs. 1, 2). In a general case (3) is not fulfilled, however an analysis shows that equation (4) represents some well-known models of the upper atmosphere density quite accurately, namely in the altitude range from 180 km to 600 and for some of the models up to 1500 km is accurate within 2% to 12%, the standard deviation here is 1.6% to 8.6%.

In particular, a three-layer official Russian model of the upper atmosphere density (Ref. 3), averaged over daily, six-month and geomagnetic effects, the solar activity level assumed to be $F_0 = 150\times10^{-22}$ W/m^2/Hz, was approximated by Eq. 4 in the altitude range 160 to 1500 km. The maximum approximation error is 9%, the standard deviation is 6%. Since this approximation will be used below to study the environment evolution due to atmospheric drag we offer values of the parameters derived for this case:

$$r_1 = 6531 \text{ km}, \quad \rho_1 = 1.36\times10^{-9} \text{ kg/m}^3, \quad E = 206.4 \text{ km}, \quad c = 7.5316$$

3. SUBSTITUTION OF ELLIPTIC ORBITS BY CIRCULAR ONES

Taking as a basis the fact that vast majority of space debris orbits are near circular (Ref. 4), with eccentricities of $e \le 0.01$, assume that the particles are moving over circular orbits. However such a substitution of near-circular orbits by circular ones for the analysis of influence of atmospheric drag needs a substantiation, because even for small eccentricities the atmosphere density at the perigee and apogee can be different in several times. Let us give the substantiation based on the following reasoning.

Let a, r_p be the semi-major axis and perigee distance of an object's orbit and consider two circular orbits which radii are equal at an initial time to $r_1 = a$, $r_2 = r_p$ respectively. It is known that $|\dot{r}_1| \le |\dot{a}|$, $|\dot{r}_2| \ge |\dot{r}_p|$ (see Fig.1); hence for the object a reference circular orbit exists which radius r_{ref} satisfies to the unequality $r_p \le r_{ref} \le a$ (see Fig.1). This means that for the analysis of the orbit decay the considered elliptic orbit can be substituted by the circular one of radius r_{ref}; at any time r_{ref} value can be easily found.

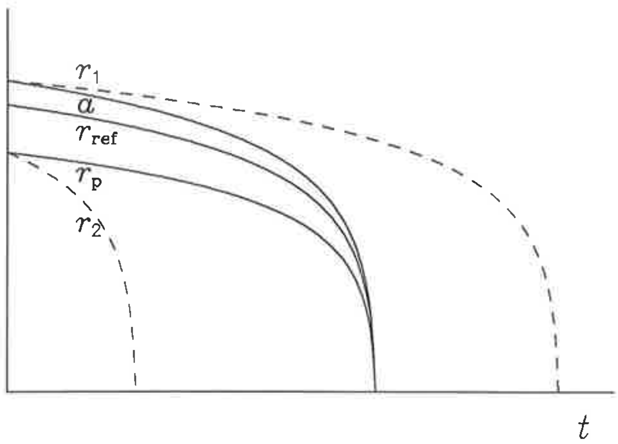

Figure 1. Degradation in the atmosphere of the elliptic and circular orbit parameters.

4. INITIAL SPATIAL DENSITY

Assume that only Earth central gravity and atmospheric drag influence the particles. One can easily obtain the following equation for the circular orbit radius of a particle:

$$\dot{r} = -2\, B \sqrt{\mu r}\, \rho, \qquad (5)$$

where $B = \frac{c_x S}{2m}$ is the ballistic coefficient, c_x is a dimension-less constant, S and m are effective area and mass of the particle, μ is the Earth gravitational constant. Assuming the particle to be spherical having density σ, one can obtain

$$B = \frac{3}{4} \frac{c_x}{s\sigma} \qquad (6)$$

Introduce the dimensionless variables

$$x = \frac{s}{s_0}, \quad y = \frac{\sigma}{\sigma_0}, \quad z = \frac{B}{B_0} = \frac{1}{xy}, \qquad (7)$$

where s_0 and σ_0 are some reference values of the corresponding parameters, $B_0 = B(s_0, \sigma_0)$, and assume that at the initial time $\tau = 0$ the spatial density of the particles having the same values of the parameters x, y within a thin spherical layer Δr_0 at the radius r_0 is given and equal to

$$\Delta(x, y, r_0, 0) = p(r_0)\, \varphi(x)\, \psi(y) \qquad (8)$$

where $p(r_0)$ is the total number of the particles in the layer, $\varphi(x)$ and $\psi(y)$ are the distribution functions which should satisfy the equality

$$\int_{x_{min}}^{x_{max}} \varphi(x)\, dx = \int_{y_{min}}^{y_{max}} \psi(y)\, dy = 1 \qquad (9)$$

where minimum and maximum values of x, y correspond to the minimum and maximum values of s, σ . Assume that $\varphi(x)$ is inversely proportional to x to the power δ and $s_{min} \ll s_{max}$; taking $s_0 = s_{min}$ in Eq. 7, we obtain:

$$\varphi(x) = \frac{\delta - 1}{x^{\delta}} \qquad (10)$$

For the calculations below let us take $\delta = 2.5$ (the modern assessment gives $\delta = 2.52$ for $s \le 2$ cm, Ref.5).

We have no information about the mass density distribution of the particles; the only what we can do is to take a model function $\psi(y)$, such as one represented at Fig. 2 and looking more or less likely. The curve at Fig. 2 reflects the equation

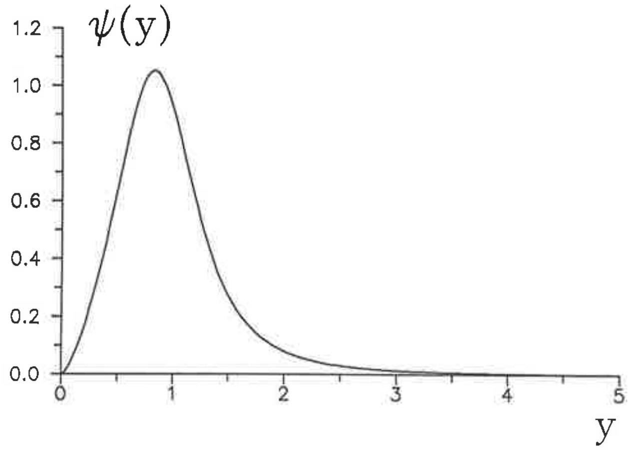

Figure 2. Mass density distribution function

$$\psi(y) = \frac{\gamma y^{3/2}}{y^6 + 1}, \qquad (11)$$

where $\gamma = 1.86$ is normalizing factor providing fulfilment of Eq. 9. The average mass density for particles 1 cm in diameter and smaller is 2.8 g/cm^3 (Ref.6). To obtain this value from Eq.11, one should put $\sigma_0 = 3.1$ g/cm^3 in Eq. 7.

To analyze the evolution of the debris orbits due to atmospheric drag it is convenient to consider the initial spatial density

$$d_0 = d(z, r_0, 0) \qquad (12)$$

of the particles having the same value of the ballistic coefficient B (or of the equivalent variable z), within the spherical layer Δr_0, because variation of height with time is the same for all the particles. Using Eq. 8, one can find d_0 from the equation:

$$d_0 = p(r_0) \int_{x_{min}}^{x_{max}} \varphi(x)\, \psi[y(z)] \left|\frac{\partial y}{\partial z}\right| dx \quad (13)$$

Substituting for $\varphi(x)$, $\psi(y)$, $y(z)$ from Eqs. 10, 11, 7 into Eq. 13 with $\delta = 2.5$, we obtain

$$d_0 = p(r_0) \frac{\gamma\sqrt{z}}{4} \left[\sqrt{3}\, \mathrm{Arcctg}\, \frac{2z^2-1}{\sqrt{3}} - \ln \frac{z^2+1}{\sqrt{z^4-z^2+1}} \right] \quad (14)$$

5. CURRENT SPATIAL DENSITY

In some time τ all the particles having the same value of z will move from the layer Δr_0 into a spherical layer Δr at a radius r, determined from Eq. 5, and their spatial density will be equal to

$$d = d(y, r, \tau)$$

Taking into account Eq. 5 and the evident relations

$$d_0 = \frac{n}{2\pi r_0^2 \Delta r_0}, \quad d = \frac{n}{2\pi r^2 \Delta r},$$

where n is the number of the particles with the same value of z within the layers, and

$$\frac{\Delta r}{\Delta r_0} = \frac{\dot{r}}{\dot{r}_0}$$

for sufficiently small Δr_0, Δr, we obtain:

$$d = d_0 \left[\frac{r}{r_0}\right]^{\frac{5}{2}} \frac{\rho}{\rho_0} \quad (15)$$

where ρ and ρ_0 are the values of atmosphere density at the distances r and r_0 respectively. Let us consider time variations of the spatial density d of the particles with the given value of z at a given radius r. To do this, it is necessary to determine from Eq. 5 the radius r_0, at which the particles were at the initial time $\tau = 0$. Let us assume that $|r_0 - r_1| \ll r_1$, where r_1 was defined in section 2. It is shown in Ref. 7 that in this case

$$r_0 \approx r + E\,\eta\,(q-1), \quad (16)$$

where

$$\eta = 1 + \frac{r - r_1}{E}, \quad q = (1 + Az)^{\frac{1}{c+1}},$$

$$A = 2\,\frac{B_0 \rho_1 (c+1)\sqrt{\mu r_1}}{E\eta^{c+1}}\,\tau, \quad (17)$$

ρ_1, E and c are the parameters of Eq. 4. Then from Eq. 15

$$d \approx d_0\,[1 + \varepsilon\,(q-1)]\, q^{-c}, \quad (18)$$

where

$$\varepsilon = \frac{5E}{2r_1}\,\eta.$$

Let us assume that at the initial time $t = 0$ the debris are situated only within the range $r_1 \le r_0 \le r_2$; then in Eq. 18 d_0 is defined by Eq. 14 for $r_0 \le r_2$ and $d_0 = 0$ for $r_0 > r_2$, where r_0 is determined from Eq. 16.

Now let us find the integral spatial density of all the particles at the time τ and radius r:

$$D(r, \tau) = \int_0^{z_{max}} d(z, r, \tau)\, dz, \quad (19)$$

where

$$z_{max} = \frac{1}{A}\left[\left[1 + \frac{r_2 - r}{E\eta}\right]^{c+1} - 1\right]$$

corresponds to the value $r_0 = r_2$. In general case the integral (19) has to be calculated numerically, but in some particular cases it can be found in an analytical form. For instance, it has been calculated analytically in Ref. 7 for $\delta = 2$, $\sigma = \mathrm{const}$ and $p(r_0) = p$ if r_0 lies within the given limits r_1, r_2 and $p(r_0) = 0$ if r_0 is outside the limits.

6. RESULTS

Let us assume that space objects larger than 0.5 mm in size are distributed between the altitudes 160 km and 1500 km according to the altitude profile of the spatial density of debris given in Ref. 4 (actually the profile represents only trackable objects, but it is not very important for the model calculations). Fig. 3 shows the prediction of the spatial density of the exixting population (not taking into account future inputs) for different time intervals; the numbers near the curves at Fig. 3 gives the time τ in years.

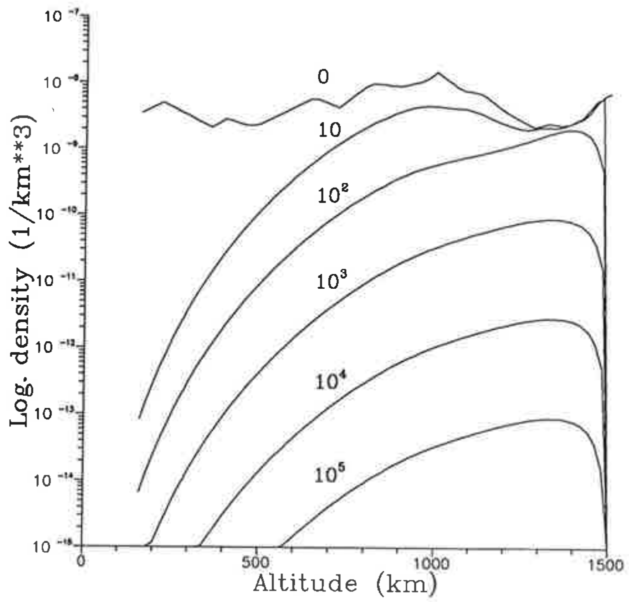

Figure 3. Initial and predicted spatial density

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