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SPACE DEBRIS EVOLUTION MATHEMATICAL MODELLING

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Abstract

A mathematical model of space debris evolution taking into account the growth, self–production of debris and space self–cleaning is worked out. The results of long-time forecast of orbital debris evolution for different altitudes are presented. Numerical simulation based on the simplified mathematical model shows that the rapid growth of the amount of debris particles takes place for different altitudes at different periods of time.

Introduction

The space activity of the population of the Earth has generated a great amount of orbital debris, i. e. manmade objects lauunched into space and fragments inactive and not serving a useful purpose any more [1–4]. The number of trackable debris (larger than 10 cm.) has been increasing since the beginnig of the spase era and now reaches 7500 objects with total mass $2,2 \cdot 10^6 kg$. The smaller objects cannot be detected from the Earth by available observing devices: radars and telescopes. The objects of dimensions 1 cm $\div$ 10 cm separating construction elements and contain fragments of the spacecrafts generated as a result of explosions or collisions. Their number reaches 17 000 that is 0,5 % of the amount of debris and the mass $\approx 10^3 kg$. The number of particles of $0,1 \div 1$ cm. originated as a result of orbital explosions and collisions of spacecrafts reaches 3 500 000 objects. All those objects remain in the orbit for periods of sufficient duration to become a hazard to space activity [5–6].

The growth of poly-sized space debris during the last years makes us face the inevitability of solution of the problem of determination of the orbital debris environment. The description of debris evolution and the determination of the amount of space debris in elliptical orbits is impossible without a profound mathematical model predicting the evolution of space debris and its growth due to realization of space programmes. The paper represents a new mathematical model of space debris evolution based on continuum mechanics multiphase models [10]. The structure of profound mathematical model takes into account: a) the influence of the atmosphere of the Earth on debris particles motion, b) the influence of the Sun radiation, c) the polydispersed character of debris particles, d) the interaction of debris particles of different size, e) the growth of the amount of debris due to the break up of satellites, f) the additional growth of debris environment in course of realization of new space programmes, g) the model of selfcleaning of low orbits (below 600 km). A conceptual block diagram for computer modelling is constructed, having a modularized structure that allows independent development of individual components (submodels) of break up, growth of debris atmospheric and radiative influence, selfcleaning etc.

Mathematical model

The whole number of particles of space debris can be devided into finite number of phases (N), including particles of close diameters $d \in S_\epsilon(d_i)$. Then the mass equation for the i – th phase looks as follows

$$\frac{\partial \rho_i \alpha_i}{\partial t} + div \rho_i \alpha_i \vec{v_i} = I_i, \quad i = 1, ..., N; \quad (1)$$

where $\rho_i, \vec{v_i}, \alpha_i$ – are the density, velocity and volumetric concentration of the i – phase of debris; $I_i = \sum_{j=1}^{N} \kappa_{ij} + M_i - \mu_i$ – is the mass flux to the i - th phase due to the destruction of the other (j – th) phases in the course of collisions (κ_{ij}), the destruction of lardge space object and realisation of new space programmes (M_i), and diminishing of the mass of the i – th phase (μ_i) as a result of slowing-down of particles and its combustion in the atmosphere.

The momentum equation in the coordinate system fixed to the Earth has the following form:

$$\frac{\partial \rho_i \alpha_i \vec{v_i}}{\partial t} + div \rho_i \alpha_i \vec{v_i} \vec{v_i} =$$

$$= \vec{F_i} + \vec{P_i} + \vec{K_i} + 2[\vec{\omega} \times \vec{v_i}], \quad i = 1, ..., N. \quad (2)$$

The second additional in the left side of the equation (2) represents a vector with the components $\{D^k\} = \{\sum_{j=1}^{3} \frac{\partial}{\partial x^j} \rho_i \alpha_i v_i^j v_i^k\}$; $\vec{F_i}$ characterises mass forses (gravitational and magnetic); $\vec{P_i}$ characterises the pressure of the Sun radiation; $\vec{K_i}$ – the additional impulse of the i-th phase, appearing as a result of mass flux to the i – th phase I_i.

Components $\varkappa_{ij}$ practically determine the velocities of mass fluxes to the i-th phase as a result of destruction of particles of the j-th phase and form the antisymmetrical tensor

$$\varkappa_{ij} = -\varkappa_{ij}.$$

The values of the components of this tensor depend upon probabilities of collision of the particles of the j-th phase with other particles, the mechanism of interaction and destruction of colliding objects and the number of particles of the i-th phase among all the particles originated as a result of breakup. The probability of collision of two objects can be determined as a multiplication of the probabilities of the location of each object at one and the same moment in a small volume with dimensions less than the sum of characteristic dimensions of the objects ($d_k + d_j$):

$$p_{kj} = p_k p_j.$$

Probabilities p_j and p_k depend upon the distribution of objects in space and are proportional to the number of objects per unit volume n_j, n_k and characteristic crossections $s_{jk} \sim (d_j + d_k)^2$,

$$p_{kj} = p_k p_j = \\ = k_j n_j (d_j + d_k)^2 k_k n_k (d_j + d_k)^2. \quad (3)$$

The probability of collision of an object belonging to the j-th phase with objects belonging to all other phases $k = 1, ..., N$ is the sum of probabilities

$$\widetilde{P}_j = \sum_{k=1}^{N} p_{kj}.$$

Thus the velocity of mass flux to the i-th phase as a result of breakup of objects of the j-th phase can be determined as follows:

$$\varkappa_{ij} = \sum_{k=1}^{N} k_k k_j n_k n_j (d_k + d_j)^4 \times \\ K^i_{kj}(|\vec{v}_k - \vec{v}_j|, d_k, \rho_k......) - \\ \sum_{k=1}^{N} k_k k_i n_k n_i (d_k + d_i)^4 \times \\ K^j_{ki}(|\vec{v}_k - \vec{v}_i|, d_k, \rho_k......), \quad (4)$$

where functions K^i_{kj} characterise the mass of particles of the i-th phase appearing as a result of fragmentation of objects of the j-th phase in the course of collision with the k-th phase. It depends upon the velocity of collision, mass, density, matherial structure etc. The function K^i_{kj} is an external parameter for this problem determined by solving problems of hypervelocity impact [11–14]. The first additional in the formula (4) characterises the positive mass flux to the i-th phase and the second additional characterises the negative mass flux, i.e. the mass flux from the i-th to the j-th phase as a result of collision of particles of the i-th phase with other particles and its fragmentation. It should be marked separately that function K^i_{kj} is equal to zero if $d_i \geqslant d_j$.

The rate of mass loses of the i-th phase μ_i as a result of slowing down the particles and combustion in the dense layers of the atmosphere can be determined as a number of particles descending annually on the orbits with perigeous height $R \leqslant R_{i*}$. From those orbits further deceleration and motion along ballistic trajectory begins in the atmosphere of the Earth. The value of R_{i*} differs for the particles of different phases.

The mean volumetric mass force $\vec{F}_i$ is determined by a formula:

$$\vec{F}_i = -\rho_i \alpha_i g(\vec{x}) \vec{e}_r - \\ \frac{1}{2} c^i_f \rho_a(\vec{x}, t) \vec{v}_i |v_i| \frac{3\alpha_i}{2d_i}, \quad (5)$$

where $g(\vec{x})$ is the gravitational acceleration; $\vec{e}_r$ - the physical component of a radial basis vector in a spherical system of coordinates; $\rho_a(\vec{x}, t)$ - density distribution in the atmosphere, depending on altitude and taking into account 11 year cycle of the Sun activity; c^i_f - drag force coefficient for a particle moving in a rarefied gas. For spherical particles this coefficient can be determined by a formula:

$$c^i_f = \frac{2e^{-\beta_i^2}}{\sqrt{\pi}\beta_i^3}(2\beta_i^2 + 1) + \\ \frac{erf(\beta_i)}{\beta_i^4}(4\beta_i^4 + 4\beta_i^2 - 1) + \frac{4\sigma\sqrt{\pi}}{\beta_{iw}}; \quad (6)$$

where

$$\beta_i = v_i \sqrt{\frac{m_a}{2\kappa T}}, \quad \beta_{iw} = v_i \sqrt{\frac{m_i}{2\kappa T_i}},$$

$$erf(\beta_i) = \frac{2}{\sqrt{\pi}} \int_0^{\beta_i} e^{-x^2} dx;$$

m_a - the mean molar mass of gases in the upper layers of the atmosphere; κ - the Boltsman constant; T_a - the temperature of gas; T_i - the surface temperature of the particle; σ - the interaction coefficient, taking into account the character of interaction of gas molecules with a moving particle.

The movement of particles of relatively small dimensions in Space is influenced by the pressure of Sun radiation p_r. The mean volumetric force $\vec{P_i}$ in the equation (2), characterising this pressure, can be determined in the following way:

$$\vec{P_i} = p_r \frac{\pi d_i^2}{4} n_i \vec{s}(\vec{x},t) H(-\vec{s} \cdot \vec{R}) =$$

$$= \frac{3}{2} p_r \frac{\alpha_i}{d_i} \vec{s} H(-\vec{s} \cdot \vec{R}); \qquad (7)$$

where $\vec{s}(\vec{x},t)$ is a vector of a unit length characterising the direction from the Sun to the particle, $\vec{R}$ - radius vector from the centre of the coordinate system to the particle;

$$H(x) = \begin{cases} 0, & if \ \ x \leqslant 0, \\ 1, & if \ \ x > 0. \end{cases}$$

The function H in the formula (7) equals to zero if the particle is on the dark side of the Earth. The formula (7) shows that for the objects of smaller size the mean volumetric force $\vec{P_i}$ in the equation (2) grows up. The symbol n_i in the formula (7) means a number of particles of the i-th phase per unit volume:

$$n_i = \frac{6\alpha_i}{\pi d_i^3}. \qquad (8)$$

Momentum flux to the i-th phase from the other phases can be determined by the following formula

$$\vec{K_i} = \sum_{j=1}^{N} \varkappa_{ij} \vec{v}_{ij} + \sum_{j=1}^{N} \dot{M}_{ik} \vec{v}_{ik} - \mu_i \vec{v}_i; \qquad (9)$$

where $\vec{v}_{ij}$ is the velocity of particles of the i-th phase originated in course of fragmentation of the objects of j-th phase; $\vec{v}_{ik}$, $\dot{M}_{ik}$ - velocity and mass of particles of the i-th phase originated either as a result of breakup of a k-th macroobject or in course of realisation of a new Space programs (launches, separations, etc.). Parameters $\vec{v}_{ik}$, $\dot{M}_{ik}$ are external for the model.

The last additional in the formula (9) characterises the momentum loses of the i-th phase as a result of slowing down and combustion of the particles in the atmosphere.

Thus the mathematical model is worked out that makes it possible to follow the evolution of clouds of fragments of different size taking into account the interactions, breakups, radiation, gravitational and aerodynamic drag forces etc.

Simplified models

Sometimes to have a longterm forecast the mathematical model can be simplified to determine the number of particles at different altitudes, neglecting the distribution of particles in space.

The mass conservation equation (1) can be rewritten in the following form to determine the

number of particles of the i-th phase per unit volume n_i:

$$\frac{\partial n_i}{\partial t} + div n_i \vec{v}_i =$$

$$= \sum_{j=1}^{N} \psi_{ij} + q_i - \frac{6\mu_i}{\pi d_i^3 \rho_i}; \quad (10)$$

where ψ_{ij} is a number of particles of the i-th phase originated as a result of fragmentation of the j-th phase per time unit and q_i is the rate of origin of particles of the i-th phase as a result of destruction of large Space objects and realization of new Space programs.

The possible simplification is the assumption that most of particles move along a circular trajectories [8]. This assumption has enough reason because most of trackable objects have trajectories with very small eccentricities.

The momentum equation as an equation of vector–type can be rewritten in two projections: on the radius–vector and on the tangent to the trajectory.

Let the velocity physical components be $\vec{v} = (v_{ri}, v_{\theta i}, v_{\varphi i})$ and let us name the radial component $v_{ri} \equiv w_i$ and the tangential component $|v_{\theta i}, v_{\varphi i}| \equiv v_i$. The following relation is valid for debris particles evolution : $w_i/v_i \ll 1$, as the velocity of descent of debris due to atmospheric drag is very low in the upper layers of the atmosphere. So for the equation (2) projection on the r–axis we may use quasi–stationar approximation.

The influence of Sun radiation can also be neglected. The equation (2) looks as follows under these assumptions

$$\alpha_i \rho_i \frac{dv_i}{dt} = F = \frac{1}{2} c_f \rho_a(r,t) v_i^2 n_i \frac{\pi d_i^2}{2}; \quad (11)$$

$$\frac{\alpha_i \rho_i v_i^2}{r} = \alpha_i \rho_i g(r) \approx \alpha_i \rho_i \frac{\gamma M}{r^2}; \quad (12)$$

where γ is the gravitational constant and M is the mass of the Earth. Then the velocity of debris particles descent can be determined as follows: $w_i = dr/dt$.

Averaging in longitude and latitude gives us the following form of the equation (10)

$$\frac{\partial N_i}{\partial t} = -w_i \frac{\partial N_i}{\partial r} - N_i \frac{\partial w_i}{\partial r} + \dot{N}_i, \quad (13)$$

where N_i is the number of debris particles of the i-th phase per unit altitude; $\dot{N}_i$ - the rate of the increase of particle number. The equation (13) with some additional assumptions is similar to the equation used in [9] for modelling of Space self–cleaning of debris.

To solve the equation (13) the velocity w_i can be determined independently from the equations (11), (12) :

$$w_i = -\frac{3}{2} \frac{c_f \sqrt{\gamma M r}}{\rho_i d_i} \rho_a(r,t), \quad (14)$$

where function $\rho_a(r,t)$ can be obtained from one of the models of a standart atmosphere or its approximations [8].

Parameters ψ_{ij} in the equation (10) can be determined by the same formula (4) as $\varkappa_{ij}$ but instead of functions K_{kj}^i we must use functions N_{kj}^i determining the number (and not mass) of particles of the i-th phase appearing as a result of fragmentation of objects of j-th phase in the course of collision with the objects of the k-th phase. The function $\dot{N}_i$ is obtained as a result of steradional summing up the right part of the equation (10).

The annual rate of origin of new debris particles per unit altitude due to space activity of the Mankind Q_i can be determined by a modification of formula suggested in [7].

$$Q_i = L\big[(1 - F_e)A_1 D_1 + F_e D_e A_e\big]\frac{N_i}{N^*}, \quad (15)$$

where $N^* = \sum_{i=1}^{N} \int_{R_0}^{R_1} N_i dr$; L -the annual number of launches; A_1, A_2 - the average amount of objects originating in the orbit due to one successful launch A_1 and an exploding one A_e; D_1, D_e - part of the originated objects remaining in the orbit for a long period after a successful launch and after an explosion on the orbit correspondingly; F_e - the coefficient characterising the amount of explosions among all the launches.

The initial conditions are obtained in the following way. The whole number of trackable objects ($d_i \geqslant 10\ cm$) and its distribution vs altitude are introduced. In addition the numbers of untrackable objects ($1cm \leqslant d_i < 10cm$ and $0.1cm \leqslant d_i < 1cm$) are introduced. The distribution of untrackable objects vs altitude is estimated and the character of its distribution is supposed to be the same as that for trackable ones.

The calculations were carried out for a simplified problem to estimate the whole number of particles of different size. For this purpose the effective diameter d was introduced and the whole number of particles was reguarded as one phase. This simplification was done just to obtain the illustrative material and the introduction of several phases that differ from one another by the diameters of objects included and inclinations of the orbits does not make a serious difficulty.

Under these conditions the rate of fragmentation of particles in course of collisions was introduced by the formula

$$\Psi = \mathcal{F} n^2 d^4. \qquad (16)$$

Results and discussion

The calculations were carried out for the following values of parameters: $L = 120$ launches per year; $A_1 = 4$ objects per launch; $A_e = 125$ objects per launch; $D_1 = 0.63$; $D_e = 0.82$; $F_e = 0.03$; the number of trackable objects was estimated as 7000.

The results of numerical modelling are shown in Fig.1 illustrating the plots of number of debris particles vs time and altitude. The initial distribution for the year of 1990 is not shown in the Fig.1 but it is similar to that shown for the year of 2000. It is clearly seen from the Fig.1 that the distribution of the number of particles vs altitude has two zones of maximum: $650km \leqslant h \leqslant 1000km$, $1400km \leqslant h \leqslant 1500km$, - and this picture is qualitatively preserved for rather a long period of time. The increase of the population of the orbits within the first two hundreeds years is rather slow but steady. Nowadays the number of debris particles in the orbits is far from being critical. After the critical conditions are reached the number of collisions of debris particles grows rapidly and the self–production of debris takes place [3]. After the critical concentration of particles per volume unit n is achieved the orbit can no longer be used for space flights. Since then the new launches on the orbit are considered to be stopped. The critical conditions are reached on different altitudes at different periods of time. In the orbits of higher altitudes the process of debris self–production begins later. The results obtained have rather simple explanation within the model applied. On the first stage of debris evolution the collisions of particles are rare and the average diameter d remains stable.

The growth of the number of debris particles in course of collisions is proportional to the particles concentration in the second power $\Psi \sim n^2$. After the concentration increases to a critical value and the number of particles grows rapidly the new space programmes are stopped and since then the volumetric concentration of debris in the orbit remains stable

$$\alpha = (\pi d^3/6)n = const.$$

Thus the growth of particles turns to be proportional to $\sim n^{2/3}$ (see formula (16)). Besides the drag force $F = (1/2)c_f \rho_a v^2 (3\alpha/2d)$ increases with the diminishing of size of particles.

Thus the mathematical model of space debris evolution is worked out. The possible simplifications of the model are reguarded. Numerical results of long–time forecast of the orbital debris distribution show the possibility of description of evolution of debris particles with the help of suggested model taking into account the influence of the atmosphere, collisions of particles, the growth of debris due to realization of new space programmes.

References

1. Chobotov V.A. Dynamics of orbital debris clouds and the resulting collision hazard to spacecrafts. *FBIS, Journal of the British Interplanetary Society.* Vol. 43, N 5, 1990, pp. 187–195.

2. Flury W. Space debris: a European view. *Proceedings of Space Debris Forum, JSASS, Tokyo, May 15, 1992,* pp. 37–54.

3. Kessler D.J. Collisional cascading: the limits of population growth in low Earth orbit. *Advances in space Res.* Vol.11, N 12, 1991.

4. Loftus J.P., Anz–Meador P.D., Reynolds R. Space debris minimisation and mitigation plans and practices. *Space Debris Forum, JSASS, Tokyo, May 15, 1992,* pp.16–27.

5. Yasaka T., Ishii N. Breakup in geostationary orbit: a possible creation of a debris ring. *IAA-91-596. 42nd Congress of the International Astronautical Federation,* Montreal, October 5–11, 1991, pp.1–12.

6. Perek L. Space debris as an issue for the international community. *Space Debris Forum, JSASS, Tokyo, May 15, 1992,* pp.1–15.

7. Talent D. Analitic model for orbital debris environmental management. *AIAA Paper No 1363,* 1990, pp.1–10.

8. Nazirov R.R., Ryazanova E.E., Sagdeev R.Z., Sukhanov A.A. Analysis of space self–cleaning of debris due to atmospheric drag. *Space Research Institute Pr–1670,* Moscow, 1990.

9. Nazarenko A.I. The technique of Near–Earth Environment self–purification prediction process. *Orbital Debris Monitor, Fairfax.* Vol.5(2), 1992, pp.5–6.

10. Smirnov N.N., Dushin V.R., Panfilov I.I. The space debris evolution model. *Orbital Debris Monitor, Fairfax.* Vol.5(2), 1992, p.4.

11. Chobotov V.A., Spancer D.B. Debris evolution and lifetime following an orbital breakup. *Journ. Spacecraft.* Vol. 28, N 6, 1991, pp.670–676.

12. McKnight D.S. Examination of possible collisions in Space. *Orbital Debris Monitor, Fairfax.* Vol. 5(2), 1992, p.5.

13. Fortov V.E. Physics of high–velocity impact. *Orbital Debris Monitor, Fairfax.* Vol.5(2), 1992, p.16.

14. Zelensky A.S., Kiselev A.B., Maksimov V.Th., Yumashev M.V. The mathematical modelling of dynamic reaction of spacecraft under crushing load from artificial objects. *Orbital Debris Monitor, Fairfax.* Vol.5(2), 1992, p.16.

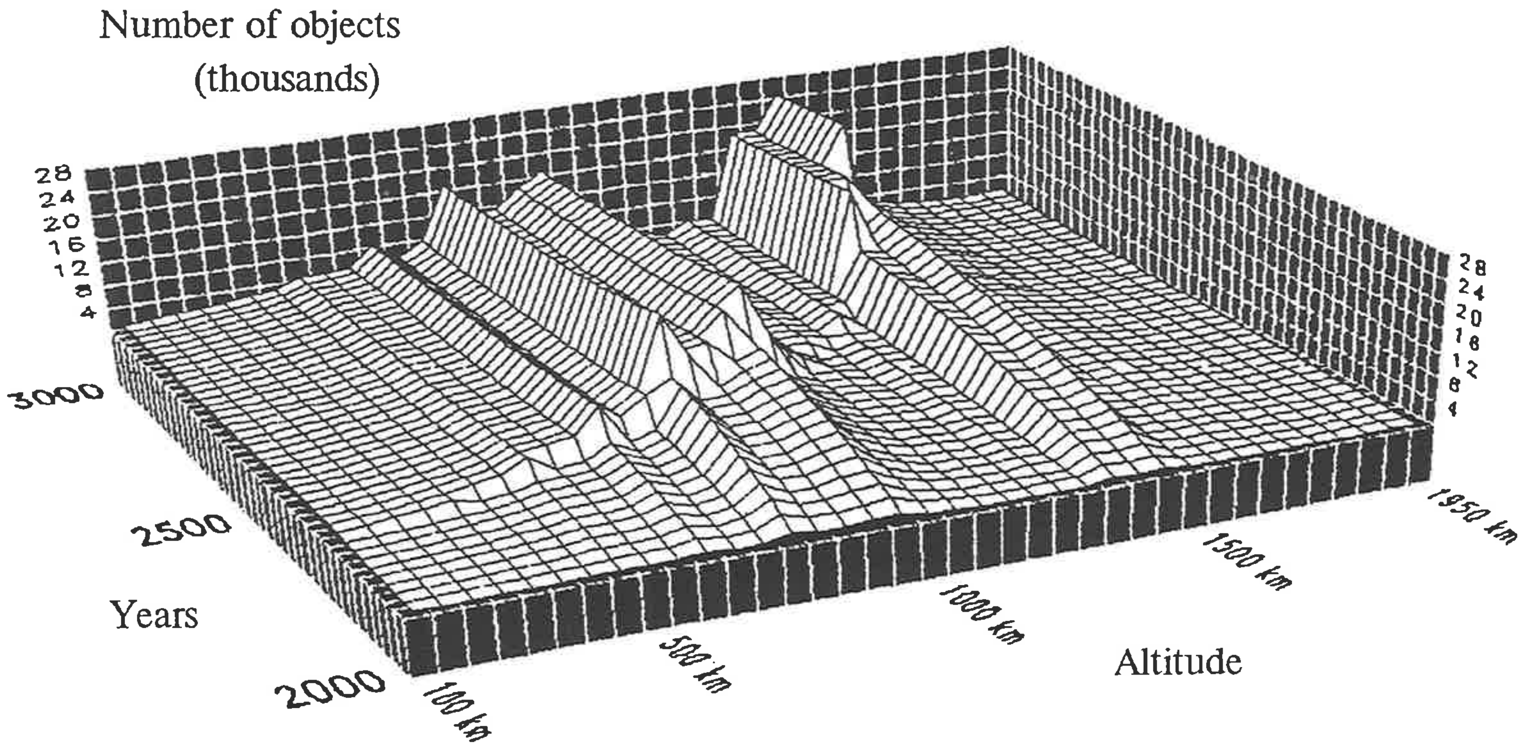

Fig. 1. Long-term forecast of the orbital debris distribution