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THEORETICAL AND PRACTICAL ISSUES IN SOFTWARE FOR SPACE DEBRIS MODELING

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Abstract

This paper describes critical issues in the modeling of space debris. The U.S. is tracking thousands of objects currently, and assessing the threat to planned space activities by computing hazard probabilities. Fundamental parts of these calculations are the programs that model different kinds of space interaction events and the resulting hazard, both to current and planned space activities and to people and structures on the ground.

We describe results of a detailed mathematical analysis of a prominent model for space debris, discuss modeling issues that confront any space debris model, and some separate issues that confront implementors of such models.

1 Introduction

Space hazards come from old satellites, old rockets and boosters, and other debris. The U.S. is tracking thousands of space debris particles currently, and assessing the threat to existing satellites and planned launches by computing the hazard, often in terms of collision probabilities and collision geometry. One of the most important of these hazards is to existing satellites or planned launches from the debris already in space. Fundamental to these calculations are the programs that model different kinds of impact between satellites and particles, the breakup of a satellite due to collision or explosion, and the short-term and long-term propagation of the resulting debris clouds. This paper concentrates primarily on computer software for modeling this kind of space object breakup, and describes some common difficulties with writing and using such programs. We do not generally address the problem of finding better debris models; we only describe improvements in the use of ones that already exist or that will be developed.

As an example, we will describe results of a detailed mathematical analysis of a widely used breakup model, the FAST models of D. McKnight [3], and show how some simple changes in the published model tremendously improve the reliability of programs written to implement it. This distinction between the program and the model is important, and does not seem to be made sufficiently clear in this application domain.

We also discuss modeling issues that confront any model of space debris, and some separate issues that confront an implementor of any such model. We want to emphasize in general the difference between a space debris model and a program that implements that model, since many of the difficulties of using those programs are not at all difficulties of the model, they are deficiencies in the program.

2 The FAST Models

The FAST (Fragmentation Algorithms for Satellite Targets) models, as described in the available references [2] [3], consists of separate models of several different aspects of space object breakup. We describe certain modifications to these models that integrate them more fully, as well as some suggestions for further research in the individual models. We are focussing on this model because it is one of the better ones, and therefore makes quite a good case study.

It should first be mentioned that the FAST report does not describe an algorithm at all. It describes alternative sets of equations for various component models of the breakup problem, with little or no guidance about when to use which equation or how to set some of the parameters. It is essential in using these equations to have access to the model developer, or someone else who has experience with these models, to guide these selections. The models are incomplete, and will therefore be misleading, without that access.

To be fair, it is clear that there was no intention to describe a single consistent model of satellite breakup. Instead, there are five different models that describe important aspects of the breakup problem, each of which has been tuned separately to the available data. There are models described for breakup conditions, mass distribution, velocity distribution, fragment density, and ballistic coefficient. The most detailed model is for mass distribution. In order to turn these disparate models into an integrated model of satellite breakup, some model integration is required; that entails changes in the individual models. We have made one set of choices along these lines, in order to write a program for the combined models, including some useful changes to the FAST models themselves. We will not describe all the models in detail, but we will describe some of the changes we made in order to write a program to implement the models. We will not describe the velocity distribution at all, since it is the most intricate and requires the most assumptions and discussion. More details will be made available in a forthcoming report.

It is apparent that the FAST models have been developed by fitting data from actual experiments. It seems that part of the reason for the peculiarities in the model equations, and in equations for many other models in this problem domain, is that the form of the equations has been chosen to facilitate fitting, not to facilitate understanding of the problem. Each partial model has a number of parameters that can be adjusted to fit a particular problem, and presumably enough parameters are adjustable to fit many kinds of data. There should be some theoretical justification for the form of each equation, so that the parameters can correspond to some real physical phenomena. This justification should be the direction of future research in this area. Until that can be done, we will have to rely on the empirical models. Even in this case, coefficients that are derived from fitting data cannot be assumed valid unless both the data and the fitting algorithms used are presented, so that an independent assessment of the models can be performed.

2.1 Breakup Model

The first question is what type of breakup event is being considered, since that choice will affect the mass distribution model used: explosion or collision. If there is a collision, then there is a further question of whether or not the fragmentation is complete. We define

- M_t = mass of target (gram),
- E_p = relative kinetic energy of projectile and target (Joule).

There is an assumption here that the target is nearly stationary compared to the projectile, so that the relative velocity is nearly the same as the projectile velocity, and the relative kinetic energy of the projectile can usefully be computed from the relative velocity of the projectile and target. This assumption and others like it need to be made explicit as part of the model if they are to be used to compute kinematics, for momentum or energy conservation. The assumption does not affect the empirical equations, and this fact, too, should be made explicit.

The criterion for complete fragmentation in a collision is given as a lower bound CF on the *energy ratio* $ER = E_p/M_t$ of the collision, but the bound is not the same in all the references. If we take M_e to be the *ejecta mass* in grams, then when $ER \leq CF$, so that the fragmentation is incomplete, the ejecta mass is a proportional amount of the total, decreasing to 0 as ER increases to CF. There is a difficulty in the formula used when ER is small. In fact, it may be that low energy impacts (in this case, those with $ER \leq CF/2$) must use a completely different mass distribution with different rules, but these rules are not part of this model.

2.2 Mass Distribution

This section describes the models used for mass distribution.

There are two mass distribution models for target breakup: one for "ejecta mass" and one for "remaining" mass. The idea here is that some of the target mass is "blown away" by the collision (this part is called "ejecta"), and the rest simply comes apart. The model used for the "remaining" mass is the same as the explosion model.

The ejecta mass distribution takes the usual power-law form: for $M > 0$, $CN_e(M)$ is the number of ejecta fragments with mass at least M.

$$CN_e(M) = A\ (M/M_e)^{-B}, \qquad (1)$$

where A and B are (unitless) constants. As long as both M and M_e are in the same units, the model computes the same values for $CN_e(M)$.

The theoretical distribution of masses of ejecta pieces according to the $CN_e(M)$ model is that M_n, the mass of fragment number n, has

$$M_n = M_e\ (A/n)^{1/B}.$$

It should be noticed that this mass distribution does not depend at all on the energy of collision, when there is complete fragmentation. This is an important area of future research, to determine how the mass distribution depends on the excess energy.

Parameter estimation for B follows either of two empirical models, and the precise conditions under which each one was derived would greatly assist in the choice between them. The estimate agrees generally with other work in the area [1], and a model comparison would be helpful. Parameter estimation for A can be derived from mass conservation: the sum of the masses of the ejecta pieces should equal the ejecta mass. A clever mathematical simplification allows the value of A to be computed directly. These estimation methods give an improvement of [3] over [2], which simply specifies constants corresponding to A and B. It would be interesting to see the curve fitting and modeling data that was used to improve these coefficient estimates.

For the remaining mass or the explosion mass, the FAST report uses an exponential model that has several difficulties in common with other breakup models. The units used are extremely important: unlike the previous model, changes in the units affect the value of the computation (even when all mass parameters have the same units). The theoretical mass distribution for remaining mass is computed without enforcing mass conservation. The model could be changed to enforce mass conservation, but the resulting sum does not converge as the number of particles approaches infinity, so that no simplification tricks will work here as they did above. That non-convergence is also a good reason to look for a different model for this mass distribution, or a better way to estimate N. This is not a criticism solely of FAST; all debris modeling seems to use this mass distribution model, one way or another. As before, there should be some kind of "dust" model for particles not to be treated individually. Again, this is not a problem for FAST alone; all major breakup models have the same deficiency.

2.3 Fragment Density and Ballistic Coefficient

The model for fragment density is given as three different functions for mass of fragment in terms of diameter of fragment, one each in three different diameter intervals. The models are not smooth, or even continuous, at the boundaries between the diameter ranges. There is a gap at one boundary and an overlap at the other, so this density model cannot be inverted to compute diameters from masses. Since there is no way to compute the diameters needed from the masses produced by any mass distribution model, this density model cannot be used. In order to eliminate this problem, we have presumed to invert the functions using a graph of the relations given, and changed the boundaries where the different models meet to make the function continuous (though still not smooth).

The ballistic coefficient model was clearly derived by combining one of the density models above with a definition of the coefficient and an assumption of spherical particles. Therefore, any change in the density model means a corresponding derived change in this one. Moreover, since only one of the density models was used, this model is inconsistent with the density model over many ranges of diameter. The difficulty with the density model means that this model cannot be used with a mass distribution model either.

So here we completely throw out this model, except for one of the constants and the spherical particle assumption. It is replaced with the definition from first principles, which will be consistent with the other models, even if they are changed. There should be some theoretical justification for the two models used here, in the form of sample data and curve fitting algorithms if nothing else.

2.4 Discussion

The use of diameter as the fundamental model parameter in the velocity, density and ballistic coefficient models has problems. Because the density model does not define a 1-1 function, it cannot be inverted to produce a mass-to-diameter function. Therefore, those models cannot be used to provide further information about a particular sample mass distribution. If the density model can be corrected, then the set of models can be made more coherent. We have made several proposals for changing the density and ballistic coefficient models.

Most of these difficulties seem to be associated with attempting to use the equations without access to the model developer, whose experience and judgment are necessary for proper selection of parameters and equation sets to use. Without that access, any user will need much more supporting data; especially some criteria for applicability of the different partial models, description of the curve fitting model and data used, and policies for the use of the equations in the absence of sufficient data. With some judicious replacement and explanation, as noted here, the equations presented can be turned into a model of space object breakup.

3 Modeling Issues

In this section, we discuss some of the modeling issues that arose as we studied the FAST models, which we think are common to any model of space debris. Most breakup models have apparently been developed by fitting actual data [2]. The sparsity of that data has led to some questionable, but necessary, assumptions in the models. Each model has parameters that can be adjusted to fit a particular problem, and enough parameters are adjustable to fit many kinds of data. These models should have some theoretical justification for the forms of the equations, so that the parameters can correspond to some real physical phenomena. Such justifications should be a direction of future research. Until then, we will have to rely on shared empirical models. The coefficients in these models that are derived from fitting data can only be justified by presenting both the data and the fitting algorithms, so that independent assessments of the models can be performed.

3.1 Expert Knowledge

The notion of a model is changing over time. It used to be that the domain expert would have a set of programs (or even just equations) that could be used to help answer questions. Those programs were usually home-grown, closely matched the expert's own models of the phenomena, and had several parameters that the expert could adjust to match the problem.

When such a modeling program is used by a human expert (not only a domain expert, but an expert in the particular program, and in the model implemented by the program), there is an enormous amount of knowledge added to what is contained in the model, and much of the knowledge in the model is not implemented in the program. That knowledge helps determine if the model is at all appropriate for a problem, helps prepare a problem for the model, helps set up the program to apply to the problem, and helps interpret the computational results. This is especially true of programs that produce random samples from an extremely difficult probability distribution, such as the ones used in the space debris models.

This situation is changing more and more to one in which the customers, having problems that need expert advice, want to run these programs for themselves. When these models or programs are distributed to others, even experts in the field, much of that additional knowledge is lost. When they are distributed to users with problems to solve, most of it is lost. The experts are being asked to put more knowledge into the programs, and to make much more formal the difference between a model and a program implementing the model. Otherwise, the use of the model is unjustified (if not unjustifiable), and the results are unreliable.

3.2 Excess Precision

Programs that produce numbers are deceiving. Even when the program is explicitly producing a sample from a random distribution, as a way of getting a feel for the phenomenon modeled, it is only the experts who know enough to disbelieve the numbers. We have seen this process occur time and time again, even though the users say that they will treat the numbers as samples, they often show up as averages because the samples are confused with "typical" values.

The model programs have explicit notions of the variability of their models. In some modeling domains, and particularly in debris modeling, not enough is known to do that yet. Therefore, the programs should be used to compute some kind of range of possibilities, by making several runs with the same parameters and different random number seeds, so that some idea of the variability of the results can be displayed using both ranges and summary distributions.

3.3 Random Samples

When a "typical" situation is to be generated from a model, and there is not enough knowledge to do so, any of several things can be done. Usually we generate a random sample, and use the Bayes Principle of Indifference to fill in values we don't know. Such assumptions as, "All cases are equally likely", or "Assume spherical symmetry" are versions of that principle, which is generally used to impose enough extra symmetry on a problem that we can estimate the parameters.

In the debris analysis problem, it is these extra symmetries that are the most troubling. For example, a spherically symmetric expanding cloud has a fairly well-defined set of parameters (namely, the rate of radius increase) that allows the cloud extent to be predicted and tracked. We know from the parameters how fast the cloud expands, and that it does not expand faster. Now let us look again at this latter property. It gives an upper bound for the distance of particles in the cloud from the cloud center at any given time.

This upper bound is what is troubling about the symmetry assumption for the cloud. An asymmetric cloud can expand much faster in certain directions than the averaged symmetric one, and these could be the most dangerous directions for neighboring space objects. We need some way to make safety predictions about that possibility also.

What is needed here is a way of computing a better model for cloud expansion, given that we do not know how asymmetric such clouds can be. We need to find some way of estimating how asymmetric such clouds can be, and adding in calculations of that sort to the models. It seems clear that the extremely asymmetric cases are rare, though there is no data supporting that possibility, and we want to make that an explicitly modeled situation anyway, so it can be traded off to compute the hazard. We do know that the behavior of all of these models is non-linear, so the behavior of the average case is not the same as the average behavior of the cases, and yet we have no models for how much they differ.

It is the same problem when we assume spherically symmetric density of a cloud (or even uniform density in some cases), and calculate the decay to background flux on that basis. Unexpected turbulence (well, not really unexpected, only unpredictable; we *know* the clouds will be turbulent and therefore asymmetric) can lead to clusters of high density that decay much more slowly, and therefore travel much farther, than the "averaged" spherically symmetric density. This problem should be an area of active research.

4 Implementation Issues

There are choices that must be made for any implementation of any of these models. For example, all the models account for uncertainties in the basic breakup event using random variables, and then must decide whether to compute distributions for the random variables or a single sample from those random variables. The choice of which conservation laws to enforce (mass, energy, angular momentum, etc.) also affects the generation of the random variables, because they imply dependencies among the random samples. This is especially true for the mass distributions, since they are the most well-developed, and often depend critically on the assumption of independent samples.

In order to write a program to implement most mass distribution models, we needed to make a decision about the number of particles to generate. First, since the number of particles generated is finite, all of the mass conservations expressions are approximate, and the resulting masses must be normalized to add up properly. Most mass distribution models have no lower bound on particle size, which usually leads to convergence or computation problems. We could relax the assumption that the masses add up, so that the rest of the calculations will treat only the mass "accounted for" by the mass distribution model, under the assumption that the other mass is not as important. There should be some lower bound on the size or mass of particles to be treated individually, and all smaller particles treated collectively somehow. This "dust" model is not used in any of the popular breakup models, and should be an area of active research.

More generally, if the existing models and programs do anything, they should help us learn more about the space debris modeling problem. Therefore, the programs and the models will change with time: new models will be developed and new programs will be written. The programs will need to be compared and contrasted, so they will need to be written to be used in an enviroment, instead of making too many assumptions about their run-time context.

5 Summary and Conclusions

We have described some important issues in the modeling of space debris, using the FAST models as a reference point for discussion, and in the programs that implement those models. We pointed out that for many of the models, the knowledge necessary to prepare the inputs or to interpret the outputs is provided by the knowledgeable user (most often the expert or model developer) and that this undermines the ability of the program to be used by others; we suggested that this information be made explicit and organized in a number of straightforward ways now available. We also pointed out that many of the programs suffer from excess precision, given the uncertainties in their input and underlying models, and that the hazards can be seriously underestimated by computing only average behaviors.

By discussing several modeling issues and some related implementation issues, we hope to encourage experts and model developers to create debris analysis programs which are more transferable to other users, more reliable in the face of the existing uncertainties in debris modeling, and more flexibly organized so that as our understanding increases in this area, we can more easily introduce the necessary changes to our evolving models.

References

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