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APPROACHING THE EXPONENTIAL GROWTH: PARAMETER SENSITIVITY OF THE DEBRIS EVOLUTION

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ABSTRACT

We report on our ongoing modelling work for the future collisional evolution of the low-orbiting debris population. We have studied the evolving distribution of the orbiting objects with respect to mass and altitude, accounting for several source and sink mechanisms, and describing collisional outcomes through a semi-empirical model for the fragment mass distributions. We have found that the exponential growth of collisional framents always occurs in our model, whereas its timing and pace are very sensitive to some poorly known parameters related to the physical properties of the Earth-orbiting objects. However, we stress that fairly plausible parameter choices predict that the runaway growth will occur within the next century. In these cases, the sensitivity of the results to future launch and/or deorbiting policies is so weak that drastic changes will need to be made to avoid or delay in a significant way a catastrophic outcome.

1. INTRODUCTION

The low-orbiting debris population is similar to the asteroid belt, since it is subject to a process of high-velocity mutual collisions which affects the long-term evolution of its size distribution. However, the situation is more complex than for the asteroids for at least three reasons: (i) launches and explosions provide sources of material (partially) subject to human control, adding to the fragments generated by impacts; (ii) the number density of objects is a sensitive function of altitude, and so is the sink mechanism due to drag; (iii) relative speeds are dominated by mutual inclinations, which are much larger than typical orbital eccentricities and unevenly distributed (whereas among the asteroids eccentricities and inclinations have similar, fairly broad distributions, with average values ≈ 0.15).

To model the debris evolution process, we have numerically integrated a large set of coupled, nonlinear, first-order differential equations, with each equation giving the rate of change of the population present in a discrete size bin and in a given altitude shell (Cordelli *et al.*, 1993). Initial conditions are provided from the (limited) knowledge of the existing population, which has to be extrapolated to the smaller size range of untrackable particles. The frequency of collisions is derived in each altitude shell by a computation of the average intrinsic collision probability of the corresponding population of catalogued orbiting bodies (Rossi and Farinella, 1992); this probability is then multiplied by the cross-section of the assumed target and the number of existing projectiles. The same method provides the average impact speed vs. altitude (actually, the speed is found to be approximately constant at ≈ 10 km/s).

2. COLLISIONAL AND DRAG MODEL

Each collision can provoke either localized target damage (with a fraction of target mass M fragmented and ejected from a "crater" proportional to the impact energy, up to a maximum of 1/10), whenever the projectile's kinetic energy per unit of target mass E/M is smaller than a given threshold value S, called *impact strength* and assumed to be of the order of 10^3 – 10^4 J/kg; or complete target break-up, with a largest fragment including less than one half of the target mass, if the S threshold is exceeded. In either case the fragment mass distribution is modelled by truncated power laws, in agreement with the results of laboratory impact experiments (see e.g. Fujiwara *et al.*, 1989; Petit and Farinella, 1993). In the cratering case, the characteristic exponent q of the incremental fragment mass distribution $dN \propto m^{-q}dm$ (where dN is the number of objects having masses in the interval $[m, m + dm]$) has been fixed to 1.8; mass conservation implies that in this case the largest fragment comprises 1/4 of the total excavated mass. In the break-up case, an empirical relationship between the largest fragment to target mass ratio m_1/M and the specific impact energy E/M normalized to S has been used ($m_1/M = (E/SM)^{-1.24}/2$), whereas q has been derived from the relationship $q = (2+m_1/M)/(1+m_1/M)$ — implying that q ranges from 5/3 in a barely catastrophic collision ($m_1/M = 1/2$) to 2 in a supercatastrophic event ($m_1/M \to 0$), the latter case being such that equal logarithmic mass bins contain equal masses.

In the altitude range between 700 and 900 km, explosion events are also taken into account as a fragment source, with the resulting mass distribution modelled according to an empirical exponential law (Su and Kessler, 1985): the cumulative number of fragments of mass larger than m resulting from the explosion of an object of mass M is $0.1708\ M \exp(-0.65\sqrt{m})$ for $m \geq 1.936$, and $0.870\ M \exp(-1.82\sqrt{m})$ for $m < 1.936$, all masses being given in kg.

Drag-induced orbital decay has been taken into account by introducing, for each altitude shell, a characteristic residence time, inversely proportional to the (average) atmospheric density at the corresponding height and to the cross-section of the considered object. Drag provides the only mechanism causing an exchange of bodies between different shells, as collisions are assumed to produce fragment swarms remaining in the same shells as the parent bodies (this is consistent with the fact that typical fragment ejection speeds are $\approx$ 100 m/s, much smaller than the orbital velocities) and the contribution to the evolving populations of objects having sizeable orbital eccentricities is neglected (a reasonable approximation, as these objects contribute only a few percent of the catalogued population).

We stress that many areas of uncertainty remain in this modelling work. The most critical ones appear to be: (i) the relationship between mass (m) and cross-section (A) of the orbiting objects (as a default, we have used the simple empirical relationship $(m/\mathrm{kg}) = 62\ (A/\mathrm{m}^2)^{1.13}$, introduced by Kessler and Cour-Palais, 1979); (ii) the value of the (average) impact strength S of the targets and the fragment mass distribution function, which may depend on the shape, structure and material properties of the typical spacecraft; (iii) the existing Earth-orbiting populations as a function of mass and altitude, which are the initial conditions for the future evolution; (iv) the future launch and explosion rates, the two dominant sources of new objects before disruptive impacts become frequent enough; (v) the time dependence (neglected in our model) of drag in the high atmosphere, which provides the most important sink for small, low-orbiting bodies. The prospects for more reliable estimates of the corresponding parameters depend upon new observational and laboratory work, to be carried out for this specific purpose.

3. NUMERICAL RESULTS

The complex algorithm described above has been implemented by integrating numerically 150 coupled equations, referring to 15 altitude shells (six 50-km thick shells between 400 and 700 km, plus nine 100-km thick ones up to 1600 km) and 10 logarithmic mass bins (centered at values ranging from 1 g to 6000 kg, and spanning a factor 5.664 each). The integrations have covered a time span of several centuries in the future, starting from an initial population of about 57,000 bodies exceeding 0.4 g in mass: the vast majority of these objects have masses smaller than a few tens of grams, and only about 1650 exceed 10 kg; the altitude distribution of the small untrackable particles is assumed to mimic that of the tracked population. The net current rate of insertion into orbit of new massive objects, such as satellites and rocket bodies, is assumed to be about 60/year; this takes into account that a significant fraction of the launched objects (e.g., U.S. space shuttles and many Russian reconnaissance satellites) re-enter intentionally at the end of their short-lived missions. The altitude distribution of newly launched spacecraft in the future is assumed to always resemble the current one; for each launch, a few tens of bodies are inserted into the same altitude shell and in the mass bins up to 1 kg, to simulate the loss of small non-operational objects and devices during the early orbital phases. We have used a default value of one explosion/year, involving a body of 1500 kg in mass, in each of the two shells between 700 and 800 and between 800 and 900 km heights. For the impact strength S, we have adopted values ranging from a lower limit of 10^3 J/kg, similar to that found experimentally for natural stony targets, to an upper limit 50 times higher.

The qualitative evolution pattern is surprisingly similar to that predicted by a much simpler model, based on two equations only (Farinella and Cordelli, 1991). The typical features are shown in Figs. 1 and 2, which refer to our standard case, obtained by extrapolating to the future the current launch and explosion rates and assuming $S = 10^3$ J/kg. After a period of slow and steady population growth ranging from decades to centuries, depending on the altitude, the generation of collisional fragments exceeds the insertion into orbit of non-collisional debris and significantly increases the frequency of catastrophic impacts. As a consequence, the growth of the small-size population becomes exponential, while the abundance of larger objects (including the operational satellites) reaches a maximum and then rapidly drops. Later on, the environment is dominated by collisional fragmentation, with more satellites being destroyed than launched. It is plausible to infer that this will force the end of any space activity to be carried out in the corresponding altitude shells. The most critical altitude range for the early onset of runaway fragment growth corresponds to the crowded shells between 700 and 1000 km (see Fig. 1); but somewhat later the process is triggered also between 1400 and 1500 km (Fig. 2).

4. PARAMETER SENSITIVITY

We have explored the parameter sensitivity of the results of our model, in order to look for possibilities of preventing the occurrence of the runaway fragment growth. The most important parameter appears to be the average impact strength S of the targets (i.e., the threshold energy density resulting into catastrophic break-up), which we have assumed to be independent of size and altitude. In the standard case with $S = 10^3$ J/kg, the exponential fragment growth and the corresponding rapid disruption of large objects occur only about 50 and 100 years in the future between 900 and 1000 and between 1400 and 1500 km, respectively; with $S = 10^4$ and 5×10^4 J/kg, the "catastrophe" in the 900 to 1000 km shell is delayed until about 250 and 400 years in the future, respectively (see Fig. 3).

The sensitive dependence on S highlights the need for further experimental work to obtain reliable estimates of this parameter for the existing orbiting objects, rather than the potential utility of hardening future satellites. Indeed, we have found that, with $S = 10^3$ J/kg, stopping altogether the launching activity 50 years in the future — or, better, assuming that since that time an old satellite is de-orbited for any newly launched one — does not delay the "catastrophe" in any significant way, but just changes the subsequent trends in fragment abundances (which reach a peak and then drop as all the large objects are eliminated by collisional break-up). Even in the scenario of zero net launch rate and no more explosions since the year 2000, the ongoing collisional process will trigger an exponential fragment growth phase and a corresponding rapid decrease of the spacecraft population within the next century (see Fig. 4). A more realistic option is possibly that of deorbiting half of the satellites after 10 years since their launch. If this practice were started now (and preventing also all the explosions), the catastrophe would be delayed only to about 70 yr in the future (Fig. 5); the delay with respect to the standard case would be even smaller if the deorbiting of old satellites were started in 2000. Assuming that our standard parameter choice were realistic, these results indicate that a fairly drastic deorbiting policy needs to be adopted soon to avoid a catastrophic outcome during the 21st century.

The atmospheric density profile assumed to compute the rate of drag-induced orbit decay has some influence on the results only in the lowest few shells, where the abundance of fragments, albeit reduced, is not prevented from reaching the exponential growth stage. The explosion rate affects the abundance of fragments in the most dangerous shells in the immediate future, before collisional fragmentation takes over, and therefore plays some role in its timing; however, this role is in most cases marginal, and typically much less important than that of S.

Finally, we have found that the pace of the evolution is fairly sensitive to the assumed relationship between the mass and the cross-section of the orbiting objects: Fig. 5, obtained with the relationship $(m/\text{kg}) = 37.97\ (A/\text{m}^2)^{1.86}$, derived by Badhwar and Anz-Meador (1989), shows that the exponential fragment growth is significantly delayed in this case. This is due to the smaller collisional cross-section — hence longer lifetime — of the large bodies, playing the role of targets (about a factor 2 at $m = 10^3$ kg) and even more to the opposite effect for the small "projectiles" (the cross-section at $m =$ 1 g is larger by a factor 60 with the Badhwar and Anz-Meador formula, enhancing both collisional and drag-induced elimination of small fragments).

5. REFERENCES

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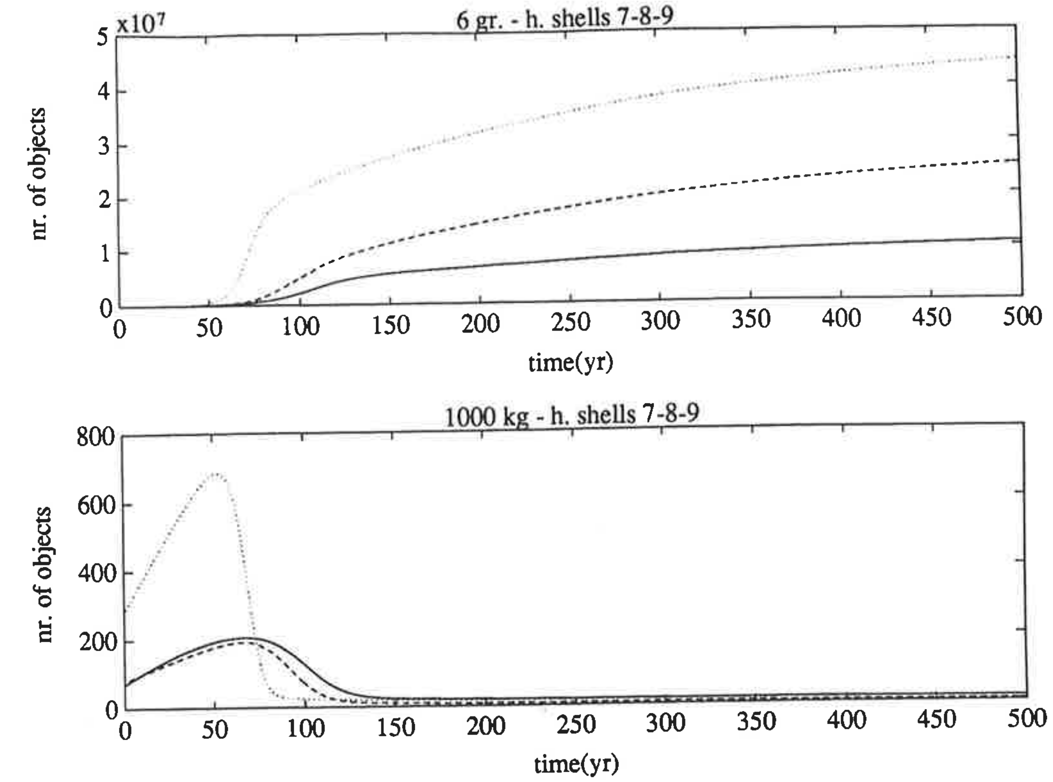

Figure 1. The time evolution of the abundance of objects in the mass bins centered at 5.66 g and 1059 kg (and ranging between 2.38 and 13.5 g and between 445 and 2521 kg, respectively), for the 100–km thick altitude shells centered at 750 (solid line), 850 (dashed line) and 950 km (dotted line). Here $S = 10^8$ J/kg, the Kessler and Cour–Palais mass vs. cross–section relationship is adopted, and the current rate of launches and explosions is assumed to be maintained in the future.

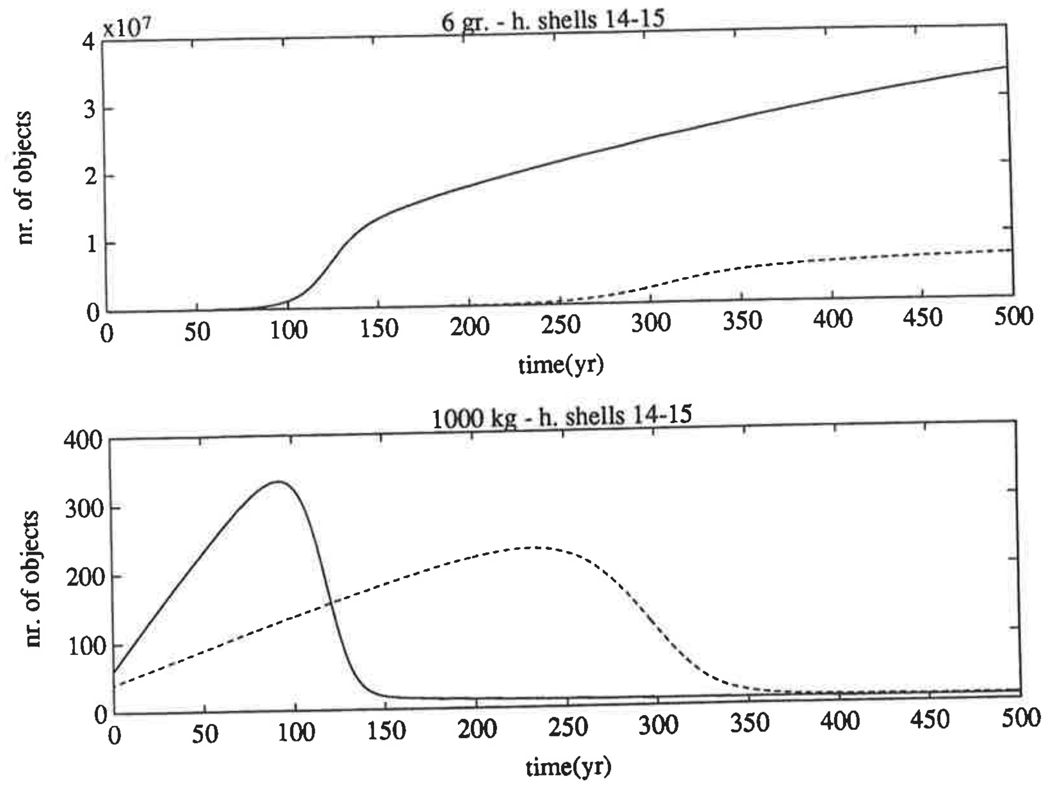

Figure 2. The same as Figure 1, but for the 100–km thick altitude shells centered at 1450 (solid line) and 1550 km (dashed line).

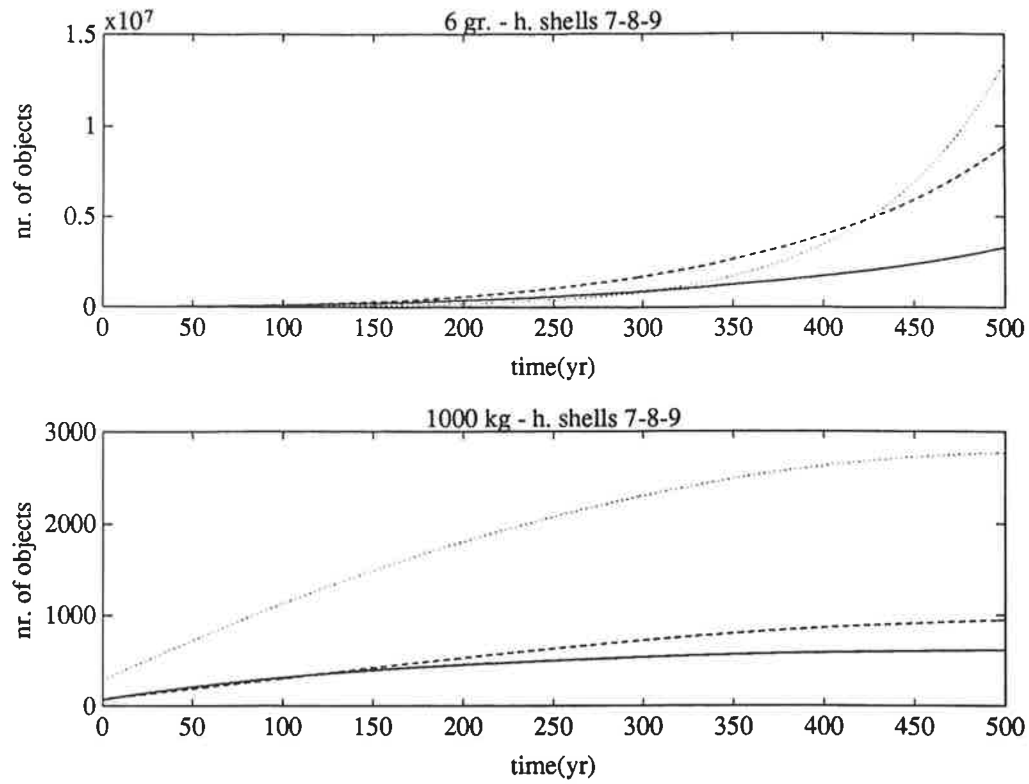

Figure 3. The same as Figure 1, but with $S = 5 \times 10^4$ J/kg.

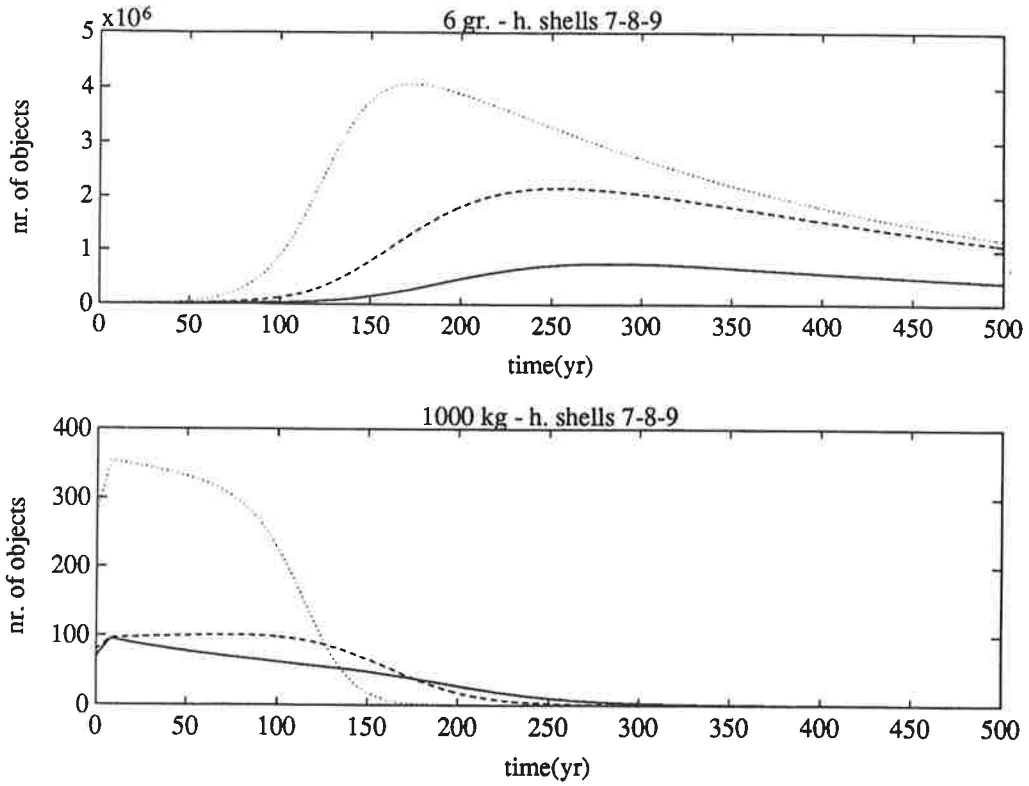

Figure 4. The same as Figure 1, but assuming a complete stop of all new launches and explosions after year 2000.

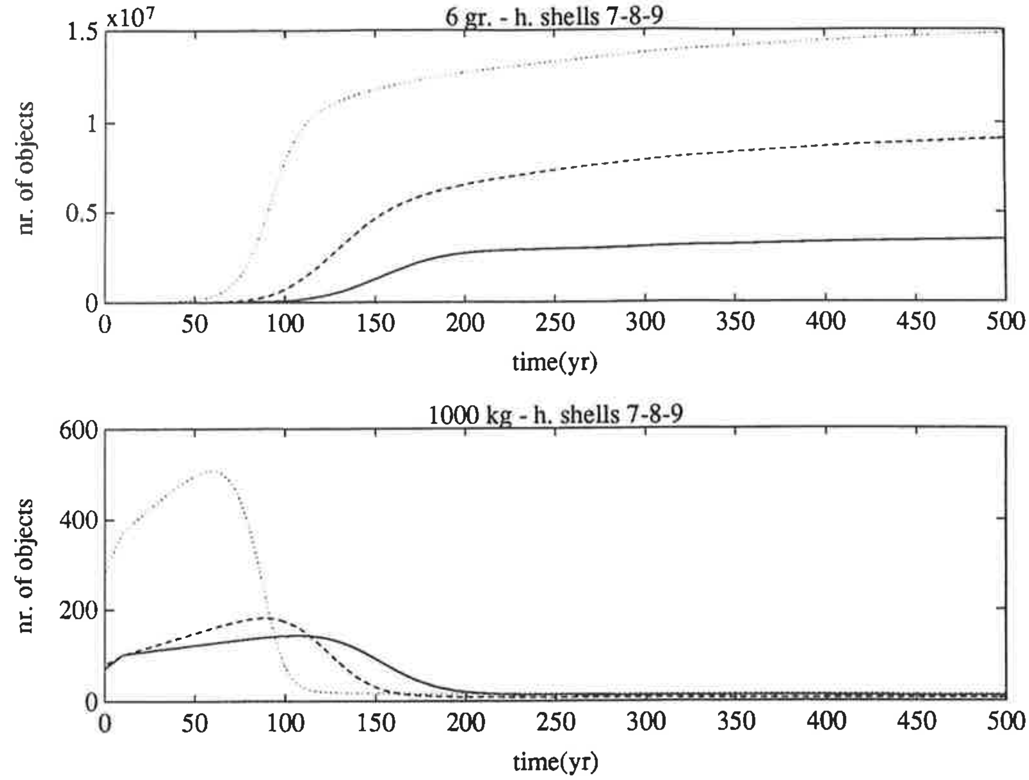

Figure 5. The same as Figure 1, but assuming no more explosions and that half of the satellites are deorbited after 10 years since launch, starting at present.

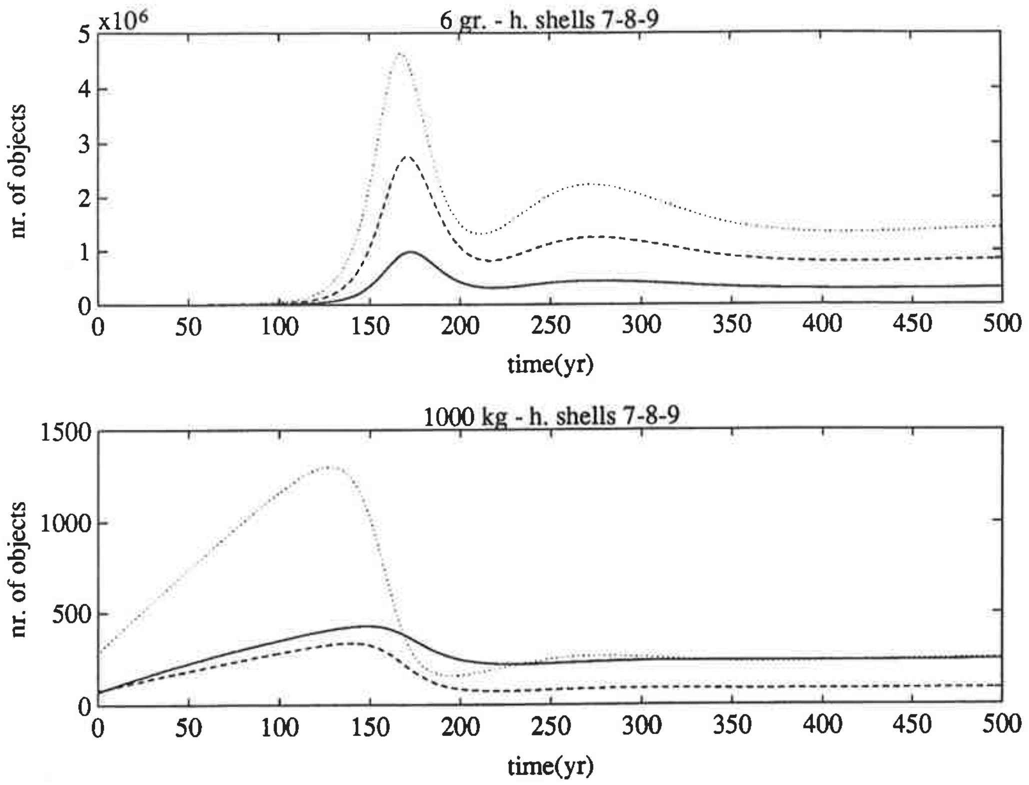

Figure 6. The same as Figure 1, but adopting the mass vs. cross-section relationship derived by Badhwar and Anz-Meador (1989).