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THE EFFECTS OF SATELLITE BUNCHING ON THE PROBABILITY OF COLLISION IN GEOSYNCHRONOUS ORBIT

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ABSTRACT

The rapid increase in the satellite population in geostationary earth orbit (GEO) is a matter of international concern, in part because of increased collision hazard. Colocated satellite pairs in GEO experience natural drift requiring periodic stationkeeping impulses, leading to similar trajectories and close encounters. To assess this risk, a procedure was devised which ranks satellite pairs in GEO according to the highest number of encounters over an extended time interval. Probability of collision was determined by a geometric and a stochastic approach. It was found that many pairs of satellites in GEO remain in close proximity and experience many close approaches over time. The top 10 pairs in terms of closest encounters were identified and mean time to collision based on encounter statistics was determined. Results of the study suggest that the bunching of active or inactive satellites at certain longitudes is a significant effect to be considered in the assessment of the collision hazard in the geosynchronous ring.

1. INTRODUCTION

The geostationary orbit (GEO) is widely used for communication, broadcasting, weather observation, and surveillance. Physical congestion is an international concern in terms of radio frequency and position allotment as well as in terms of collision hazard and environmental conservation (Ref. 1). The collision hazard is considered as a future issue, and there is neither uniform understanding nor a policy to control it at this time. To reduce the chances of an accidental collision and free desired longitudinal positions, some organizations, including agencies of the U.S. Government, have moved satellites to higher orbits beyond GEO just prior to retiring a satellite from service. Just how effective this practice is and what the altitude of the disposal orbit should be is currently under study. The urgency of the issue is, however, considerable in view of the fact that the population of GEO spacecraft is increasing at a rate of at least twice that of the general population and may escalate even more in the future (Refs. 2 and 3).

Another important issue is that of colocated satellites in GEO. Active geosynchronous satellites remain fixed over a point on the earth's surface. Due to the always-present natural perturbations, however, these satellites tend to drift away from their initial earth-fixed longitudinal positions in time. When two or more of the GEO satellites are placed in or near a specified longitudinal slot, they must be held within a given longitudinal band or "window" by periodic stationkeeping impulses. As independent orbit control by stationkeeping impulses leads to similar trajectories, the probability of collision at close encounter may become significant. The placement of four satellites at a common longitude, for example, results in an expected time of close encounter (50 m or less) of 0.6 yr assuming uncoordinated stationkeeping strategy (Ref. 4). The coordinated stationkeeping of four satellites located in the 18.8°W to 19.2°W longitude arc, as illustrated in Figure 1 (Ref. 5), precludes the possibility of such collision.

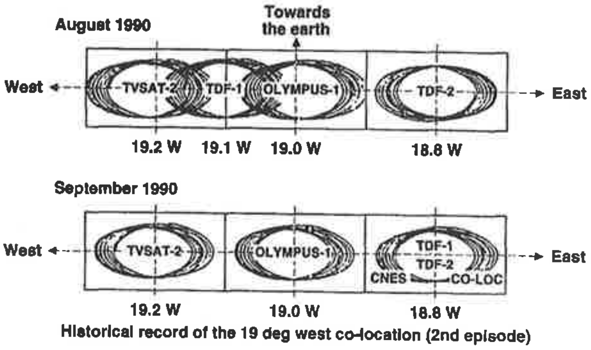

Figure 1. Example of Co-located Satellite Pairs

A procedure is described which identifies satellite pairs that experience large numbers of close approaches. The PC-based process makes use of the USSPACECOM catalog of geosynchronous satellites to calculate the longitudinal position of all satellites at a common epoch. Pairs which are located within a specified longitude band of 0.5° and an orbit inclination band of 1° are identified. The orbits of the "co-located" satellites are then propagated, and the number of close approaches within a given range determined. A ranking of satellite pairs according to the number of encounters is obtained over a time interval of approximately 16 mo beginning 16 November 1990. The probability of collision is assessed for the 10 pairs with the highest approach frequencies by a geometrical and a stochastic approach.

2. STUDY APPROACH

A typical longitudinal distribution of the geosynchronous population of objects is illustrated in Figure 2. It is apparent from Figure 2 that a significant "bunching" of objects occurs at certain longitudes. The nonuniform distribution of the population is the result of placing active spacecraft at preferred locations for the purpose of improving communication or earth coverage performance. The maintenance of tight longitudinal boundaries (e.g., $<0.1°$) is accomplished by a variety of stationkeeping strategies which sometimes ignore the presence of other objects in overlapping regions. In the cases where no coordinated stationkeeping is performed, a distinct probability of collision exists.

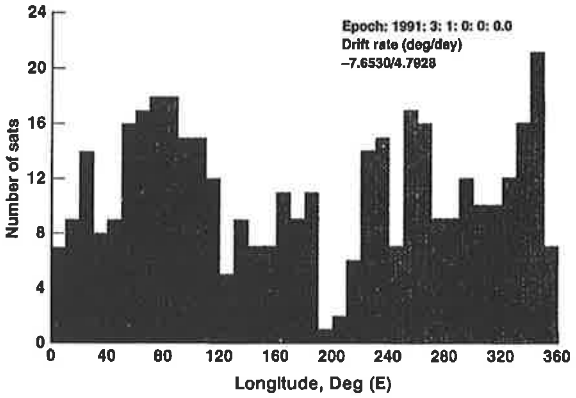

Figure 2. Geosynchronous Satellite Longitude Distribution

This study examines closely located or bunched satellite pairs which exhibit large numbers of close approaches over a period of time. The following steps describe the methodology which has been used to perform the study.

1. Using a monthly USSPACECOM database of GEO satellites, propagate all objects to a common epoch.
2. Compute the longitude of all objects at the common epoch.
3. Identify all satellite pairs within 0.5° delta longitude and 1° delta orbital inclination.
4. Propagate all satellite pairs over a specified time interval such as 2 wk before and after a given date.
5. Rank satellite pairs according to the number of close approaches on a biweekly basis.
6. Select absolute minimum distance as a random variable on a biweekly basis.
7. Compute maximum collision probability by a geometric approach based on the total number of encounters and the position uncertainty of the satellite pairs at encounter.
8. Fit a Weibull distribution function to the absolute minimum values and compute probability of collision.
9. Summarize and compare results.

2.1 Encounter Statistics

Typical results obtained by the study approach described previously are illustrated in Table 1. The results show that the pair of satellites 15237/16482 experienced the largest number of close approaches within 10 as well as 100 nm range over a period of time between 16 November 1990 to 16 March 1992. The smallest (absolute minimum) distance observed on 3/01/92 was 0.296 nm based on a biweekly propagation of each satellite. The simulation of close encounters was performed using a compatible NORAD propagator for each of 32 biweekly intervals. No stationkeeping maneuvers were simulated, but their effects were assumed to be included in the USSPACECOM two–line element sets supplied on the first of each month for the GEO satellite population.

The sample satellite owner report given in Table 2 gives the longitudinal location and interval between the satellite pairs as well as the description and launch date information.

Table 1. Colocated Satellite Encounter History, from 16 Nov 90 to 16 March 92

Satellite No. 1	Satellite No. 1	Count of Min. < 10 nm	Count of Min. < 100 nm	Date	ABS Min. (nm)
15237	16482	287	735	03/01/92	0.29600
19621	20705	177	535	12/01/90	0.71400
13069	20872	160	433	11/01/91	0.11600
4902	5587	124	160	10/17/91	0.42900
15826	20946	123	521	12/17/91	0.44100
15235	20873	122	419	08/17/91	0.44400
9047	12309	104	488	05/01/91	0.33400
9852	10365	103	129	05/17/91	0.32100
20193	20762	81	268	06/16/91	1.37400
18384	19344	65	671	11/16/90	0.15300
18316	19874	64	280	03/01/92	2.10000
16101	20776	53	162	01/01/91	6.05700
19330	19684	48	48	02/15/91	0.43800
20107	20217	44	289	04/01/91	0.85400
19397	20693	30	193	07/01/91	0.36000
15642	21222	29	48	08/17/91	1.02900
19621	20122	26	406	08/17/91	0.71900
12967	13984	25	109	02/15/91	0.66000
8697	15643	22	185	11/16/90	4.10300
16597	20771	20	254	04/01/91	3.11500
19548	20777	19	35	01/17/92	0.23200

Table 2. Sample Satellite Owner Report

Sat ID	Long. Deg. (E)	Delta Long. Deg	Description	Owner	Launch Date
19621	340.98	0.0159	TDF 1	FR	10/28/88
20122	340.96	0.0159	OLYMPUS	ESA	07/12/89
15237	274.98	0.0185	TELSTAR 3C	US	08/30/84
16482	274.96	0.0185	SATCOM KU-1	US	01/12/86
19548	297.57	0.0291	TDRS C	US	09/29/88
20777	297.60	0.0291	EUTELSAT II F1	EU	08/30/90
19621	340.98	0.0883	TDF 1	FR	10/28/88
20705	341.06	0.0883	TDF 2	FR	07/24/90
18384	346.20	0.0922	COSMOS 1888	USSR	10/01/87
19344	346.29	0.0922	COSMOS 1961	USSR	08/01/88
9478	182.26	0.1196	MARISAT 3	US	10/14/76
10669	182.38	0.1196	OPS 6391	US	02/09/78
15642	250.73	0.1291	ANIK C1	CA	04/12/85
17561	250.60	0.1291	GOES 7	US	02/26/87
20122	340.96	0.1331	OLYMPUS	ESA	07/12/89
20168	340.83	0.1331	TV SAT 2	FRG	08/08/89
14133	250.93	0.1935	ANIK C2 (TELESAT-7)	CA	06/18/83
15642	250.73	0.1935	ANIK C1	CA	04/12/85
15826	234.98	0.2026	TELESTAR 3D	US	06/17/85
20946	235.18	0.2026	GSTAR IV	US	11/20/90
20107	140.22	0.2102	GORIZONT 18	USSR	07/05/89
20217	140.01	0.2102	GMS 4	JPN	09/05/89
20083	48.60	0.2479	RADUGA 1-1	USSR	06/21/89
21038	48.85	0.2479	RUDUGA 1-2	USSR	12/27/90
14077	341.51	0.4434	INTELSAT 5 F-6	ITSO	05/19/83
20705	341.06	0.4434	TDF 2	FR	07/24/90
20193	328.38	0.4436	BSB R-1	UK	08/27/89
20762	328.83	0.4436	BSB R-2	UK	08/18/90

Figures 3 and 4 illustrate the time history of close approaches for the 15237/16482 satellite pair and the global (absolute) minimum distance for each 2-wk period plotted as a function of time, respectively. The frequency distribution of the latter is shown in Figure 5.

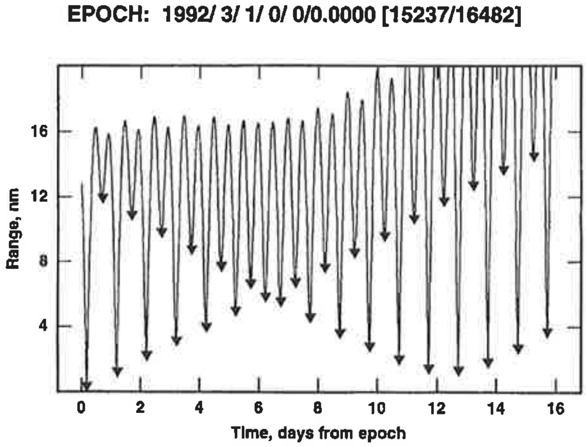

Figure 3. Range versus Time

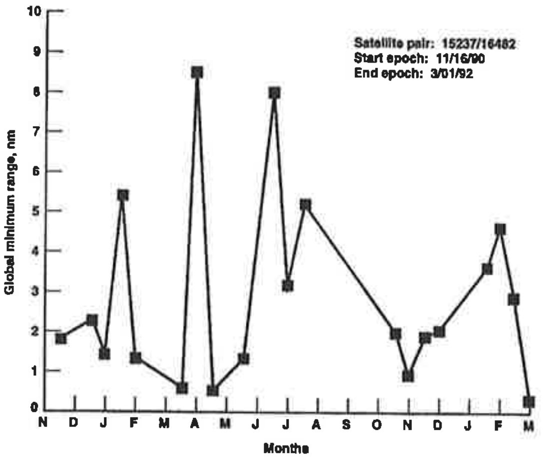

Figure 4. Global Minimum Range versus Time

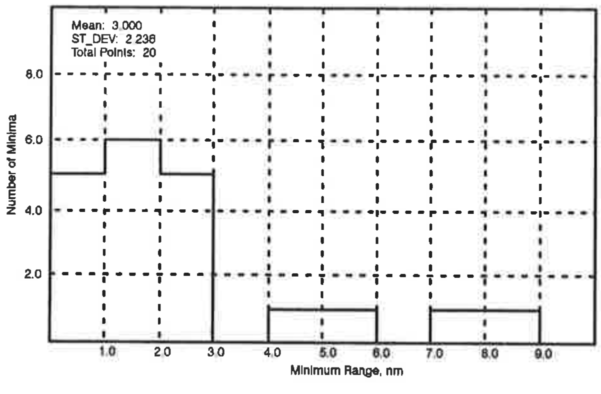

Figure 5. Satellite Pair 15237/16482 Frequency Distribution of Global Minima

2.2 Collision Probability

The probability of two satellites colliding at the time of closest approach is computed by a "geometric" and a "stochastic" method using the encounter statistics described above. The geometric approach is used to compute the maximum probability of collision at each encounter and the results are summed for all encounters for each satellite pair. The "stochastic" method, on the other hand, employs the Weibull probability distribution fit of the minimum distance between satellites considered as the random variable. This approach represents the application of the asymptotic theory of extreme order statistics to the distribution of minimum distances between different satellite pairs. This approach has been used in Ref. 6, for example, to estimate the collision probabilities for satellites in low earth orbit.

2.3 Geometric Approach

The probability that two satellites will collide during an encounter depends on the distance of closest approach R_{min} and the physical size of the satellites. The latter may be represented by the sum of the satellite equivalent radii. Thus, for example, the collision radius for two satellites of equivalent radii R_{s1} and R_{s2} is

$$R_s = R_{s1} + R_{s2} \qquad (1)$$

If it is assumed that the position uncertainties associated with the three dimensions (coordinates) of a nominal miss distance R_{min} at an encounter are Gaussian (normal) with zero biases and equal variance σ, and are uncorrelated, a probability of collision for $R_s \ll R_{min}$ is of the form (Ref. 7)

$$P(col) = \left(\frac{2}{\pi}\right)\left(\frac{R_s}{\sigma}\right)^2 \exp\left[\frac{-R_{min}^2}{(2\sigma^2)}\right] \qquad (2)$$

Due to tracking and ephemeris modeling errors there is an uncertainty in each satellite's position at the time of ephemeris update. The in-track, cross-track and radial component uncertainties can be combined into a common separation between satellites (e.g., 1 σ) which grows as a function of time from the last ephemeris update. Typical uncertainties may be on the order of 0.75 km within 2 wk, 1.6 km within a month or 5.0 km within 2 mo. The position uncertainties may, however, be much greater if no recent ephemeris updates exist. This suggests that a worst case uncertainty can be assumed for analysis which yields the maximum probability of collision at the time of closest approach. The result may be regarded as an upper bound which may be used to determine the relative vulnerability of different satellite pairs.

The maximum value of Eq. 2 occurs when $\sigma = R_{min} / \sqrt{2}$ and is given by

$$P(col)_{max} = \frac{4}{\pi e}\left(\frac{R_s}{R_{min}}\right)^2 \qquad (3)$$

This result is similar to that obtainable as the ratio of the effective cross-sectional area πR_s^2 to the circular area πR_{min}^2. It is thus a geometric representation of the collision hazard based on the assumption of an equal likelihood that two satellites may be anywhere within a cross-sectional area of radius R_{min} at the time of the closest approach.

The upper bound probability of collision over an extended time interval (Δepoch) is the sum of the maximum collision probabilities for all encounters Nt, i.e., $\sum_{Nt} P(col)_{max}$, which is valid when it is less than unity. The corresponding mean time to collision is its reciprocal; i.e.,

$$Tc = \left(\sum_{Nt}\frac{P(col)_{max}}{\Delta epoch}\right)^{-1} \qquad (4)$$

Tc is summarized in Table 3 where the mean time to collision is seen to be of the order of a few thousand years for the pairs examined. A ranking may thus be obtained as an indication of the relative collision potential for the satellite pairs of interest.

Table 3. Geometric Collision Probability for All Encounters

Pair Number	Satellite Pair/ Owner	Longitude Deg (E)	Nt<10 nm (in 32-2 wk per)	Rs (M)	$\sum_{Nt} P(col)_{max}/\Delta epoch$	Tc (yr)
1	15237 Telstar (US) 16482 SATCOM (US)	275	267	11	5.16E-05	1614
2	19621 TDF1 (FR) 20705 TDF2 (FR)	341	177	19	4.63E-05	1799
3	13069 Westar (US) 20872 SBS6 (US)	261	160	7	4.12E-05	2023
4	4902 NATO2 5587 OPS9431 (US)	252	123	7	7.31E-06	11396
5	15826 Telstar (US) 20946 GSTR (US)	235	122	11	3.19E-05	2609
6	15235 SBS4 (US) 20873 Galaxy (US)	269	124	7	3.06E-05	2727
7	9047 COMSTR2 (US) 12309 COMSTR4 (US)	284	103	7	1.44E-05	5772
8	9852 KIKU (JPN) 10365 EKRAN (RUS)	99	104	15	5.89E-05	1415
9	20193 BSB R-1 (UK) 20762 BSB R-2 (UK)	329	81	10	4.59E-05	18165
10	18384 COS1888 (RUS) 19344 COS1961 (RUS)	346	65	10	4.22E-05	1975
ΔEpoch = 16 months						Average 4950

2.4 Stochastic Approach (Weibull Fit)

The distribution of absolute (global) minimum distances of closest approach obtained for each 2-wk time interval for all 10 satellite pairs is shown in Figure 6. A Weibull probability distribution fit to the data in Figure 6 has been approximated in the form of

$$f(x) = \left(\frac{\tau}{\beta}\right)\left(\frac{x}{\beta}\right)^{\tau-1} \exp\left[-\left(\frac{x}{\beta}\right)^{\tau}\right] \qquad (5)$$

where x is the random variable (minimum range) and τ, β are the shape and scale parameters, respectively. The curve fit for β = 3.67 nm and τ = 1.5 is illustrated in Figure 6.

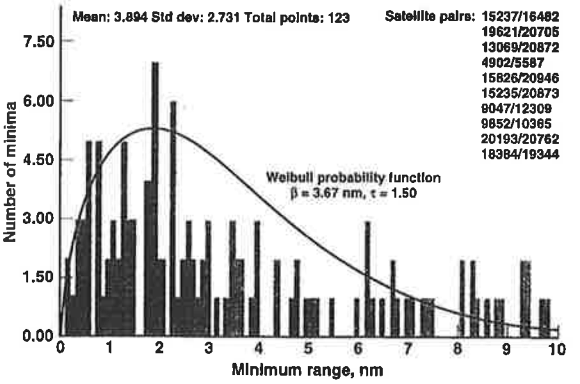

Figure 6. Global Minima for Top 10 Satellite Pairs

The Weibull mode or value of x having the largest associated probability (peak) value is

$$\begin{aligned} m &= \beta\left(1 - \frac{1}{\tau}\right)^{\frac{1}{\tau}} \\ &= 1.76\ nm \end{aligned} \qquad (6)$$

The mean or expected value is

$$\begin{aligned} \mu &= \beta\Gamma\left(1 - \frac{1}{\tau}\right) \\ &= 3.31\ nm \end{aligned} \qquad (7)$$

where Γ = gamma function.

The standard deviation is given by

$$\begin{aligned} \sigma &= \beta\left[\Gamma\left(1 + \frac{2}{\tau}\right) - \Gamma^2\left(1 + \frac{1}{\tau}\right)\right]^{\frac{1}{2}} \\ &= 2.25\ nm \end{aligned} \qquad (8)$$

Comparison of the m, μ, and σ values for the data and the Weibull probability distribution is an indication of the goodness of fit obtained.

The cumulative Weibull probability function is of the form

$$\begin{aligned} F(x) &= \int f(x)dx \\ &= 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{\tau}\right] \\ &\approx \left(\frac{x}{\beta}\right)^{\tau} \text{ for } \frac{x}{\beta} \ll 1 \end{aligned} \qquad (9)$$

F(x) is illustrated in Figure 7 for the parameters selected (solid line) and the data (points). The lower end of Figure 7 is expanded in Figure 8 where the function fit curve appears to be in good agreement with the data.

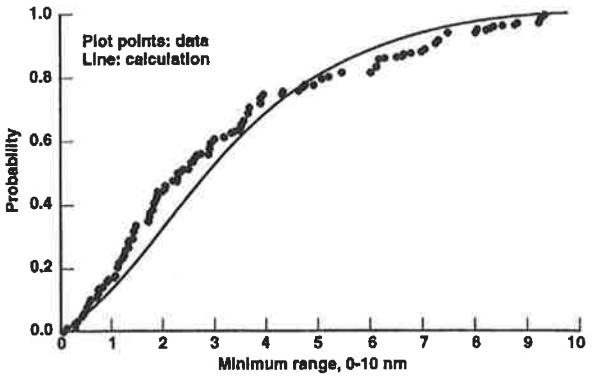

Figure 7. Weibull Probability Function F(x), (β = 3.67 nm, τ = 1.50)

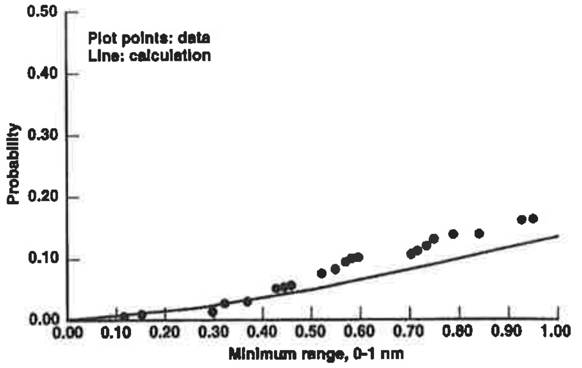

Figure 8. Weibull Probability Function F(x), (β = 3.67 nm, τ = 1.50)

The probability of experiencing a given minimum distance during an encounter can thus be obtained either from Figures 7 or 8, or can be calculated from Eq. 8. For example, if a collision radius Rs = 10 m then $F(x) = 5.62 \times 10^{-5}$ which is the probability of collision for a close approach encounter.

Assuming an equal probability of collision at each encounter, the total collision probability is proportional to the number of close encounters Nt during an extended time interval (16 mo). The results for the 10 satellite pairs are summarized in Table 4 with the expected time to collision

$$T_c = \left(\frac{NtF(x)}{\Delta epoch} \right)^{-1} \qquad (10)$$

An average value to Tc is 2728 yr which compares with 4950 yr based on the geometric approach discussed above.

Table 4. Weibull Collision Probability ($\beta = 3.67$ nm, $\tau = 1.5$)

Pair Number	Satellite Pair/ Owner	Longitude Deg (E)	Nt<10 nm (In 32-2 wk per)	Rs (M)	Pw(col)/enc $F(x) = (Rs/\beta)^{\tau}$	Pw(col)/mo NtF(x)/16	Tc (yr)
1	15237 Telstar (US) 16482 SATCOM (US)	275	20	11	6.52E-05	8.15E-05	1022
2	19621 TDF1 (FR) 20705 TDF2 (FR)	341	20	19	1.48E-04	1.85E-04	451
3	13069 Westar (US) 20872 SBS6 (US)	261	15	7	3.30E-05	3.10E-05	2691
4	4902 NATO2 5587 OPS9431 (US)	252	6	7	3.30E-05	1.24E-05	6729
5	15826 Telstar (US) 20946 GSTR (US)	235	15	11	6.52E-05	6.11E-05	1363
6	15235 SBS4 (US) 20873 Galaxy (US)	269	10	7	3.30E-05	2.06E-05	4040
7	9047 COMSTR2 (US) 12309 COMSTR4 (US)	284	9	7	3.30E-05	1.86E-05	4489
8	8852 KIKU (JPN) 10365 EKRAN (RUS)	99	6	15	1.04E-04	3.89E-05	2145
9	20193 BSB R-1 (UK) 20762 BSB R-2 (UK)	329	10	10	5.62E-05	3.52E-05	2370
10	18384 COS1888 (RUS) 19344 COS1961 (RUS)	346	12	10	5.62E-05	4.22E-05	1977
ΔEpoch = 16 months							Average 2778

3. CONCLUSIONS

A procedure was described which ranks satellite pairs in geosynchronous orbit according to the highest number of encounters over an extended time interval. The probability of collision was determined by a geometric and a stochastic approach. The geometric method assumes a Gaussian distribution for the position uncertainty of each satellite. The stochastic approach used a Weibull distribution function to fit the absolute minimum distance between satellites over a given time interval.

The results of the study show that many pairs of satellites in geosynchronous orbit remain in close proximity to each other and experience large numbers of close approaches over time. The top 10 pairs in terms of the closest encounters were identified as operational satellites which may or may not be subject to coordinated stationkeeping. The mean time to collision based on the encounter statistics examined was found to range from a few hundred to a few thousand years. These findings are significant in that they identify all satellite pairs undergoing close encounters and present a relative assessment of the collision hazard for such pairs. Moreover, the results suggest that the bunching of active or inactive satellites at certain longitudes is a significant effect which should be considered in the assessment of the collision hazard in the geosynchronous ring. Although the results obtained are approximate in view of the many simplifying assumptions made, they show that the collision hazards for colocated satellites are in general significantly higher than those for the population of objects in general.

4. REFERENCES

1. Yasaka, T. and S. Oda, "Classification of Debris Orbits with Regard to Collision Hazard in Geostationary Region," 41st Congress of IAF, Paper No. IAA-90-571, Dresden, GDR, 6–12 October 1990.
2. Perek, L., "The Scientific and Technical Aspects of the Geostationary Orbit," 38th Congress of IAF, Paper No. IAA-87-635, United Kingdom, 10-17 October 1987.
3. Bird, A. G., "Special Considerations for GEO–ESA," AIAA/NASA/DOD Orbital Debris Conference, Paper No. AIAA-90-1361, Baltimore, MD, 16–19 April 1990.
4. Flury, W., "Collision Probability and Spacecraft Disposition in Geostationary Orbit," *Advances in Space Research*, (COSPAR 1990).
5. Space Debris Study Group, Japan Society for Aeronautical and Space Sciences, Interim Report, 17 January 1992.
6. Vedder, J. D. and Talbot, J. L., "New Method for Estimating Low-Earth-Orbit Collision Probabilities," *J. Spacecraft and Rockets*, *28*, No. 2, March–April 1991, pp. 210–215.
7. Chobotov, V.A. (ed.) *Orbital Mechanics* , AIAA Education Series 1991. pp. 353–354.