RESOLUTION OF TRACK ASSOCIATION AMBIGUITIES USING EXPECTATION-MAXIMIZATION ALGORITHM

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ABSTRACT

Space object observations cannot always be unambiguously associated to catalogued objects. Multiple objects, orbiting in close proximity to each other, can pass an assignment criterion and can therefore become likely candidates for the association. If the state uncertainty of the catalogued objects is large, this ambiguity can only be resolved when new observations arrive again. In order to screen for conjunctions between space objects, the most likely associations have to be identified at all time steps. This works studies an approach where multiple objects with unresolved associations are grouped and then their orbits are determined simultaneously. The measurements are loosely associated first with a weighting factor depending on the statistical distance to the current reference trajectory. The factors are iteratively updated using an expectation-maximization approach with repeated orbit determination and association runs. The method is presented along with practicality considerations and recommendations for an operational implementation.

Keywords: tracklet association, mixed model estimation.

1. INTRODUCTION

Passive optical telescopes typically collect short arcs of observations, called tracklets, that contain a series of right ascension (α) and declination (δ) values. The observations from different images are associated to each other with high confidence assuming e.g. a linear movement between frames [5]. However, associating tracklets to each other or to catalogued objects cannot always be achieved without ambiguities [12], i.e. a new tracklet fits well with multiple other tracklets or with multiple objects in the catalogue. The issue is common for observations of co-located satellites in geostationary orbits.

A considerable amount of research has been published on the association of optical measurements to catalog objects, e.g. [6, 10, 7, 8]. They mostly deal with the proper representation of state and measurement uncertainties to increase the true positive rates, while allowing only few false associations. Notably, references [6] and [8] propose using different coordinate systems for the association. Reference [10] represent the uncertainty with a Gaussian mixture to achieve better performance.

All approaches aim at reducing the number of false positives, but given the close proximity operation of some spacecraft, they will nevertheless find multiple matching object candidates for a new measurement. A hard selection of the best fitting candidate could then lead to wrong associations and create catalog states unusable for collision avoidance services. A soft or loose association, e.g. for all objects where an association criterion falls below a threshold, can be resolved later on when more observations are available. The ESA correlation software as described by [14] uses a statistical distance measure between actual measured tracklet and one modeled from the catalog object state for the correlation. Alternative distance metrics are provided in [7, 10, 11]. All objects that create modeled measurements close enough to the actually measured one are then loosely linked to the tracklet. Once all possible object candidates have been identified and more than one observation is available, the ambiguities can be resolved for groups of objects.

Multiple-hypothesis tracking (MHT) frameworks are used to combine several observations to one track in the presence of multiple objects and possibly also clutter. Reference [2] provides a summary over current approaches and references for different implementations. The main drawback of a MHT approach is that the full enumeration of combinations of different observations can lead to an explosion in the number of hypotheses that have to be maintained. This can partly be overcome by pruning (deleting) unlikely hypotheses and merging different object state hypotheses.

Probability hypothesis density filters, e.g. as described by [3] for cataloging applications, represent all hypotheses in one multi-object probability density function. In reference [3] it is proposed to use a mixture of Gaussians for that purpose. However, this leads to the same difficulty of creating a large number of mixture components that need to be updated when new measurements arrive. The same pruning and merging clean-up steps need to implemented to overcome the problematic growth and keep the problem computationally feasible.

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Reference [13] proposes the so-called probabilistic multi-hypothesis tracking (PMHT), which treats the association decisions not as an integer-programming problem (i.e. each tracklet can only originate from only one object), but instead defines continuous assignment weights. These weights are then estimated jointly with the states using an expectation-maximization approach [4]. The algorithm uses the fact that the trajectory estimation is a straight-forward task if the assignment vector is known (they propose a Kalman filter and smoothing process). Equivalently, the most likely assignment vector can be estimated if the trajectories of the objects are available. The expectation-maximization approach iteratively improves the association probabilities and then the object states.

Based on the work in [13], reference [9] proposes a similar approach using a batch least-squares adjustment for the state and association weight estimation. Batch non-linear least-squares is commonly used for spacecraft orbit determination [1]. This is the reason why this work takes up the latter formulation to solve the tracklet assignment problem for closely-spaced objects.

The paper is structured as follows: first the combined association probability and trajectory estimation process adapted from [9] is presented, then it is applied for a test case of two satellites flying in close formation with each other, and lastly, possible challenges and practical considerations are outlined.

2. MIXED-MODEL LEAST SQUARES ESTIMATION

It is assumed that the individual angular observations of a tracklet are associated to each other with high confidence. The angles are therefore combined into one observation vector

$$\mathbf{z} = (\alpha_1, \delta_1, \alpha_2, \delta_2, \dots)^\top \ . \tag{1}$$

As the sky is regularly scanned by a limited number of telescopes and limiting weather conditions, each object can only be tracked for a few minutes. The information of such short tracklets can be compressed by fitting a simple model (e.g. linear) to the observations and using the fitting parameters as the observation. The angles and angular rates vector $\bar{\mathbf{z}} = (\alpha, \delta, \dot{\alpha}, \dot{\delta})$ at a mean epoch, also called attributable vector, is often used in association (cf. [11]) as it smoothes out noise of the individual observations. It will be later used for the computation of the association weights instead of directly using the individual observations.

For the batch estimation problem, all tracklets are summarized in one vector

$$\mathbf{Z} = \left(\mathbf{z}_1^\top, \mathbf{z}_2^\top, \dots, \mathbf{z}_n^\top\right)^\top \ , \tag{2}$$

where n denotes the number of tracklets. These tracklets have been loosely associated to m objects using one of the association metrics discussed in the introduction.

Each object trajectory is described with a state vector $\mathbf{y}$, e.g. using orbital elements or position and velocity, at a certain epoch (not necessarily at the same). The state vectors for all m targets are summarized in the vector

$$\mathbf{Y} = \left(\mathbf{y}_1^\top, \mathbf{y}_2^\top, \dots, \mathbf{y}_m^\top\right)^\top \ . \tag{3}$$

The assignment vector

$$\mathbf{A} = (k_1, k_2, \dots, k_n)^\top \quad \text{where} \quad k_j \in (1, \dots, m) \tag{4}$$

links every tracklet to one object, with k_j corresponding to the target numbers as used in (3).

Given all measurements $\mathbf{Z}$ and a measurement function $\boldsymbol{h}$ which maps the object states $\mathbf{Y}$ to the measurement space considering the assignments in $\mathbf{A}$, the least squares problem can be described with

$$\mathbf{E} = \mathbf{Z} - \boldsymbol{h}(\mathbf{Y}, \mathbf{A}) \ , \tag{5}$$

where $\mathbf{E}$ is a vector of residuals. Assuming the errors between modeled tracklets and observed ones ($\mathbf{Z}$) are zero-mean and the uncertainty is described with a covariance $\mathbf{C}_\mathbf{E}$, the weighted least-squares solution can be computed with

$$\left[\hat{\mathbf{Y}}, \hat{\mathbf{A}}\right] = \underset{\mathbf{Y},\mathbf{A}}{\arg\min}\, \mathbf{E}^\top \mathbf{C}_\mathbf{E}^{-1} \mathbf{E} \, . \tag{6}$$

This problem is a high dimensional (m state vectors each at least 6 dimensional and n assignments) mixed-integer nonlinear optimization problem and difficult to solve directly. The assignment vector $\mathbf{A}$ contains integer values and the number of measurements n can become very large.

The residuals are rewritten as a finite mixture

$$\mathbf{E} = \sum_{i=0}^{m} \mathbf{w}_i (\mathbf{Z} - \boldsymbol{h}_i(\mathbf{y}_i)) \tag{7}$$

where the association weights $\mathbf{w}$ are introduced. The components of the weights vectors are either 0 if the tracklet is not assigned to the object, or 1 if it is. As proposed by [13, 9], the hard assignment is replaced with a soft assignment using the continuous range between between 0 and 1. In order to count each measurement once, the following constraint must be fulfilled for each weight vector

$$\sum_{i=0}^{m} \mathbf{w}_i = \mathbf{1} \, . \tag{8}$$

The loss function in equation (6) can be rewritten with

$$\sum_{j=0}^{m} \mathbf{w}_i^2 (\mathbf{Z} - \boldsymbol{h}_i(\mathbf{y}_i))^\top \mathbf{C}_\mathbf{E}^{-1} (\mathbf{Z} - \boldsymbol{h}_i(\mathbf{y}_i)) \, . \tag{9}$$

If the weights are known, the non-linear least squares problem can be solved with a method such as Gauss-Newton or Levenberg-Marquardt. In fact, each component of the sum can then be individually minimized, leading to m separate orbit determination runs for each object.

Likewise, when the state estimates are known, the weights are computed that minimize the loss function considering the constraint in equation (8) (cf. [9]):

$$\mathbf{w}_i = \frac{[(\mathbf{Z} - \boldsymbol{h}_i(\mathbf{y}_i))^\top \mathbf{C}_\mathbf{E}^{-1}(\mathbf{Z} - \boldsymbol{h}_i(\mathbf{y}_i))]^{-1}}{\sum_{j=0}^{m}[(\mathbf{Z} - \boldsymbol{h}_j(\mathbf{y}_j))^\top \mathbf{C}_\mathbf{E}^{-1}(\mathbf{Z} - \boldsymbol{h}_j(\mathbf{y}_j))]^{-1}} \,. \tag{10}$$

Iterative optimization process The iterative optimization process is described in figure 1. First, the spacecraft states are initialized using prior knowledge, e.g. from previous orbit determination runs or from external sources such as publicly provided two-line elements (TLEs) obtained from `www.space-track.org`. If the prior knowledge comes with an uncertainty estimate, it can be incorporated into the loss function [9]. For the sake of notational simplicity, this work does not consider the prior state uncertainty, but just initialize the target trajectories.

Based on the initial trajectories of the objects, the target association weights are computed using equation (10). However, it turned out for the test case (in next section) that the convergence performance could be improved when using the mean angles and angular rates $\bar{\mathbf{z}}$ instead of the direct observation vector $\mathbf{z}$. Of course, this requires a different measurement function $\bar{\boldsymbol{h}}$ and measurement covariance $\bar{\mathbf{C}}_\mathbf{E}$. The attributable $\bar{\mathbf{z}}$ is also commonly considered beneficial for classical association tasks, e.g. for the attribution penalty in [11].

Once the vectors $\mathbf{w}_i$ are computed, the weighted nonlinear least squares adjustment can be started using the association weights in addition to the already existing weighting computed from $\mathbf{C}_\mathbf{E}$. The result of each adjustment is the updated state vector of the corresponding object. The process of computing weights and updating the object states is repeated until convergence in the loss function (9) is detected.

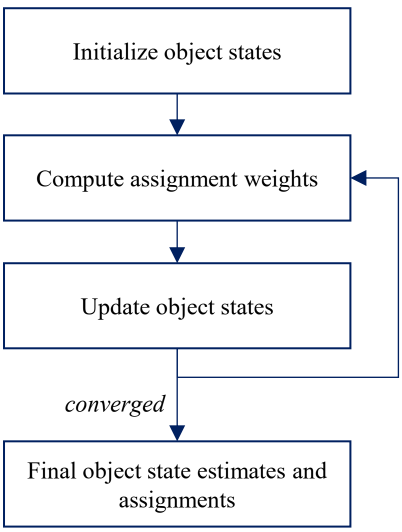

Figure 1. Illustration of the iterative mixed-model least-squares estimation process

3. TEST CASE

As a test case, two satellites from the Eutelsat Hotbird 13 constellation are observed (COSPAR ids: 2006-032A and 2008-065A) from the Zimmerwald observatory in Switzerland. The available measurement set is from the first week of January 2015 and contains around 17 measurements for 2006-032A and 11 for 2008-065A. The data has been loosely associated to the objects using TLEs and the approach described in [6]. The selected cluster is the same as the test case presented in [15], however, using different observations.

The iterative process is started using a TLE with an epoch close to the latest tracklet. The state vector at estimation epoch is obtained by using SGP4. From thereon a high-fidelity force-model is used for the state propagation, considering a 32x32 gravity field model, third body perturbations (Sun and Moon), and solar-radiation pressure. The initial discrepancy between trajectory and observations is quite large as can be seen in figure 2. The figure shows the hour angle (i.e. local sidereal time - right ascension) and the declination of the modeled objects and the measurements (×).

The updates of the association weights for each iteration is illustrated in figure 3. Most weights quickly converge towards 1 or 0, however, a few converge to a constant level of around 0.2 or 0.8 respectively. One tracklet has even a weight value larger than 0.3/less than 0.7. This tracklet could be either an outlier or generated by a different and not considered object. Outliers can corrupt the solution for the individual orbit determination runs and should be avoided. This can be efficiently tackled in the PMHT formulation, when accounting for an additional clutter model as e.g. presented in [16].

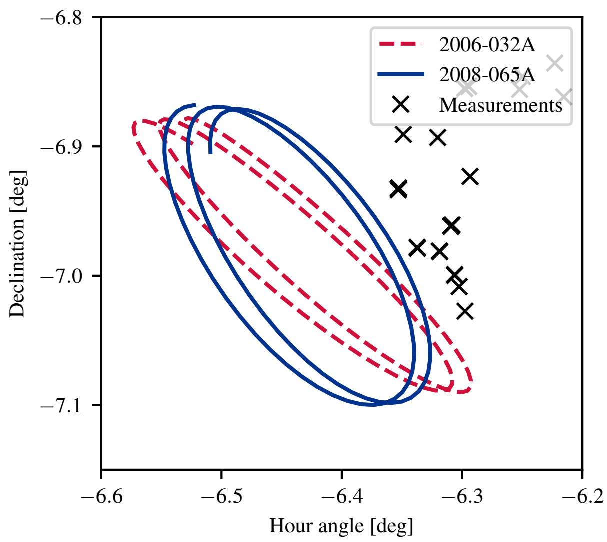

Figure 2. Initial guess of the spacecraft trajectories together with the measurements.

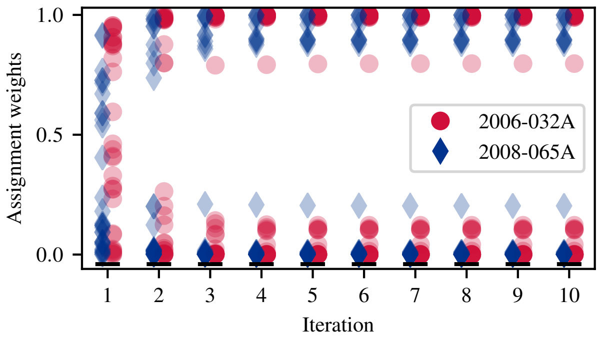

Figure 3. Evolution of association weights during optimization.

The evolution of the loss function value is shown in figure 4. It clearly shows that the process converges to a solution. It should be noted that this solution could be also a local-minimum as global convergence in not guaranteed by the presented approach. Reference [16] summarizes an approach, namely covariance inflation and deflation, to increase the chances of finding the global minimum.

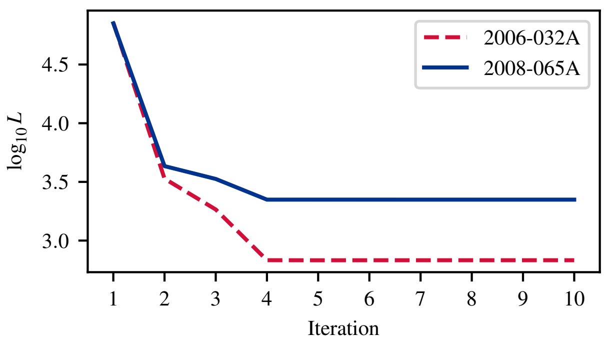

Figure 4. Evolution of loss function during optimization.

The estimated trajectories of the two spacecraft after convergence is reached are shown in figure 5. The observations match well with the trajectories, except for one mentioned outlier.

4. CONCLUSIONS

The paper presents the current status of our efforts to apply the methods and findings of probabilistic multi-hypothesis tracking research to orbit determination problems for closely spaced spacecraft formations. The method is shown to work in principle, but it requires further testing and implementation efforts to use it in an automatic way on larger data samples.

Practically, it is unfeasible to process all objects in space simultaneously with one loss function. Therefore, satellites need to be grouped or clustered into subproblems. This grouping can be effectively achieved by combining objects with an overlap in associated observations. If these groups become too large, they can be furthermore reduced by limiting the group size and use a clutter model to account for measurements that have been wrongly associated to the group.

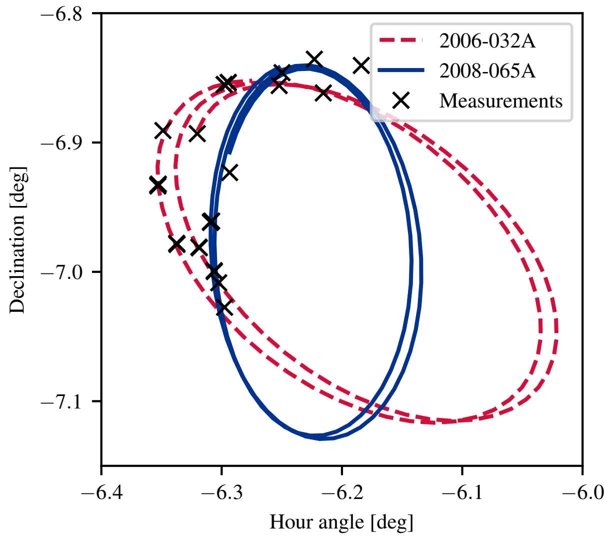

Figure 5. Final estimate of the spacecraft trajectories together with the measurements.

One of the main drawbacks of the approach is its sensitivity to an unfavorable initial guess for the object orbits, which then leads to a local minimum being found instead of the global one. The presented test case shows an extreme situation of using poor initial orbits, but nevertheless finds the proper associations. Further research is required to characterize the radius of convergence. It is expected that the previous orbit determination result stored in the catalog should be a sufficiently good starter, but this needs to be confirmed using a moving window processing of larger data sets.

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