RE-ENTRY PREDICTIONS FROM TLE TIME SERIES, WITH THE STELA S/W

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ABSTRACT

The long term evolution of orbital parameters of a trajectory is efficiently driven following semi-analytical approaches. The STELA s/w is designed to check the compliance of choices of storage orbits with the IADC guidelines. By propagating the mean variational equations jointly to the equations of motion, it is even convenient to adjust the averaged model to TLE times series.

We focus in this paper on the ballistic coefficient and the area-to-mass ratio determination that have to be evaluated jointly to the initial (mean) state vector of the satellite. For most of space debris, their a priori-value deduced from individual TLEs is frequently far from the one leading to the best compatibility between theory and observations. We show how to calibrate these coefficients as well, and we give an example taken from the last days spent out in space by the chinese space station Tiangong-1 that reentered the atmosphere early April 2018.

Keywords: orbit propagation; atmospheric reentry; space debris.

1. INTRODUCTION

The space debris population has steadily increased since the early 1960s, and spacefaring nations and organizations have recognized the mounting risk to space operations posed by orbital debris. Studying the evolution of the space debris population is a major issue, over short as well as over long time scales, in view of the safety of space operations, and to prevent a too strong rise of the collision probabilities between artificial satellites, operational or space debris. For many purpose, over long time scales it is very convenient to use semi-analytical approaches, such as the one developed in the STELA/SATlight french s/w in the framework of the French Space Operation Act, as part of the recommendations provided by the IADC. A time scale can be considered “long enough” in the case of an atmospheric reentry, and those s/w can be used over years of propagation, or only over the last few days before such a reentry. This paper shows an application of the use of the CNES/IMCCE s/w to predict the time of reentry of objects belonging to the TLE data sets, with an application on the reentry of the chinese space station Tiangong-1. In this last case, the space weather was quiet and stable enough at the beginning of Spring 2018 to enable an analysis of the error budget of the method mainly involving the well-known accuracy of the TLEs.

2. ORBIT PROPAGATION MODEL: EQUATIONS OF MOTION AND VARIATIONAL EQUATIONS

Over long time scales, the question is to perform an orbit modeling accurate enough to provide a good estimate of a satellite lifetime. By using the STELA software developed by CNES and its fortran prototype jointly developed by CNES and IMCCE [? ], the propagation model is based on averaged equations of motion that are valid in all dynamical configurations, in particular for all values of the eccentricity (small or large), or the inclination (small or large). Such an averaging approach allows to use a large integration step size (about 2 days, typically), reducing significantly the total time of computation w.r.t. the classical numerical integration, w/o inducing significant errors.

The averaging approach follows methods developed in the theory of mean orbital motion in [? ]. All the equations have been formulated through a set of orbital elements that is suitable to describe orbits with high eccentricities and any inclination (except 180 degrees) : (a, Ω + ω + M, e cos(ω + Ω), e sin(ω + Ω), sin 2 cos Ω, sin 2 cos Ω), where (a, e, i, Ω, ω, M) stand for the traditional Keplerian elements. They form a mean state vector $\vec{E}(t)$ governed
by the fundamental principle of dynamics:

\[
\frac{d\vec{E}}{dt} = \mathcal{F}(\vec{E}, \sigma_{\text{dyn}}) \\
\vec{E}(t) = \vec{E}(t_0)
\]

(1)

(2)

where \( \mathcal{F} \) stands for the dynamical model, and \( \sigma_{\text{dyn}} \) the dynamical parameters characterizing it, and with \( \vec{E}(t_0) \) the initial mean state vector. They are propagated jointly with the variational equations that provide the sensitivity of the orbit to a \( \sigma \) parameter:

\[
\frac{d}{dt} \left( \frac{\partial \vec{E}}{\partial \sigma} \right) = \frac{\partial}{\partial \sigma} (\mathcal{F}(\vec{E}, \sigma_{\text{dyn}})) + \frac{\partial}{\partial \vec{E}} (\mathcal{F}(\vec{E}, \sigma_{\text{dyn}})) \frac{\partial \vec{E}}{\partial \sigma}
\]

(3)

Corresponding Planetary Lagrange and Gauss equations have been written for conservative and non-conservative perturbations respectively. Concerning conservative perturbations, the mean averaged potential is computed analytically from the expression of the osculating potential. The effects of non conservative perturbations on mean orbital parameters are computed through a Simpson quadrature method. For the atmospheric drag, quadrature are equally spread in true anomaly on the orbit part where altitude is below a given threshold (so that the average is more accurate). For solar radiation pressure, Earth shadow entry and exit points are computed analytically following the assumption that Earth’s shadow is a circular cylinder. Note that all equations (except those for tesseral terms of the Earth gravity field) are written without any development in power of eccentricity.

To put it in a nutshell, the dynamical model is a simplified one since the idea was to consider only the perturbations that have a significant effect on the orbit evolution, with a minimum model to be considered:

- zonal terms: \( J_2, J_2^2, J_3, J_4, J_5, J_6, J_7 \)
- tesseral terms in case of a commensurability between the satellite period of revolution and the sidereal revolution of the Earth
- solar and lunar gravity developed up to degree 3
- atmospheric drag
- solar radiation pressure, including the Earth shadow.

The osculating state vector \( \vec{E}(t) \) can even be deduced from the mean state vector by adding the short periodic terms, of gravitational or non gravitational origin (\( \mathcal{L}(\vec{E}(t)) \) and \( \eta(\vec{E}(t)) \) respectively):

\[
\vec{E}(t) = \vec{E}(t) + \mathcal{L}(\vec{E}(t)) \frac{\partial W}{\partial \vec{E}}(\vec{E}(t)) + \eta(\vec{E}(t))
\]

(4)

By using such a simplified but convenient propagation model, a time series of mean orbital elements can be built accounting for all the significant perturbations. Figure 1 shows the decay of the semi-major axis of a Tiangong1-like orbit, with DTM2013 as the reference atmospheric drag model. Here, no parameter is adjusted to the tracking data, and this is clear that the atmospheric drag is underestimated.

3. SPACE DEBRIS ENVIRONMENT ANALYSIS

The main source of data about the resident space objects come from the USSTRATCOM with the TLE catalog available online on the gate way www.space-track.org. It gives the position of satellites, rocket-bodies or space debris updated for a part of them several times per a day. To deal with this important amount of data we created a database named ODIN handled by a Python environment and the package CelestialPy which was first developed at the University of Namur (Belgium) and now at the IMCCE (Observatory of Paris, France). The database ODIN contains TLE data, space weather data, and informations about historical fragmentations and launches. The Python package CelestialPy update this database, and use tools like orbit propagators, source model to provide results about space object populations. One of the main results produced are the calibration of the balistic coefficients and the determination of the atmospheric reentry dates.

4. ATMOSPHERIC REENTRY

The reentry prediction is a challenge because it needs tools able to perform orbit propagation with a strong atmospheric drag, and an accurate determination of the ballistic coefficient. In this Section we describe the algorithm followed.

4.1. Algorithm

The algorithm is based on an orbit determination by differential correction. In order to estimate the reentry date we have first to compute the ballistic coefficient which in not known for the majority of case. Moreover, this value can change with time and need to be adjusted over the last TLE data. Once the ballistic coefficient known by
fitting process, we propagate the orbit until the reentry to determine the date. The window uncertainty is computed performing Monte-Carlo runs taking into account an uncertainties of several hundred meters over the position.

### 4.2. Retroprediction for the case of Tiangong-1

Tiangong 1 was a Chinese space station, launched on September 29, 2011. It was a test module for the future Chinese space station scheduled in 2020. Tiangong 1 received a crew in 2012 and 2013. Since 2013, the orbit of Tiangong 1 is regularly enhanced and the last maneuver occurred on November 2015. Finally, on April 2, 2018, Tiangong 1 started a non-controlled reentry in the dense layers of the atmosphere, burnt, and fell in the Pacific ocean at the position given by the coordinates (13.6°S 195.7°E) at 0:15 AM UTC. The case of the reentry is particularly interesting because the Chinese authorities confirmed they lost the control in 2016, and thus, the reentry prediction was a great challenge.

In Figure 2 we provide the evolution of the reentry windows in function of the date of prediction. We plot also the evolution of the mean ballistic coefficient computed at each date by a fitting process over the last 30 days of TLE data.

Figure 3 and the corresponding Table 1 show the best results obtained by adjusting a STELA trajectory to the available TLE up to a month prior to the reentry.

### 4.3. Prediction

Taking into account the last updates of the TLE data we provide a service of reentry date prediction automated. In Figure 4, we plot the reentry windows computed at the date of November 30th, 2018, for the period of the year 2019.

### 5. CONCLUSION

It appears, from the Tiangong-1 atmospheric reentry, that it is worth benefiting from a while time series of TLE to accurately provide an accurate estimation of the time of reentry: single TLEs do not have such a possibility, and with stable space weather conditions not inducing strong changes in the atmospheric density because of changes of the solar activity, we feel that the best approach consists in adjusting an averaged trajectory to tracking data. It is also required to adjust, at least, the initial state vector as well as the ballistic coefficient.

### REFERENCES

Table 1. Differences on the semi-major axis between the TLE time series data set, and the best fit computed with STELA/SATlight. The column $\Delta a$ gives an estimate of the altitude decay (that is growing with time, as expected), from the propagated orbit. The column "$\Delta a$ wrt best fit" provides, for each observation epoch, the difference between the semi-major axis deduced from the TLE, and the propagated one: the maximum value is of the order of a few kilometers, and the minimum is not reached at the initial epoch. The "mean" of the differences is here a time dependant quantity, and is obtained by accounting for a number of observation epochs that does correspond to the number of the lines in the table: the first value is the same as the column $\Delta a$ wrt best fit, the second one is the average between 75 and 315, and teh final value of 1550 is obtained by averaging 21 values : this column makes it possible to identify two time slots in the ad-justement: positive values correspond to a semi-major axis that is higher in the observations than in the propagation (first period), whereas negative values do correspond to the opposite (second period). The rms (that is here time-dependent as well) is obtained following classical formulas, and that enables to roughly quantify the accuracy of the TLE data set.

<table>
<thead>
<tr>
<th>TLE Epoch (UTC)</th>
<th>a (TLE) (km)</th>
<th>$\Delta a$ (km/d)</th>
<th>$\Delta a$ wrt best fit (m)</th>
<th>Mean (to date)</th>
<th>rms (to date)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 3 2018 7h 35min 7.9996sec</td>
<td>6604.41502</td>
<td>2.180</td>
<td>75</td>
<td>75</td>
<td>0</td>
</tr>
<tr>
<td>22 3 2018 6h 28min 23.8927sec</td>
<td>6602.26125</td>
<td>2.210</td>
<td>315</td>
<td>195</td>
<td>120</td>
</tr>
<tr>
<td>23 3 2018 5h 58min 13.0103sec</td>
<td>6600.12605</td>
<td>2.350</td>
<td>661</td>
<td>350</td>
<td>240</td>
</tr>
<tr>
<td>24 3 2018 5h 36min 15.3007sec</td>
<td>6597.28956</td>
<td>2.570</td>
<td>706</td>
<td>439</td>
<td>259</td>
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<tr>
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<td>408</td>
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<td>232</td>
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<tr>
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<td>6.400</td>
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</tr>
<tr>
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</tr>
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<td>1550</td>
<td>-509</td>
<td>1040</td>
</tr>
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Figure 4. Reentry prediction for the year 2019.