EMPLOYING FAST ORBIT PREDICTION FOR OPTIMISATION OF SATELLITE VISIBILITY COMPUTATION

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ABSTRACT

The development of space technology and the growth of the number of missions since the beginning of the Space race has caused a serious problem. The uncontrolled spread of space debris that jeopardises active satellite networks has posed several challenges to routine space operations. In this sense, building/populating and updating a catalogue of man-made space objects is a mission of vital importance that requires great accuracy and computational efficiency.

In this context, it was more than fifty years ago that the development of algorithms to determine the visibility of artificial satellites was extended allowing progressively improve the accuracy of their results and the computation time required.

The topic of this task is to analyse the visibility problem and to test the efficiency of different types of existing approaches by using a visibility tool developed in python. Starting from the basic concepts, the resolve originates with the simplest study "geometrical propagation", where the movement of the satellite is governed solely by the Keplerian motion. From this point, it will be possible to increase the difficulty of the problem and continue this work in the future by understanding how perturbations affect the satellite visibility.

1 INTRODUCTION

The typical satellite visibility problem consists of determining the rise and set times, and from these values the viewing periods of an Earth-orbiting object, referred to an Earth-fixed ground station tracking sensor, is computed.

Traditionally, the resolution of this problem has been done using the numerical approach, also conventionally called brute force method. This procedure starts with the initial values of the satellite ephemeris according to which its trajectory in the orbit is evaluated at certain instance and check if the longitude and latitude falls under the station visible cone. Although this method is the most accurate in its results, its main drawback is that it requires a great computational effort. The computational cost arises from evaluating thousands of intermediate orbital positions and the subsequent transformations of the elevation, azimuth and distance values with respect to the fixed position of the ground station to determine if the satellite is visible or not. This clout increases significantly when working with several satellites simultaneously. With the aim of overcoming this disadvantage, especially undesirable for onboard real-time mission planning, many researchers have developed rapid algorithms for determining the visibility periods sacrificing accuracy by virtue of computational speed.

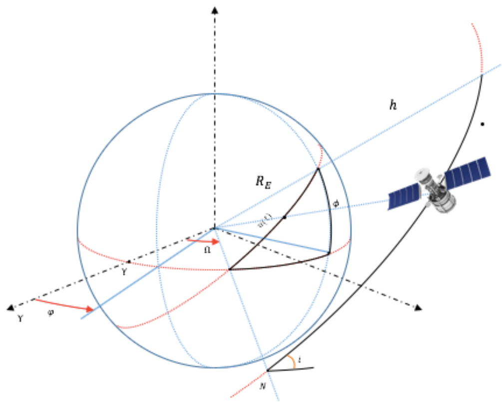

Figure 1. Satellite trajectory in an ECI system.

In this way, the algorithms applied to this task are divided into three categories: numerical, already mentioned; the analytical ones, faster than the previous but with the detriment of results accuracy and the semi-analytical ones that combine the advantages of the previous ones.

Escobal was one of the first authors to develop a successful analytical method when he formulated a closed form solution for unrestricted visibility periods of satellites orbiting an oblate Earth (Escobal, 1963) [1]. The proposed approach consisted in transforming the geometry of the satellite propagation into a single transcendental equation as a function of eccentric anomaly for each moment of time.

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Other authors more recently have studied this problem by applying different approaches that are also successful. For example, Lawton solved the problem by using an unrestricted function by means of Fourier Series (Lawton, 1987) [2]. While this method is valid only for circular or nearly circular orbits and not being effective for elliptical orbits, because the function developed is not periodic. Also, Mai and Palmer presented a coarse-to-fine refinement strategy examining the closest satellite ascending pass over the ground station latitude (Mai and Palmer, 2001) [3].

Alfano presented a numerical method that employ a fast determination of the rise and set time using the line of sight corrected for an oblate Earth, technique called parabolic blending (Alfano, 1992) [4]. Subsequently, Ali exposed a solution based on approximating the ground trace of the satellite during an interval of the order of in-view period, by a great circle arc. And recently, Han developed another new numerical method that significantly reduces computation time using self-adaptive Hermite interpolation algorithm (Han, 2016) [5].

These methods show different approaches to the problem of determining the visibility periods for obtaining different grades of precision in the results and multiple execution times depending on the assumed hypotheses.

2 THEORETICAL CONCEPTS INVOLVED IN THE RESOLUTION OF THE TASK

Ground station to satellite visibility periods are typically determined by evaluating Earth centered inertial position vectors of the site and the target object.

2.1 Two-body Problem

As in other astrodynamics problem, the two-body problem serves as the starting point for more complex studies. In this initial approach Newton's law of gravitation provides the means to find the components of the force if only gravity affects the body, i.e. no perturbations are considered. According to these premises the two-body equation can be formulated as follow,

$$\ddot{\vec{r}} = -\frac{\mu}{r^2}\hat{r} \tag{1}$$

2.2 Kepler's Equation and Problem

Kepler´s equation allows determining the mathematical relationship between the time and angular displacement throughout the orbit. Specifically, the time of flight t, or the mean anomaly M, from the perigee to certain value of eccentric anomaly.

$$M = E - e\sin E = n(t - t_0) \tag{2}$$

Which resolution requires the application of some numerical analysis method to solve the transcendental equation in terms of the eccentric anomaly.

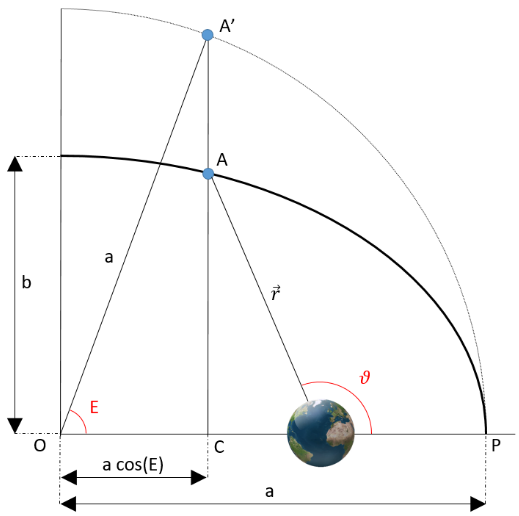

Figure 2. The eccentric anomaly of point A.

Two types of problems arise from Kepler's Equation:

- Measure the time to travel between two points of the orbit.
- Find a future location given the last known state vector at certain epoch and the time increment.

This second task is called propagation, which is a fundamental key piece in the resolution of satellite visibility periods

2.3 Satellite State Representations

To define the state of the satellite in space it is necessary to specify six quantities that are collected in many equivalent forms. There are two typical ways of expressing these initial parameters:

- The state vector, associated with the position and velocity of the satellite at a given initial epoch.
- An element set which defines the shape and orientation of the orbit at a given time that are called orbital elements.

The Keplerian orbit is normally specified by a set of six orbital elements

$$[a, e, \Omega, i, \omega, \nu] \tag{3}$$

Where the semi-major axis $[a]$ and the eccentricity $[e]$, describe the orbit size and shape. The right ascension of the ascending node $[\Omega]$, the inclination of the orbit $[i]$ and the argument of perigee $[\omega]$, define the orbit plane orientation. And finally, the true anomaly [ν] (or other expression of the anomaly), determines the satellite's current angular position relative to the perigee.

Currently, the data format of the most widespread satellite state representation is the Two-Line Elements (TLE) set. TLE data is used as input files for propagate the trajectory of satellites and space debris orbiting the Earth. Also, it is used as source for Simplified General Perturbations (SGP) and Simplified Deep Space Perturbations (SDP) models.

2.4 Coordinate Reference and Time Systems

One of the first requirements for describing an orbit through certain satellite state representation is to define a consistent and relevant reference system.

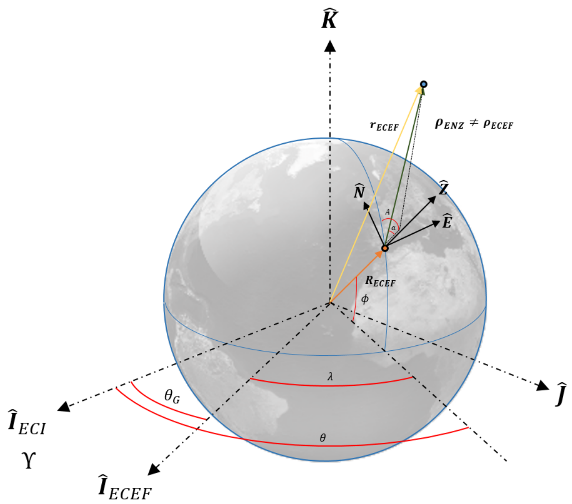

Figure 3. Relationship between Geocentric and Topocentric systems.

In practice, the choice of the right coordinate frame can substantially reduce the complexity of the given problem.

The main difficulty affecting the visibility problem is the fact that the ground station and the propagated satellite states are referred in different coordinate systems.

2.4.1 Earth-based systems

The necessary knowledge in geodesy about shape, size and gravity field, allows to specify the correct location of ground stations through the appropriate coordinates. In this context, a basic problem for the treatment of the position of the station as input of the algorithms is the conversion between geocentric and geodetic coordinates.

In the algorithms for determining visibility, it is common to convert between geocentric parameters, referred to Earth-Centered Inertial System (ECI), Earth-Centered Earth Fixed (ECEF) and Topocentric Horizon Coordinate System (SEZ), which finally allows to express the elevation angle results for a given satellite, ground station and epoch.

The precision in the time conversions is of vital importance for the application of this type of algorithms due to the high speed of the satellites, where an error of one second assumes a position difference of the order of kilometres.

2.5 Geometrical Problem

Generally, this problem involves relating coordinates of the satellite in space with its projection point on the surface or subsatellite point and the fixed coordinates of a ground station.

In this way, it is possible to define the angular relationship between the Earth-orbiting object, the ground station and the Earth Center. In Fig. 4, this relationship can be observed in the case of the approximation of a spherical Earth.

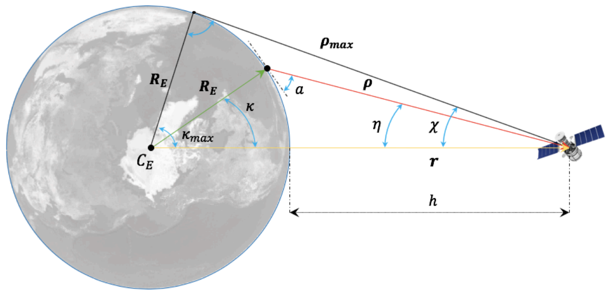

Figure 4. Geometrical problem.

3 SATELLITE VISIBILITY PERIOD COMPUTATION TOOL

The objective of this tool is to compute the period of visibility of satellites and compare the effectiveness, accuracy of results and computational time, applying each one of the three methods presented: analytical, numerical and semi-analytical.

The visibility tool has been developed in Python using some procedures from existing algorithms from each one of the three referred methods. Fig. 5 shows an outline of the organization of the tool.

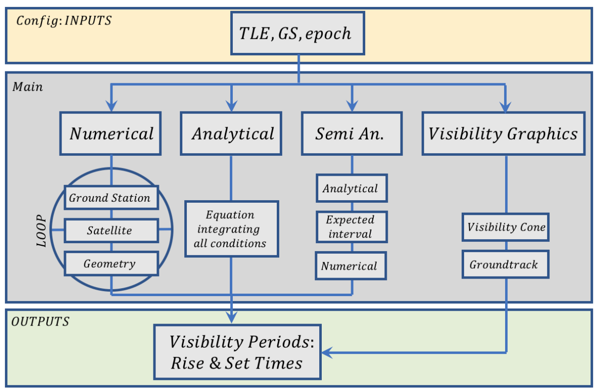

Figure 5. Scheme of the visibility tool.

It starts from a main menu in which the first step available is to adjust in the *CONFIG* file the program's according to the desired study parameters: satellite TLE data, ground station coordinates and epoch.

After setting the initial configuration, in the *MAIN* function it is possible to select each one of the sub-tools available for the computation of the visibility periods (numerical, analytical or semi-analytical) or to plot the visibility graphics. Once the run is completed, it provides the value of execution time, which makes it easier to compare the fastness of the results obtained. The steps in the implementation of each algorithm are explained in the subsections below. Meanwhile, Figs. 6, 8 and 10 show the conceptual differences between the three methods.

3.1 Numerical

Among other authors it has been used as reference some procedures developed by D. Vallado (Vallado, 2013) [6]:

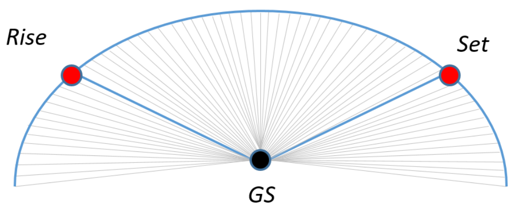

Figure 6. Scheme of a numerical method.

- **Ground Station**

  $\left(\phi_{gd}, \lambda, h_{ellp} \rightarrow \vec{r}_{GS,ECEF}\right)$

  The first task is, starting from the geodetic coordinates, to determine the ground station vector in ECEF system using the auxiliary terms of oblateness of the Earth.

- **Satellite**

  $(\vec{r}_{ECI}, \vec{v}_{ECI}, \Delta t \rightarrow \vec{r}(t)_{ECI}, \vec{v}(t)_{ECI})$

  Starting from the state vector referred to Earth-Centered Inertial System and defining an arbitrary step time, the integration is made along all the points of the trajectory of the orbit. In the implementation of the algorithm a step of 1s has been taken.

  $(\vec{r}_{ECI}, \vec{v}_{ECI}, \text{epoch} \rightarrow \vec{r}_{ECEF}, \vec{v}_{ECEF})$

  The next step is to convert the state vector to Earth Centered-Earth Fixed System referred to the epoch of observation.

- **Solve the Geometrical Problem at each time**

  $\left(\vec{\rho}_{ECEF}(t) = \vec{r}_{ECEF}(t) - \vec{r}_{GS,ECEF}(t)\right)$

The next step of the algorithm is to implement the loop to calculate the position vector referred to the station for each instant of time.

$(\vec{\rho}_{SEZ}(t) = [ECEF\ to\ SEZ]\ \vec{\rho}_{ECEF}(t))$

The transformation matrix is applied to express the position vector in the topocentric reference system South-East-Zenith.

$$\left(\rho(t) = |\vec{\rho}_{SEZ}(t)| \ \ ; \ \ el(t) = \sin^{-1}\frac{\rho_z(t)}{\rho(t)}\right)$$

Finally, in topocentric coordinates it is possible to decompose the components of the vector and calculate the distance and the angle of elevation. Thus, the rise and set point with zero elevation angle can be verified. In Fig. 7, the elevation angle for the next case study is shown:

- Satellite data:
  - $a = 9701.13\ km$
  - $e = 0.0078478$
  - $i = 111.84°$
  - $\Omega = 180°$
  - $\omega = 0°$
  - $T = 0\ s$

- Ground Station data:
  - $\phi_{gd} = 36.4616$ N
  - $\lambda = 6.2055$ W
  - $h_{ellp} = 0\ m$

- Epoch: $00^h\,00^m\,00^s$ - January, 1st 1980

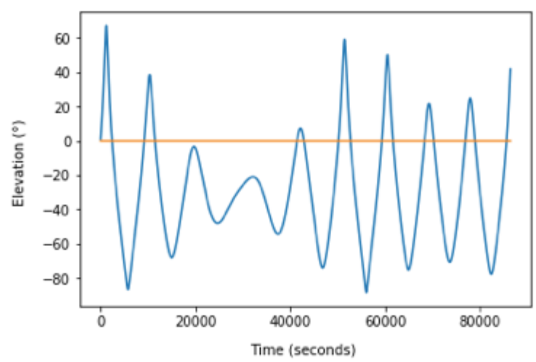

Figure 7. Elevation angle versus time.

3.2 Analytical

The algorithm presented by P. R. Escobal [1] has been taken as the main reference to represent this type of procedure.

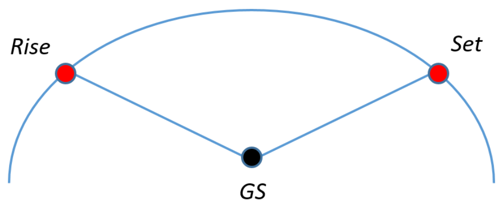

Figure 8. Scheme of an analytical method.

The fundamental part of this approach is the formulation of an equation, which is called controlling equation, and in which the eccentric anomaly is considered as an independent variable. This equation integrates the geometric conditions of propagation and allows determining the rise and set point through its roots. It also determines the visibility periods of the satellites according to condition $F > 0$.

It is necessary to solve this equation for each complete revolution of the satellite. In Fig. 9, the controlling equation for the first revolution of the same example used previously is graphically represented.

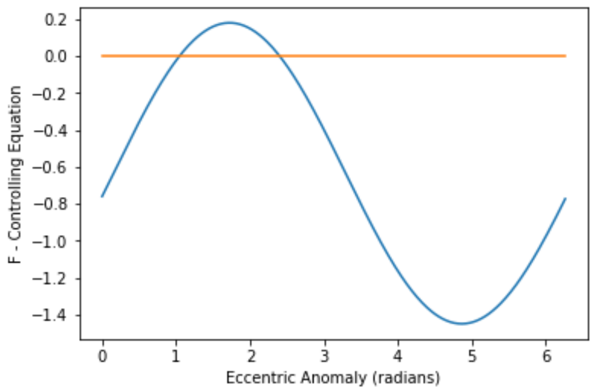

Figure 9. Graphic representation of the controlling equation versus the eccentric anomaly.

3.3 Semi-Analytical

The semi-analytical method combines the positive aspects of the numerical, its accuracy and also the analytical rapid computation time.

The procedure followed for its application is as follows:

- Apply the selected analytical method to find estimated values of Rise and Set.
- From this central value, an increment of time is defined to calculate values before and after.
- Finally, the numerical method is applied in the environment of the previous values to determine the definitive Rise and Set values.

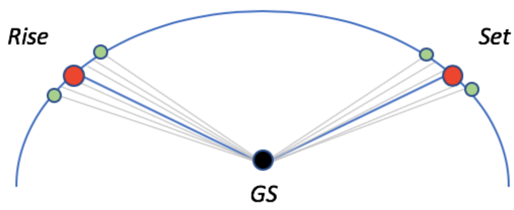

Figure 10. Scheme of a semi-analytical method.

3.4 Visibility Cone and Ground Track

In addition, a specific tool has been developed in order to represent the projection over the terrestrial surface of the visibility cone of certain ground station, Fig 11. And also, the ground track of the satellite along successive steps according to the parameters of the example of the following paragraph, Fig 12.

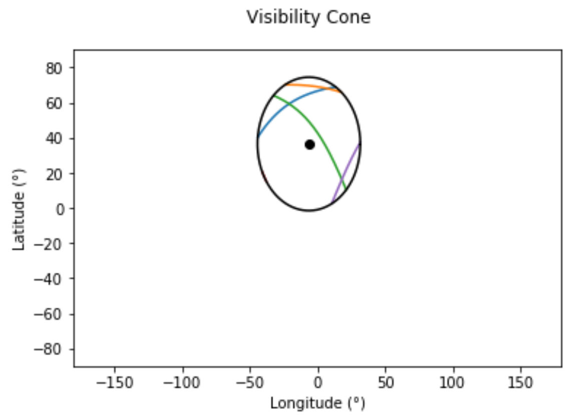

Figure 11. Projection of the visibility cone at ROA station for LAGEOS 1.

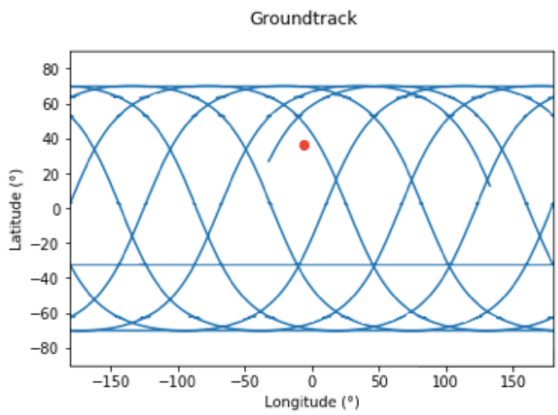

Figure 12. Ground track of LAGEOS 1.

4 RESULTS

As a use case for the tool, it has been computed the visibility periods of an object from a ground station at a given epoch.

The results obtained for the LAGEOS-1 TLE data and the position of the Laser Tracking Station of the Royal Observatory of Spanish Navy in San Fernando are gathered in Tab. 1. So that, the geometrical problem responds to the following inputs:

- Satellite data (from TLE):
  - $a = 12271.19\ km$
  - $e = 0.0044$
  - $i = 109.84°$
  - $\Omega = 137.53°$
  - $\omega = 329.02°$
  - $M_A = 30.78°$

- Ground Station data:
  - $\phi_{gd} = 36.4616$ N
  - $\lambda = 6.2055$ W
  - $h_{ellp} = 0\ m$

- Epoch: $00^h 00^m 00^s$ - October, 1st 2018

	Numerical	Analytical	Semi-Analyt.
1st Rise	$1^h20^m31^s$	$1^h17^m46^s$	$1^h20^m31^s$
1st Set	$2^h30^m55^s$	$2^h27^m25^s$	$2^h30^m55^s$
2nd Rise	$4^h37^m16^s$	$4^h36^m16^s$	$4^h37^m16^s$
2nd Set	$5^h43^m7^s$	$5^h46^m18^s$	$5^h43^m7^s$
3rd Rise	$11^h47^m46^s$	$11^h45^m54^s$	$11^h47^m46^s$
3rd Set	$12^h42^m10^s$	$12^h47^m45^s$	$12^h42^m10^s$

Table 1. Results comparison of the three methods

Computation time observed:

- Analytical (evaluated for 12 days): approx. 2 s
- Numerical (evaluated for 1 day): approx. 30 s
- Semi-An. (evaluated for 1 day): approx. 4 s

5 FUTURES IMPROVEMENT IN DEVELOPMENT

In addition, while is taking place the submission of this paper some improvements continue with the work started. Among others are to develop polar plots of azimuth/elevation that allow the user to make a presentation of the situation more clearly and develop a code to allow the ingestion of a TLE catalogue to compute the visibility periods for all objects from a ground station. Multiple possibilities of progress would be enabled from this last one, such as the elaboration of satellite density maps.

All this set of improvements could define a complete visibility solution for a specific ground station.

6 USE CASES OF THE ALGORITHMS

The application of this kind of algorithm opens up to prefiltering objects for correlation in observation processing within a cataloguing system, also the implementation and improvement of them would enable the standardization of formats and the optimization of results with high accuracy and little processing time. Likewise, it would ease the stations scheduling, deploying more automated procedures, as well as sensor tasking applied by the Expert Centres when coordinated observation campaigns. As a particular application to a use-case in a SLR network, the visibility outcomes derived from the algorithm implementation would allow to improve the development of the high-level design and the architecture of a future Laser Tracking and Momentum Transfer Network.

7 REFERENCES

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