CORRELATION OF OPTICAL OBSERVATIONS TO CATALOGUED OBJECTS USING MULTIPLE HYPOTHESIS FILTERS

O. Rodriguez Fernandez(1)**, J. Utzmann**(1)**, and U. Hugentobler**(2)

(1)*Aribus Defence and Space GmbH, Friedrichshafen, Germany, Email:*
{oscar.rodriguezferandez,Jens.utzmann}@airbus.com
(2)*Institut für Astronomische und Physikalische Geodäsie, Technische Universität München, Munich, Germany, Email:*
urs.Hugentobler@bv.tum.de

ABSTRACT

One of the key challenges for catalogue maintenance of space debris is to correctly correlate new measurements to their originating objects. This paper describes a workflow to associate optical observations to object for which a prior information is available in a robust manner. For efficiency, observations are compressed into attributable vectors and compared against known objects in the measurement space. A pre-filter is used to early eliminate non feasible candidates based on physical constraints and on information theoretic metrics. Suitable candidates are used in a Multiple Hypothesis Bayesian framework to perform the association based on the highest likelihood. The use of a Multiple Hypothesis Filter improves the robustness of the process for the case of ambiguous association of closely spaced objects. Different pre-filter algorithms are described. Results of applying this workflow are shown for GEO and MEO targets.

Keywords: SST; Correlation; Catalogue Maintenance.

1. INTRODUCTION

The Space Surveillance and Tracking (SST) segment is responsible for the detection and prediction of space debris in orbit around the Earth to avoid the degradation of space activities due to collisions. With this purpose, the US Space Surveillance Network (USSN) maintains a catalogue of more than 19000 Resident Space Objects (RSO) [1]. The information of this catalogue must be updated regularly by including observations coming from external sensors (radar, optical, etc). This process of updating the orbital information of catalogued objects is also known as catalogue maintenance while catalogue build-up refers to all the activities needed to include a new object in the catalogue. The former will be the main topic of this paper.

For catalogue maintenance the correlation process is critical. New observations must be linked together in the form of a tracklet and correlated with the object that originated them before attempting orbit determination. A common problem in this process is the cross-tagging or the association of an observation to the wrong object, this is specially relevant for closely space-object or debris clouds originated after a break-up. Cross-tagging will degrade the orbital information for the object, increasing the risk of operations and degrading the capability to re-observe the object. In this paper, we propose a work flow for catalogue maintenance that relies on the use of Multiple Hypothesis Filter and probabilistic data association for the correlation of optical observations.

This paper is organized as follows. First, the complete workflow for catalogue maintenance is presented, together with a review of the basic concepts. Then, the algorithm for tracklet to catalogue correlation using MHF is described. Finally, some results using both simulated and real data are shown for MEO and GEO objects.

2. BACKGROUND

Figure 1 shows the general workflow for catalogue maintenance for optical observations. Observations are generated by telescopes in the form of images. These images must be processed by image processing algorithms to produce the astrometry reduction, i.e. generate Right Ascension and Declination measurements. Measurements belonging to the same object in consecutive frames are joined together in a tracklet in the tracklet building step. While each single optical measurement provides only angular information, from a tracklet we can derive the angular rates in the form of an attributable vector in the attributable compression step. The tracklet to catalogue correlation step performs both the association and the orbit determination in a single step. Instead of trying to correlate against the full catalogue of objects, a pre-filter algorithm selects suitable candidates based on different criterion to decrease computational time. All those tracklet not deemed correlated (UCT) are stored for further processing.

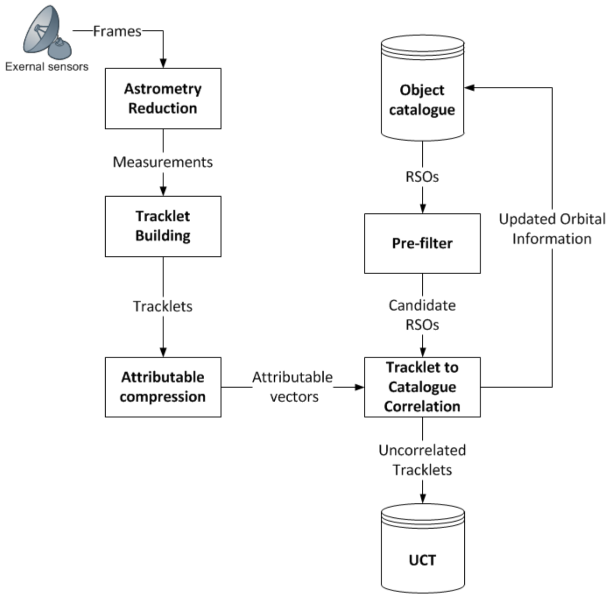

Figure 1. Catalogue Maintenance Workflow for optical observations

2.1. Astrometry reduction and Tracklet building

The astrometry reduction step is performed by external dedicated software. It must detect all the objects present in the frame and compare them against the known star background to extract angular information in the form of Right Ascension and Declination. Even in tracking mode (objects appear as points and start as trails), several closely space objects can appear on the same image. Figure 2 was obtained by the Airbus Robotic Telescope (ART)[2] tracking one ASTRA satellite (2011-041A). Another 4 satellites were detected in the same tracking session (3 ASTRA satellites and ARABSAT-5C). A total of 20 consecutive images were obtained for this tracking session.

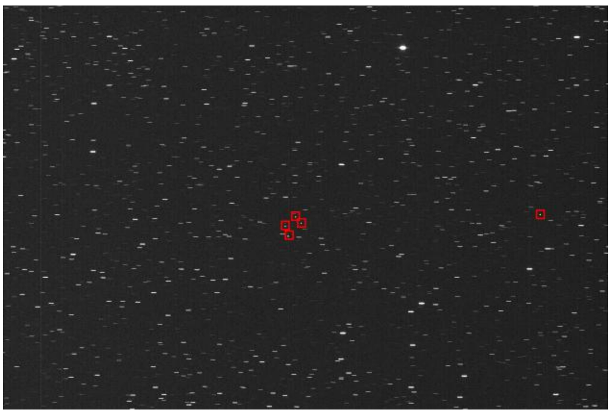

Figure 2. Optical observation of ASTRA satellites made with ART on the 08-11-2018 in tracking mode. 4 ASTRA satellites can be seen on the left of the images. On the right, an extra satellite is detected.

In order to be able to derive rate information in the next step, we must link together consecutive measurements belonging to the same object to build the tracklets. The tracklet building is performed by assuming linear motion during the duration of the tracklet. The linear motion assumption was found to be good enough for tracklets spawning only a few minutes, typical duration of tracklets obtained by sensors in survey mode. For longer tracklets, a quadratic model might be used instead.

Initially all possible pairs of detections between two consecutive images are linked together as long as their angular velocity is below a predefined value to form candidate tracklets. Detections from subsequent images are either linked with pre-existing candidate tracklets if the fit to the linear motion is below a predefined threshold or left unlinked. After each frame, new candidate tracklets might be created from unlinked detections (tracklet spawn), unfeasible combinations might be deleted (tracklet pruning) or tracklets might be closed if it they not been updated during a predefined number of frames. It is assumed that at least 4 observations must be joined together to obtain a tracklet. This algorithm will also filter out any possible false detection coming from the image processing as, in general, it will not be possible to consistently fit them to the assumed linear motion. Figure 3 shows the result of applying the tracklet building algorithm to the same tracking session showed in figure 2. It was able to succesfully detect the 5 different tracklets on the images as well as discard all the other point features detected in the frame that do not belong to a RSO.

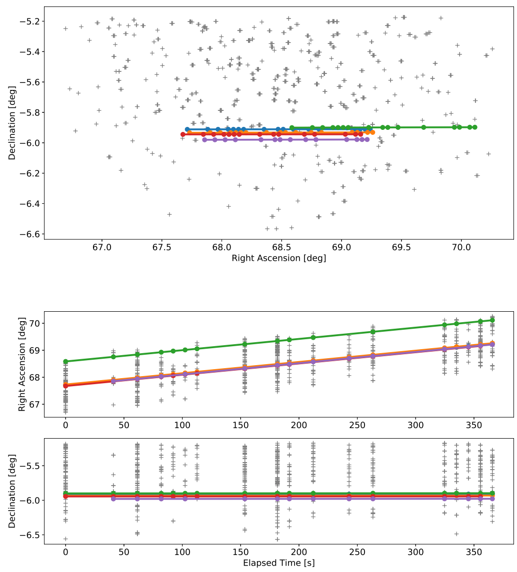

Figure 3. Tracklets detected from a series of 20 consecutive tracking images. All the points that do not belong to a line are false detections.

2.2. Deriving attributable information

Each one of the tracklets can be mathematically expressed as an attributable vector $\mathbf{z}$ at time t_0 with it associated uncertainty $\mathbf{C_z}$.

$$\Big\{\mathbf{z} := (\alpha, \delta, \dot{\alpha}, \dot{\delta}), \mathbf{C_z}\Big\} \quad \text{at} \quad t_0 \tag{1}$$

The use of attributable vectors has two advantages. First, it extracts all the usable information from the tracklet and second itcompress all the measurement into a single point of time, decreasing the total number of propagations needed in the association step. Similar to the tracklet linking step, the attributable is extracted using either a linear or quadratic motion model. A complete derivation can be found in [3].

2.3. Coarse catalogue initialization from external sources

When no a priori information is available in the form of a catalogue (with state vector and uncertainty information), tracklet to catalogue correlation might still be attempted using publicly available Two Line Element (TLE) information. TLEs do not have associated covariance information, necessary for the algorithm proposed here. However, a coarse covariance might be estimated by analysing the consistency of the TLE predictions over time. This approach is suitable for a quick catalogue initialization for correlation purposes but should not be used to offer other services such as conjunction assessment. The methodology described here follows [4]. A series of N TLEs spawning between 15 days and one month for a single satellite are obtained. The TLE corresponding to the latest epoch will be used as the prime, defining the epoch at which the covariance matrix is estimated. The rest of TLEs are propagated using the Simplified General Perturbation 4 (SGP4) theory to the epoch of the prime TLE and the residuals computing by differentiating the state vectors. The covariance matrix $\mathbf{C}_x$ is then computed as 4.

$$\delta \mathbf{x}_i = \mathbf{x}_i - \mathbf{x}_{prime} \quad i = 1, N-1 \tag{2}$$

$$\mathbf{m} = \frac{\sum_{i=1}^{N-1} \delta \mathbf{x}_i}{N-1} \tag{3}$$

$$\mathbf{C}_{\mathbf{x}prime} = \frac{\sum_{i=1}^{N-1} (\delta \mathbf{x}_i - \mathbf{m})(\delta \mathbf{x}_i - \mathbf{m})^T}{N-1} \tag{4}$$

2.4. Success metrics

The performance of the proposed algorithm will be evaluated using the 3 following metrics regarding whether the correlation process was or successful or not:

- **True positives**. This is the total number of tracklets corectly associated with their originating object.
- **False positives**. Number of tracklets associated with a different object.
- **False negatives**. Number of tracklets not deemed to be associated with any object that should have been associated.

3. TRACKLET TO CATALOGUE ASSOCIATION USING MULTIPLE HYPOTHESIS FILTERS

Bayes' theorem describes the probability of the hypothesis W given the measurements Y and the underlying model M (dynamical and/or measurement model), also known as the *posterior density* $f(W|Y, M)$.

$$f(W|Y, M) = \frac{f(Y|W, M) f(W, M)}{f(Y, M)} \tag{5}$$

The *likelihood* of the measurements $f(Y|W, M)$ accounts for the data noise while *prior* knowledge on the hypothesis can be incorporated in $f(W, M)$. Finally, $f(Y, M)$ is referred as the *evidence*. This term, plays an important role in problems such as model selection [5]. However, it can be neglected when estimating W, as it is independent of the parameters. Here will only act as a normalization constant. This is the reason why it will be neglected, together with the Model in the following derivation.

$$f(W|Y) \propto (Y|W) f(W) \tag{6}$$

Translating this into the tracklet to object correlation domain, our hypothesis is that a certain tracklet belongs to particular object c_i while our measured data is the attributable vector $\mathbf{z}$. We can assess the probability of each catalogued object c_i given an attributable vector as the product of the probability of having observed the attributable vector if it belonged to the catalogued object times the prior knowledge on object probability $f(c_i)$.

$$f(c_i|\mathbf{z}) \propto (\mathbf{z}|c_i) f(c_i) \tag{7}$$

In the most simple case, each object is modelled with the same probability, neglecting the effect of the *prior*. However, we can introduce different criteria to model this *prior* that will allow us to early discard part of the catalogued objects e.g. visibility checks,orbital regions, etc. For the simple case, the probability of a given tracklet belonging to a catalogued object is proportional to the probability of the catalogued object having produced the measured attributable:

$$f(c_i|\mathbf{z}) \propto f(\mathbf{z}|c_i) \tag{8}$$

The next step is how to assess the *likelihood* of the measurements $f(\mathbf{z}|c_i)$. The catalogued objects are modeled as a multivariate normal distribution with mean $\mathbf{x}$ and covariance $\mathbf{C_x}$ in the state space while the attributable vector is another multivariate normal distribution with mean $\mathbf{z}$ and covariance $\mathbf{C_z}$ in the observable space. In order to compare these two functions, we must select a common reference. As there is not enough information in a single tracklet to extract a complete orbital solution, the observable space is the most suitable reference frame for this comparison. Using the non linear function 9 that estimates the observables from the state vector and suitable methods such as the unscented transformation [6] we can transform the density function of the catalogued object from the state space to the observable space. The result would be the modelled attributable vector $\tilde{\mathbf{z}}$, which is again a normal multivariate distribution.

$$\mathbf{z} = h(\mathbf{x}) \tag{9}$$
$$\mathbf{x}, \mathbf{C_x} \longrightarrow \tilde{\mathbf{z}}, \mathbf{C_{\tilde{z}}} \tag{10}$$

The probability of a given catalogued object having produced the measured attributable can be modelled as the difference between the measured attributable vector and the modelled attributable $\Delta\mathbf{z}_i$, two independent normally distributed random variables. In [5, 72-73] it is proven that this difference is given by the convolution between both density functions, which is equal to another normal distribution for the particular case where both of them are normal distributions.

$$\begin{aligned} f(\Delta\mathbf{z_i} = \mathbf{z} - \tilde{\mathbf{z}}_\mathbf{i}) &= \int_{-\infty}^{\infty} f(\mathbf{z}')f(\mathbf{z}' + \Delta\mathbf{z_i})d\mathbf{z}' \\ &= \mathcal{N}(\Delta\mathbf{z_i}, \mathbf{0}, \mathbf{C_z} + \mathbf{C_{\tilde{z},i}}) \end{aligned} \tag{11}$$

The likelihood of the measurements given the object c_i is then modelled with this normal distribution[1] combination of the observed attributable and the modelled attributable. The goal of the tracklet to object correlation is finding the object $\hat{c}_i$, between the set of candidate objects, that maximizes this likelihood.

$$\hat{c}_i = \arg\max_{c_i} \mathcal{N}(\Delta\mathbf{z_i}, \mathbf{0}, \mathbf{C_z} + \mathbf{C_{\tilde{z},i}}) \tag{12}$$

Instead of maximizing the normal distribution, we can minimize the negative algorithm, which will simplify the posterior analysis of the function.

[1] $\mathcal{N}(x|\mu, \Sigma) = \frac{1}{\sqrt{\det(2\pi\Sigma)}} \exp^{\frac{-1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$

$$\begin{aligned} \hat{c}_i &= \arg\min_{c_i} -\ln f(\mathbf{z}|c_i) \\ &= \arg\min_{c_i} \frac{1}{2}\{\ln[\det 2\pi(\mathbf{C_z} + \mathbf{C_{\tilde{z},i}})] + \\ &(\mathbf{z} - \tilde{\mathbf{z}}_\mathbf{i})^T(\mathbf{C_z} + \mathbf{C_{\tilde{z},i}})^{-1}(\mathbf{z} - \tilde{\mathbf{z}}_\mathbf{i})\} \end{aligned} \tag{13}$$

Analysing equation 13, one can interpret the first term as a term to prune large uncertainties i.e. the information about the catalogued object is not reliable enough while the second term is a statistical distance between two distributions [7].This only yields the most probable object. There still is to be addressed the case when none of the catalogued objects have produced the measurements. For that case the algorithm will still produce a correlated object between all the possible candidates. The next step is then to analyse if the difference $\Delta\mathbf{z_i}$ is in a statistically significant part of the distribution that can be done using the Mahalanobis distance. The square of the Mahalanobis distance is just a quadratic form of the normally distributed variable $\Delta\mathbf{z_i}$. As probed in [8, 57-58], this distance should follow a χ^2 distribution of n_z degrees of freedom, where n_z is the size of the variable $\Delta\mathbf{z_i}$ (4 in the case of optical attributables). This property can be used to stablish threshold values based on significance levels of the χ^2 distribution.

The main drawback of this algorithm is that several objects might be below the association threshold, especially for the case of formation-flying satellites. This drawback can be overcome by using Multiple Hypothesis Filters (MHT) [9].

Each one of the candidate objects passing the test is transformed into a single hypothesis with their a priori weights given by 14.

$$w_i^- = \frac{\mathcal{N}(\Delta\mathbf{z_i^-}, \mathbf{0}, \mathbf{C_z} + \mathbf{C_{\tilde{z},i}^-})}{\sum_j^v \mathcal{N}(\Delta\mathbf{z_i^-}, \mathbf{0}, \mathbf{C_z} + \mathbf{C_{\tilde{z},j}^-})} \quad i = 1, .., v \tag{14}$$

The a priori states and covariances $\mathbf{x}^-$, $\mathbf{C}_x^-$ for each of the hypothesis are updated with the measurements of the tracklets using any filtering algorithm to obtain the updated states $\mathbf{x}^+$, $\mathbf{C}_x^+$. The weights are updated by 15

$$w_i^+ = \frac{w_i^- \mathcal{N}(\Delta\mathbf{z_i^+}, \mathbf{0}, \mathbf{C_z} + \mathbf{C_{\tilde{z},i}^-})}{\sum_j^v \mathcal{N}(\Delta\mathbf{z_i^+}, \mathbf{0}, \mathbf{C_z} + \mathbf{C_{\tilde{z},j}^+})} \quad i = 1, .., v \tag{15}$$

Only the object with the highest weight after the update is associated. What is more, the algorithm already provides with an updated state and covariance matrix for the associated object that can be used to update the catalogue for future associations, improving the robustness of the overall process.

3.1. Pre-filter

The described algorithm requires to propagate the states and covariances of all the objects in the catalogue until to the epoch of the attributable vector. For large catalogues, this is one of the most time consuming operations. Here we proposed to use a pre-filter step to avoid unnecessary propagations of non suitable candidates reducing the search space. The use of the pre-filter algorithm is specially interesting for real time processing of observations.

This pre-filter checks have been developed with the goal of been computationally cheap to compute and to include enough safety margin in the assumptions made to avoid discarding that might be suitable candidates for correlation. Two pre-filter scores have been used for this results:

1. **Angular velocity bounds**. The relative angular velocity can be derived from the angular rates of the attributable vector. By assuming that the object is observed at zenith distance at either the perigee or the apogee, maximum and minimum values for the angular velocity can be used to discard non feasible candidates without any propagation.

2. **Line of Sight**. Checking the visibility between the ground-station and the object necessary needs to take into account the uncertainty around the state vector, requiring the propagation of the full covariance for each object. For that reason, here only it is checked if there is direct Line of Sight (LOS) between the object and the observer. With this simplification, only the state vector needs to be propagated. Other simplified visibility checks might be included as, for example, if the LOS is close enough to the actual pointing of the sensor. The definition of "close enough" should again be considered with enough margin to avoid discarding the originating object. For this test we have defined close enough as 10 times the width of the FoV.

For real-time catalogue maintenance, a full visibility check taking into account the uncertainties of the objects could be performed offline and stored before the observation starts to be used as a further pre-filter step.

3.2. Full correlation algorithm

The full correlation algorithm us shown in algorithm 1. The algorithm starts after the tracklet building step, extracting the attributable information for each one of the tracklets. For each tracklet, only pre-filter extracts only the suitable candidates that will be propagated (both state and covariance) to the attributable epoch. This step can be parallelized to reduce computational times.

Algorithm 1 Tracklet to object correlation

for all new tracklets **do**
    Extract Attributable vector at t_0
    for all Objects in Catalogue **do**
        Check pre-filter constraints
        for all Pre-filtered objects **do**
            Propagate state vector and covariance to t_0
            Extract Modelled Attributable
            Mahalabonis Distance
            if Mahalabonis $\leq$ threshold **then**
                Generate hypothesis
            end if
        end for
    end for
    if Number hypothesis ≥ 0 **then**
        for all Hypotesis **do**
            Update state, covariance and weight
        end for
        Correlate the hypothesis with the highest updated weight
    else
        Tracklet is Uncorrelated
    end if
end for

4. EXPERIMENTAL RESULTS

4.1. Simulated data

The proposed algorithm has been applied to produce tracklet to catalogue correlation using simulated data. A single ground-based observed, with characteristics similar to ART 2 is simulated in surveillance mode taking measurements of MEO and GEO targets. The surveillance strategy is a fence-like scenario, with two declination stripes close to the Earth shadow that are sequentially scanned by the Field of View. The surveillance pattern can be seen in figure 4 while table 1 shows the main characteristics of the simulated observed.

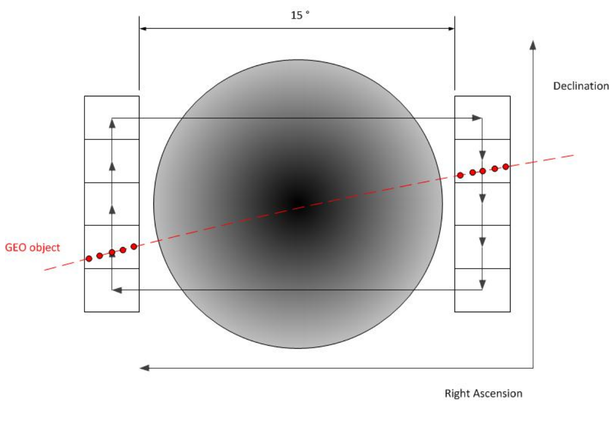

Figure 4. Surveillance set-up. Two declination stripes, with several fields per stripe close to the shadow of the Earth.

Table 1. Survey strategy characteristics

Fields per stripe	5
Frames per field	10
Frame period	10 s
FOV	2.15° x 1.43° s
Latitude	38.216°
Longitude	-6.627°
1-Sigma Sensor Noise	0.5”

The sample population simulates break-up scenarios to generate closely-space observations. The break-up will be simulated simply by adding a Gaussian distributed random component in the velocity with a 1-σ standard deviation of 2 m/s, generating a cloud of closely space objects. The objects will be defined using public TLEs and the initial uncertainty will be extracted following section 2.3. 239 objects, with altitudes between 30000 and 40000 kms and visible from the simulated sensor are selected as the base objects, generating a total of 2390 ”break-up objects” to simulate. The break-up is simulated at 2018-11-07T15:00 and a the following night is used to generate observations. A total of 1449 tracklets with between 4 and 10 observations are generated. 50 Monte Carlo runs of this scenario were performed, yielding in total 72450 tracklets to correlate in the full simulation. Figure 5 shows some of the simulated tracklets (in green) and the break-up objects projected in the measurement space (in red).

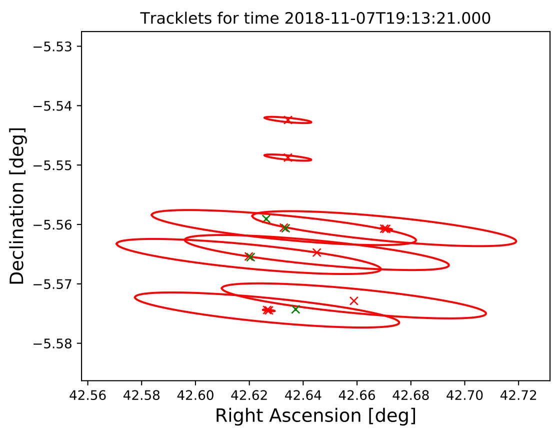

Figure 5. Generated break-up clouds and tracklets. In green it is shown the attributable vector for the generated tracklet. In red are shown some of the simulated break-up objects with their associated uncertainty projected in the measurement space.

Real world behaviour is simulated by using two different force models for propagation and introducing random errors in the state vector of the catalogued objects. One force model is used to generate the measurements and a second simplified model for the tracklet correlation and orbit determination.

Table 2. Force Model use for measurement generation and for correlation

Force	Measurement Model	Correlation Model
Gravity Field	GGM02C 15x15	GGM02C 8x8
Atmospheric Drag	DTM-2013	DTM-2013
Solar Radiation Pressure	Cannonball model	Cannonball Model
Mon and Sun Gravity	On, Legendre Expansion	On, First Order Taylor Expansion

The results of the tracklet to catalogue correlation can be seen in figure 6. Here it is depicted the temporal evolution of the tracklets against the declination. In green are represented all those tracklets correctly associated, in red those associated with a different object (false positives) and in black is shown those that were not associated any object (false negatives). In average of the 50 Montecarlo runs more than 80% of the tracklets were successfully correlated for this challenging situation. The segregated results are shown in table 3.

Table 3. Average results of the tracklet to catalogue correlation for the 50 Montecarlo

True Positives	False Positives	False Negatives
80.82%	16.81%	2.37%

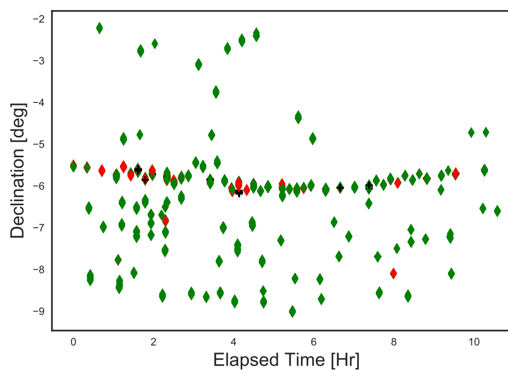

Figure 6. Results of the tracklet to catalogue correlation using synthetic data. Green dots represent correctly correlated tracklets (true positives), red dots false positives and black dots false negatives.

4.2. Real data

The algorithm has been successfully applied to real world data for closely space objects. An observation campaign was conducted the night 2018-11-08 where object 2011-041A (ASTRA 1N) was tracked at the beginning and end of the night, but several objects appear in the images (as depicted in figure 2). In total, 5 objects were always visible in the same frame, both at the beginning and end of the night.

Figure 7. The Airbus Robotic Telescope (ART).

The full workflow was applied, from the tracklet building step up to the tracklet to catalogue correlation using multiple hypothesis filters. Again, an initial catalogue was derived using the procedure depicted in 2.3. The same force model as the one used for the synthetic data case was used. The correlation results are shown in table 4. Tracklets 1-5 belong to the tracking session at the beginning of the night while tracklets 6-10 are from the same objects at the end of the night.

Table 4. Results of the tracklet to catalogue correlation using real world measurements. In bracktets is shown the percentage of objects deemed as possible candidates from the total catalogue used as input

Tracklet	Origin (COSPAR)	Pre-filter candidates	Total Hypothesis
1	2007-016A	193 (1.16%)	3
2	2011-041A	188 (1.14%)	1
3	2011-049B	186 (1.13%)	1
4	2006-012A	182 (1.10%)	2
5	2008-057A	182 (1.10%)	1
6	2007-016A	185 (1.13%)	1
7	2011-049B	191 (1.16%)	1
8	2011-041A	181 (1.10%)	1
9	2008-057A	180 (1.10%)	1
10	2006-012A	179 (1.09%)	1

Applying the described algorithm, we were able in all cases to successfully associate the tracklet. What is more interesting, the pre-filter mechanism is able to discard almost 99% of the candidates before the actual correlation is attempted, avoiding the costly propagation of the covariance matrix for most of the objects in the catalogue. In the tracklets at the beginning of the night (1-5), two or three candidates pass the threshold imposed by the χ^2 distribution on the Mahalanobis distance for some cases, that might lead to ambiguous associations. The use of Multiple Hypothesis Filters improves the robustness of the process by making sure only the most suitable candidate would be associated. As the orbit determination is performed in the tracklet association step, the uncertainty associated to each one of the catalogued objects is reduced and, at the end of the observation (time between tracklets approximately 5 hours) only one candidate object is selected as hypothesis for each tracklet.

5. CONCLUSIONS

This paper describes a complete workflow to produce the association of tracklets to objects for which state and uncertainty information is available. A tracklet building algorithm, based on simplified linear motion is described.. The tracklet buildind algorithm not only produces the initial linking needed to extract the attributable vector but it is also able to detect and discard false detections coming from the image processing software. A simplified method to produce a coarse initial catalogue of objects based on TLE consistency analysis is introduced that might be useful when no other source of information about the uncertainty (i.e. a real catalogue) to produce the tracklet association is available. The algorithm is based both on the use of the Mahalanobis distance for threshold gating as the likelihood function to initiate a Multiple Hypothesis Filter scheme. The algorithm was successfully applied for a simplified simulated case of a break-up in GEO and MEO regimens as well as tested with real data coming from the Airbus Robotic Telescope. The accuracy and uncertainty of the initial catalogue plays and important role in the correlation process, as it will determine the total number of hypothesis to start the MHF. Next steps would include testing the algorithm after a real catalogue build-up, instead of estimating a coarse initial uncertainty from TLE consistency analysis, as well as integrating it with tracklet to tracklet correlation algorithms.

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