PERTURBATIONS IN THE OPTIMIZED BOUNDARY VALUE INITIAL ORBIT DETERMINATION APPROACH

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ABSTRACT

One of the increasing threats to functional spacecraft around Earth is that of Space Debris. The risks that it poses for current and future space operations due to collisions are growing in an exponential manner. It is essential to create and maintain a catalog of space debris to monitor the space environment. Optical survey observations from the Swiss Optical Ground Station and Geodynamics Observatory Zimmerwald, Switzerland, are used to discover space debris objects. The result of these surveys are observations of objects on very short arcs (when compared to their orbital period). Optimized Boundary Value Initial Orbit Determination (OBVIOD) is one existing method to associate short-arc observations with each other and to compute initial orbits. This method is based on the solution of the Lambert problem. An important extension of this initial orbit determination method consists in including perturbations. One way to accomplish this task is by using shooting methods. In the methods hypothetical initial values are chosen at one boundary and, after integration up to the second boundary, the end values are compared with the boundary conditions. The hypothetical values satisfying the boundary conditions are accepted as the solution of the problem. We also analyzed to what extent the Keplerian model gives acceptable results and under what conditions it becomes imperative to include perturbations in the model. This was done by comparing the performance of this initial orbit determination algorithm with and without the addition of perturbations. Here performance refers to the capability of correlating the short-arc observations, which truly belong to the same object. Moreover, the analysis of different factors including computational complexity, the time taken for the algorithm to converge was also done. The tests include simulated observations for the GEO orbital regime. These observations were simulated using the same perturbation model as in the initial orbit determination. Later tests were performed using real observations obtained from the optical surveys of the Zimmerwald Observatory.

1 INTRODUCTION

Artificial debris objects, which include: non-functional spacecraft, spent rocket bodies, mission-related objects, the products of spacecraft surface deterioration, and fragments from spacecraft and rocket body breakups; orbit the Earth and will remain in orbit until atmospheric drag and other perturbing forces eventually cause their orbits to decay into the atmosphere, but only for some LEO objects. In order to avoid accident-prone proximities of active satellites and uncontrolled space debris or eventually to remove space debris, space catalogs must be maintained. The objects in GEO can only be tracked for a short duration because of the limited number of telescopes trying to cover the complete orbital region. The resulting short observation arcs, called tracklets, only contain incomplete state information and are therefore either associated to already catalogued objects or tested pairwise with other uncorrelated observations. In this work two tracklets at a time (from a group of tracklets) are examined whether they originate from the same object or not and, if they do, the common orbit solution is determined. This measurement association is a fundamental task during the catalogs build-up phase for the initial location of space objects but also later for the relocation of lost ones.

1.1 Concept

Each measurement arc contains a series of right ascension (α) and declination (δ) observations. The information of the series, which is exploited for the association, can be represented by the angles and angular rates, commonly known as the attributable vector [2].

$$a = (\alpha, \alpha', \delta, \delta') \quad (1)$$

The OBVIOD algorithm exclusively uses tracklets as it provides an advantage, which is that now also the information on the angular rates is available. Optimized Boundary Value Initial Orbit Determination method considers angular position measurements at the beginning and last epoch in the attributable vector to begin with. Next step involves range hypothesis at both the epochs. The measurements and hypothesis are used to solve the Lambert problem. One receives the orbit at the end of this step. The values of angular rates are computed from this orbit and compared with the measured angular rates from the attributable vector. The difference between these angular rates is evaluated using Mahalanobis distance. The latter is used as loss function to be minimized to find the optimum range hypothesis. The quasi-Newton methods approximate the loss function locally around some initial point $p* = (\rho 1,$

ρ2) with a quadratic function. Then, the minimum is iteratively searched by finding the root of the gradient using Newton's method. An implementation for the popular Broyden–Fletcher–Goldfarb–Shanno (BFGS) scheme is used in OBVIOD [4]. Once a value below the threshold is obtained, that particular range hypothesis is accepted. The respective pair of tracklets is associated together and initial orbit is computed. This is the method proposed by Siminski et al. in [3]. The schematic in Fig. 1 shows process flow in OBVIOD.

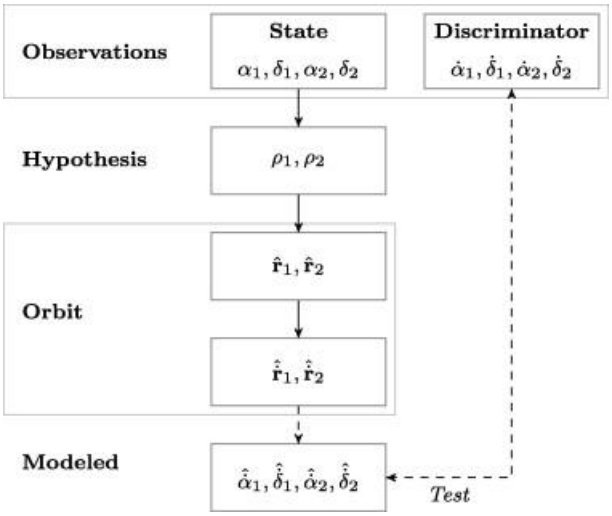

Figure 1. Process flow of the boundary-value methods. The dashed line denotes the testing metric in OBVIOD [3].

1.2 Proposed method

The method proposed seeks to add the perturbations model in the IOD. The flowchart with changes introduced by this model is given in Fig. 2. The perturbation forces are added, which include solar radiation pressure, atmospheric drag, gravity field gradient, third body attraction forces from sun and moon.

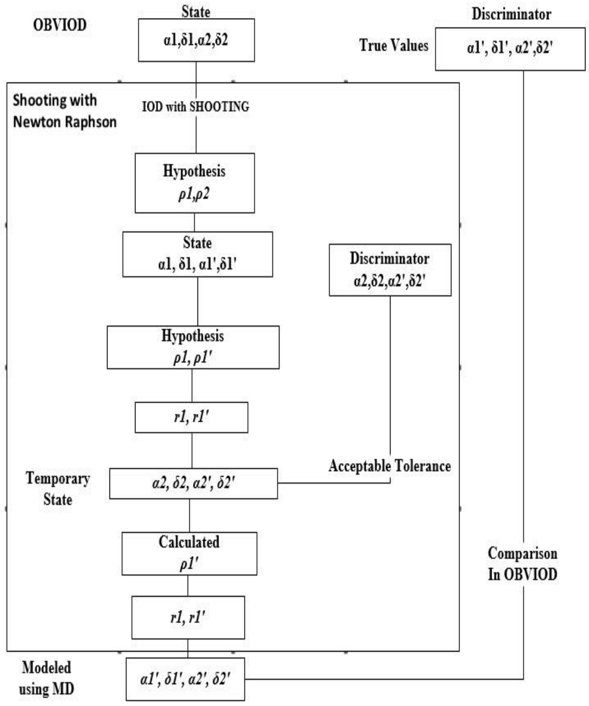

Figure 2. Process flow of OBVIOD with shooting method for IOD.

2 THE SHOOTING METHOD IN OBVIOD

In order to add perturbations, shooting method is used for IOD in OBVIOD. Shooting method is one of the approaches to solve the two-point boundary value problem (BVP). There are several approaches to solving this type of problem. It treats the two-point boundary value problem as an initial value problem (IVP). Specifically, the shooting method solves the initial value problem with initial conditions where initial value must be chosen so that the solution satisfies the remaining boundary condition. In the case it is a linear ODE, selecting the slope is relatively simple. In this work, Newton's Method is used, it requires the derivative of difference of function values with respect to time (in case of orbit propagation), during each iteration.

In the IOD part, range at initial epoch is available from the hypothesis in OBVIOD method. The angular position and rates at initial epoch are available from the attributable vector. Range rate value is assumed to get the initial orbit. This orbit is propagated to the second epoch with considering the perturbation model. From this state of the orbit, angular positions and rates at second epoch are calculated. The difference between these values and measured values available from the attributable vector form a function, which is used to iterate upon the different values of range rate. Once the difference between range rate values in consecutive iterations is sufficiently small, the iterations are stopped and that particular range rate value is accepted to determine the initial orbit.

3 TESTS DONE AND RESULTS

The main idea was to see the difference in results of IOD by introducing perturbations in the OBVIOD method. It was done for different values of Area to Mass Ratio (AMR). High AMR objects are suspected to behave differently because of the solar radiation pressure at GEO altitudes. Later tests were done for cases where observations were simulated one night apart to see if the effect of perturbations is more. For GEO case, this test becomes important if one tries to correlate observations that do not belong to the same night. Some of the parameters used for simulations are shown in the Table 1.

Interval between obs	20 sec
Right Ascension fence	249:251; 229:231 deg
Declination range	>-12,<12 deg
Optical Error	0.5 arc sec
Elevation	>20 deg

Table 1. Parameters used for simulation

3.1 Tests on Simulated Observations

The results of tests done with different values of AMR for shooting and lambert method are given in Table 2 and Table 3. The tests in these cases involved five tracklets out of which two were pairs. Tracklets in a pair belong to the same object. Time shown is the computation time for the single run for case of shooting method or lambert method. MD is the Mahalanobis distance, which is one of the criteria used for association. RMS is calculated in the orbit improvement part, which involves least squares method. True correlations refer to tracklets, which truly belong to the same object. False correlations refer to tracklets that are falsely associated together to belong to the same object. The shooting method performs very similar to Lambert method for observations only a few hours apart (in the same night).

AMR	Time(sec)	MD	RMS	True	False
1.0	190.24	0.1327 1.0406	0.9324 1.0004	2	0
0.01	51.93	0.3738	0.9770	1	0

Table 2. Correlation of observations in the same night with shooting method for different values of AMR (m^2/

AMR	Time(sec)	MD	RMS	True	False
1.0	12.24	0.1246 1.0404	0.9324 1.0004	2	0
0.01	4.27	0.3738 0.3427	0.9770 1.0067	2	0

Table 3. Correlation of observations in the same night with Lambert method for different values of AMR (m^2/kg)

Table 4 and Table 5 involved tests with three tracklets belonging to the same object, out of which two belonged to the same night and the third belonged to the consecutive night. AMR was kept at 1.0 m^2/kg. For the HAMR case, Lambert is not able to correlate the observations belonging to the same object, which are more than one revolution apart.

No. of nights	Time(sec)	MD	RMS
0.15	4.1	0.409	0.965
1.12	20.15	99.828	--

Table 4. Correlation of observations in the same night and one night apart with Lambert method for AMR 1 m^2/kg.

No. of nights	Time(sec)	MD	RMS
0.15	4.67	0.409	0.965
1.12	43.62	0.747	0.878

Table 5. Correlation of observations in the same night and one night apart with shooting method for AMR 1 m^2/kg.

3.2 Tests on real observations

The tests were also done on real observations from the Zimmerwald Observatory. The same set was used to compare the performances of old correlation tool in [5] to those of OBVIOD. The three methods were run for the Zimmerwald observations from the night of 21.06.2017, the results are shown in Table 6. This comparison was done to see the relative performance of the three methods with respect to time taken and number of correlations. The true and false positives cannot be determined because one does not know the ground truth in this case.

Method	Old Tool	Lambert	Shooting
Time(sec)	207.11	246.936	1611.5
No.of correlations	1	2	1

Table 6. Correlation of observations in the same night for real observations from Zimmerwald.

4 DISCUSSION AND CONCLUSIONS

The performance of the shooting method was compared in different cases by varying length of the observation arc, AMR values, no. of tracklets considered for simulated observations. Due to the complexity of the numerical propagator used while calculating the orbits, the time taken is higher in case of shooting method. The time taken is a factor of different parameters including tolerances, number of iterations used inside IOD with shooting. This method needs to be optimized with respect to computation time so that tests that are more extensive can be done. The robustness will also depend on the combination of stepsize and structure used inside the numerical propagator. Another factor will be working of shooting method in collaboration with MD computation in OBVIOD. Since some steps in MD optimization will also involve propagation, it will be interesting to see how well the numerical propagator keeps up with these steps. The optimized MD value for correlations in case of perturbations may vary from the simple Lambert case. Once some of the parameters are established, it will be evident which model of the perturbations is closest to the real observations case. That model will be adopted for further optimizations. Different models could also be used if some information about the AMR values of the object are available with only slight changes in the configuration used in the propagation part. This will allow one to use the shooting method for wider range of objects, orbits.

5 REFERENCES

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