Advanced methods for break-up uncertainty propagation

March 1st 2018, Vincent Morand, 4th International Workshop on Space Debris Re-entry
Increasing need for uncertainty propagation technics

- **Space Debris applications:**
  - Increasing number of Space Debris
  - Improvement of detection capability (Space Fence)
  - Bad knowledge of shapes, mass, orbit...

  → Requires efficient methods (> Monte Carlo...)

- **Regulations:**
  - French Space Operations Act, IADC, etc.. release rules or guidelines

  → Need for robust results

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*Sub-section 2 – Quantitative objectives for human safety*

*Article 20 – Quantitative objectives for human safety*

1. For the cumulative catastrophic damage risks, the launch operator must respect the following quantitative objectives, expressed as a maximum allowable probability of causing at least one casualty (collective risk):

   a) Lift-off risk
CNES activities w.r.t. Uncertainty propagation

Few R&T actions for Uncertainties propagation in Space Flight Dynamics:

- **2014**: R&T action with Thales Services: Taylor Differential Algebra (TDA) for long term orbit propagation
  
  AAS/AIAA Astrodynamics Specialist Conference, August 9–13, 2015)

- **2015 / 2016**: R&T action with Thales Services:
  TDA for atmospheric re-entry
  
  (Ref. : AAS-16-263: “Re-entry prediction and analysis using Taylor Differential Algebra”
  AAS/AIAA)

- **2016 / 2017**: R&T action with Thales Services and INRIA:
  PCE for uncertainty propagation
  
  (Ref. : AAS-17-233: “CNES ACTIVITES ON POLYNOMIAL CHAOS EXPANSION FOR UNCERTAINTY PROPAGATION”)

- **2018 - ...**: R&T action with Thales Services and INRIA
  Kriging methods

Today’s presentation limited to two example for atmospheric re-entry.
Main TDA application: image of an initial domain (uncertainty...) by $f$

TDA is:
- An intrusive method: source code has to be rewritten
- A local method: the solution is “expanded” around a reference point (Taylor Development...)

$$f(x) = f(x_0) + \nabla f(x_0)^T (x - x_0) + (x - x_0)^T \nabla^2 f(x_0) (x - x_0)^T$$
TDA : basic understanding

- Classic formulation
  \[
  \begin{aligned}
  \frac{dx}{dt} &= f(t, x(t)) \\
  x(t_0) &= x_0
  \end{aligned}
  \]

- Replace the initial condition by a first order polynomial of the state \( x \):
  \[
  X_0(x) = x_0 + (x - x_0)
  \]

- Define basic operators (+, -, *, /) for polynomials

- Rewrite complex functions (exp, sin, cos) using Taylor development
  \[
  f(x) = \sum_{k=0}^{N} D^k f(x_0)(x - x_0)^k + o(\|x - x_0\|)^N
  \]

- After integration of the ODE, the result is a polynomial of the state \( x \)
  \[
  X_f(x) = x_f + p_1(x - x_0) + p_2(x - x_0)^2 + \ldots
  \]

⇒ To get the results corresponding to another initial condition \( x_1 \) one has just to evaluate the final polynomial : \( X_f(x_1) \)
ELECTRA Tool

(Estimation de la Létalité due aux Evénements Catastrophiques sur Trajectoires Rentrant dans l’Atmosphère)

De-orbit Maneuver
Debris 1
Debris 2
Fragmentation altitude

$\mathbf{x} = f(\mathbf{x}, \dot{\mathbf{x}}, p(t))$

Numeric integration: RK6, DOPRI8 …

- Security Reentry Area
- Casualty risk

SRA
SWOT satellite test case

We retrieve the 4th order polynomials describing the position of the impact point:

\[
\begin{align*}
\text{longitude} &= \long_0 + \sum_i c_i (\varepsilon_1 - \varepsilon_{1,0})^{p_{i1}} (\varepsilon_2 - \varepsilon_{2,0})^{p_{i2}} \ldots (\varepsilon_{12} - \varepsilon_{12,0})^{p_{i12}} \\
\text{latitude} &= \lat_0 + \sum_i c_i (\varepsilon_1 - \varepsilon_{1,0})^{p_{i1}} (\varepsilon_2 - \varepsilon_{2,0})^{p_{i2}} \ldots (\varepsilon_{12} - \varepsilon_{12,0})^{p_{i12}} \\
\text{altitude} &= 50
\end{align*}
\]

Long_0 : Impact longitude without dispersion
Lat_0 : Impact latitude without dispersion
\( \varepsilon_k \): dispersed parameters (12 of them)
\( \varepsilon_{k,0} \) : nominal value of dispersed parameters
\( c_i \): 1820 coefficients for longitude, same for latitude

<table>
<thead>
<tr>
<th>Dispersed parameters ( \varepsilon_k )</th>
<th>Nominal values</th>
<th>Gaussian or Uniform law</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Position of the falling object</td>
<td>200 – 980 km orbit</td>
<td>Gaussian</td>
<td>On position and velocity: ( \sigma \sqrt{3} ) with ( \sigma = 5 \text{ m} ) and ( \sigma = 5 \text{ cm/s} )</td>
</tr>
<tr>
<td>De-orbit maneuver: thrust level</td>
<td>60 N</td>
<td>Uniform</td>
<td>( \pm 8 % )</td>
</tr>
<tr>
<td>De-orbit maneuver: thrust orientation</td>
<td>Opposite to velocity</td>
<td>Gaussian</td>
<td>3° in the direction and 3° transversal to commanded ( \Delta V )</td>
</tr>
<tr>
<td>Altitude fragmentation</td>
<td>75 km</td>
<td>Uniform</td>
<td>( \pm 5 \text{ km} )</td>
</tr>
<tr>
<td>Drag factor</td>
<td>1</td>
<td>Uniform</td>
<td>( \pm 50 % )</td>
</tr>
<tr>
<td>Lift orientation (Roll)</td>
<td>0.</td>
<td>Uniform</td>
<td>( \pm 180^\circ )</td>
</tr>
</tbody>
</table>
SWOT satellite test case: Roll dispersion

360° roll dispersion

360° roll dispersion

Reduced roll dispersion (+/-45°)

→ Computation of the polynomial 30 times faster than a 20000 Electra MC

→ Analysis of coefficients give “ANOVA-like” information

→ Singularity in the attitude law makes TDA not very well suited to this application (Further study could solve the issue)

→ At this time, TDA is being studied for “simpler” applications (Perturbed Lambert problem, covariance propagation...)
No “Chaos“ here

- Evolution of an initial domain
- Problem is assumed to be deterministic (same inputs → same outputs)
- Output is modeled as a multivariate polynomials

Non intrusive method

\[
res(x) = \sum_{k=1}^{N} c_k \Psi_k(x)
\]

- Global method (polynomial surrogate over a domain)
- Widely used in the industry (EDF, Total…)
- Strong mathematical basis
- Existing competences and toolbox

Polynomial Chaos Expansion (PCE)
Debrisk test cases

- Monte Carlo simulation as reference (5000 samples)
- PCE computation (~ 400 samples, expansion order selected)

<table>
<thead>
<tr>
<th></th>
<th>$i$ [°]</th>
<th>$\Omega$ [°]</th>
<th>$\omega$ [°]</th>
<th>$M$ [°]</th>
<th>$h_{frag}$ [km]</th>
<th>$C_D$ [-]</th>
<th>$C_p$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta-II</td>
<td>$U(95.6, 97.6)$</td>
<td>$U(0, 360)$</td>
<td>$U(0, 360)$</td>
<td>$U(0, 360)$</td>
<td>$U(73, 78)$</td>
<td>$U(0.6, 1.4)$</td>
<td>-</td>
</tr>
<tr>
<td>Sobol Indexes</td>
<td>$U(0, 180)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$U(73, 78)$</td>
<td>$U(0.6, 1.4)$</td>
<td>$U(0.6, 1.4)$</td>
</tr>
<tr>
<td>Validation</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Cas Satellite</td>
<td>$U(44, 46)$</td>
<td>-</td>
<td>-</td>
<td>$U(0, 360)$</td>
<td>$U(75.5, 80.5)$</td>
<td>$U(0.6, 1.4)$</td>
<td>$U(0.6, 1.4)$</td>
</tr>
</tbody>
</table>

- US 76 atmospheric model is used for DELTAII/Satellite (ongoing simulation with MSIS model)

- We are interested in:
  - Global ablation rate (TAG in French)
  - Total casualty area
  - Knowing what are the main contributors: ANOVA

$$\text{TAG} = \frac{\sum_{i=1}^{N} m_{g_i}}{N h_{frag}} - \frac{\sum_{i=1}^{N} m_{b_i}}{\sum_{i=1}^{N} m_{b_i}}$$

$$\text{CA} = \sum_{i=1}^{N} \text{CA}_i$$
Delta II test case

- **Global ablation rate**
  - Bi-modal distribution
  - Nice PCE approximation but limited due to the bi-modal distribution

- **Casualty area**
  - Discrete results
  - PCE is not appropriate
Sobol indexes validation

- Analyzed for global ablation rate

- Order 4 PCE

- Sobol index computation is “free” once the PCE has been computed

- MSIS model used

Polynomial Chaos Expansion allows efficient ANOVA
Small differences on the numerical values but the order of magnitude are obtained
Satellite test case

- **Global ablation rate**
  - Great PCE approximation

- **Casualty area**
  - PCE give a “smooth” distribution function

  → PCE is interesting to get the shape of the CDF faster than through Monte Carlo
Kriging Methods

- On going action
  - Interpolation method
  - Estimate of the error of the surrogate model
  - To be tested on ELECTRA and DEBRISK

**Toblers 1st Law of Geography**: Everything is related to everything else, but near things are more related than distant things!
Conclusions

Several uncertainty propagation methods are under evaluation

- TDA: Intrusive local method, maybe not the most suited one for complex atmospheric re-entry code

- PCE: non-intrusive global method, suited for smooth non-discrete problems, very efficient for ANOVA Analysis

- Kriging methods: to be evaluated, interpolation methods with an estimate of the error of the surrogate model
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