Advanced methods for break-up uncertainty propagation

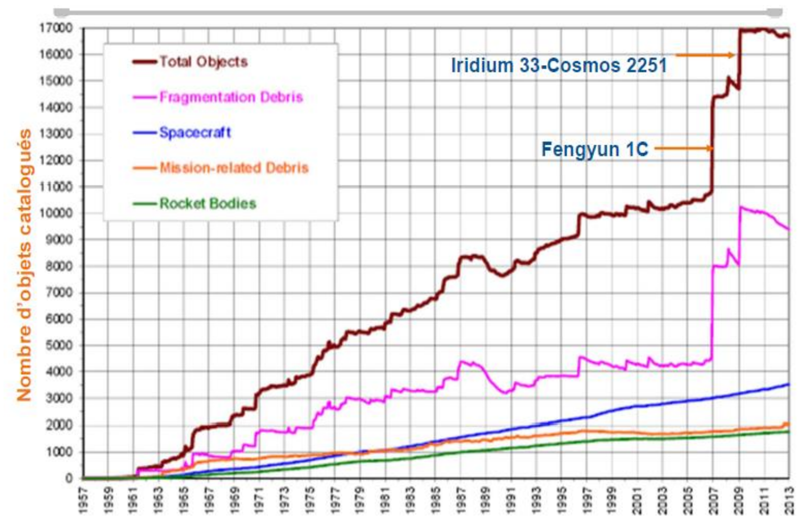
March 1st 2018, Vincent Morand,

4th International Workshop on Space Debris Re-entry

Increasing need for uncertainty propagation technics

- ❑ **Space Debris applications:**
  - ❑ Increasing number of Space Debris
  - ❑ Improvement of detection capability (Space Fence)
  - ❑ Bad knowledge of shapes, mass, orbit…

→ Requires efficient methods (> Monte Carlo…)



- ❑ **Regulations:**
  - ❑ **French Space Operations Act, IADC, etc.. release rules or guidelines**

  → Need for robust results

Sub-section 2 – Quantitative objectives for human safety

Article 20 – Quantitative objectives for human safety

1. For the cumulative catastrophic damage risks, the launch operator must respect the following quantitative objectives, expressed as a maximum allowable probability of causing at least one casualty (collective risk):

a) Lift-off risk

CNES activities w.r.t. Uncertainty propagation

Few R&T actions for Uncertainties propagation in Space Flight Dynamics:

- **2014: R&T action with Thales Services: Taylor Differential Algebra (TDA) for long term orbit propagation**

  (Ref. : AAS-15-518 :"Using Taylor Differential Algebra in mission analysis: Benefits and Drawbacks." AAS/AIAA Astrodynamics Specialist Conference, August 9–13, 2015)

- **2015 / 2016 : R&T action with Thales Services :**

  TDA for atmospheric re-entry

  (Ref. : AAS-16-263 :"Re-entry prediction and analysis using Taylor Differential Algebra" AAS/AIAA)

- **2016 / 2017: R&T action with Thales Services and INRIA :**

  PCE for uncertainty propagation

  (Ref. : AAS-17-233: "CNES ACTIVITES ON POLYNOMIAL CHAOS EXPANSION FOR UNCERTAINTY PROPAGATION ")

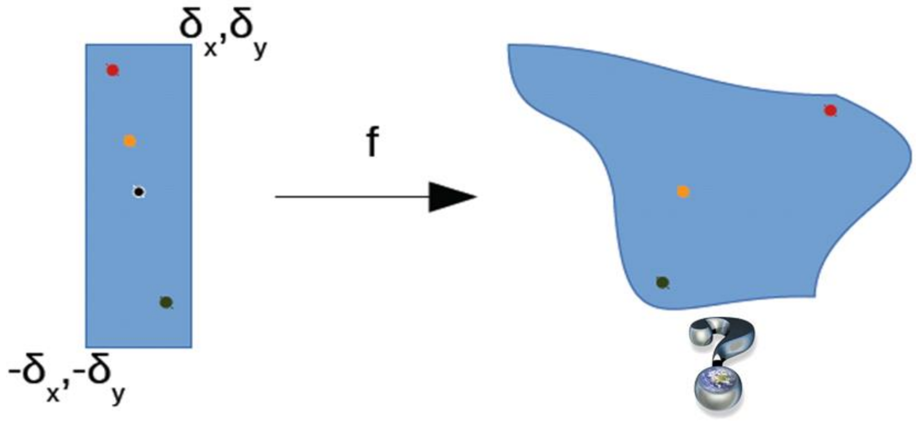
Today's presentation limited to two example for atmospheric re-entry

- **2018 - … : R&T action with Thales Services and INRIA**

  Kriging methods

Taylor Differential Algebra

- **Main TDA application : image of an initial domain (uncertainty…) by f**



- **TDA is:**
  - **An intrusive method: source code has to be rewritten**
  - **A local method : the solution is "expanded" around a reference point (Taylor Development…)**

$$f(x) = f(x_0) + \nabla f(x_0)^T (x - x_0) + (x - x_0)^T \nabla^2 f(x_0) (x - x_0)^T$$

TDA : basic understanding

- Classic formulation

$$\begin{cases} \dfrac{dx}{dt} = f(t, x(t)) \\ x(t_0) = x_0 \end{cases}$$

- Replace the initial condition by a first order polynomial of the state x:

$$X_0(x) = x_0 + (x - x_0)$$

Constant part

Polynomial part

- Define basic operators (+,-,*, /) for polynomials
- Rewrite complex functions (exp, sin, cos) using Taylor development

$$f(x) = \sum_{k=0}^{N} D^k f(x_0)(x - x_0)^k + o(\|x - x_0\|)^N$$

- After integration of the ODE, the result is a polynomial of the state x

$$X_f(x) = x_f + p_1(x - x_0) + p_2(x - x_0)^2 + \ldots$$

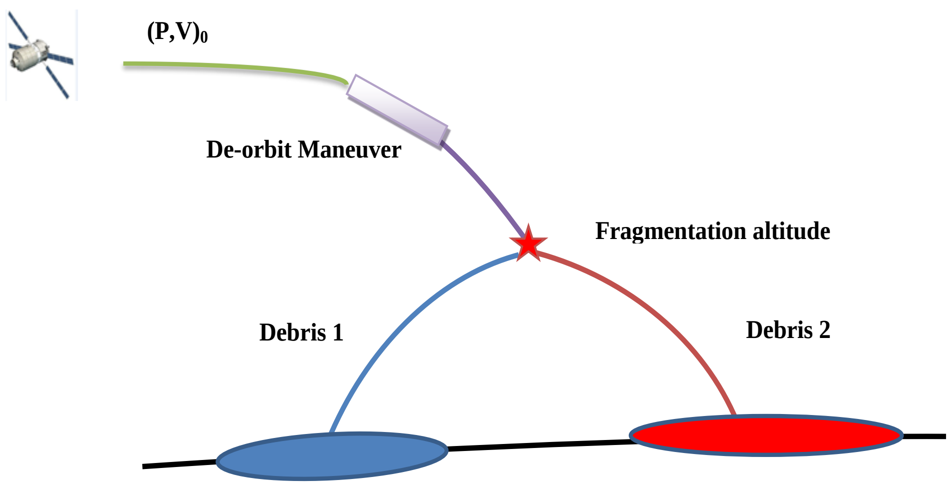
Constant part

Polynomial part (up to order N)

→ To get the results corresponding to another initial condition x1 one has just to evaluate the final polynomial : Xf(x1)

ELECTRA Tool

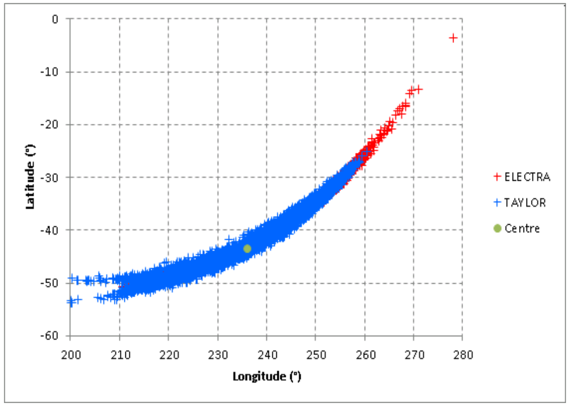
(Estimation de la Létalité due aux Evénements Catastrophiques sur Trajectoires Rentrant dans l'Atmosphère)



ODE's:

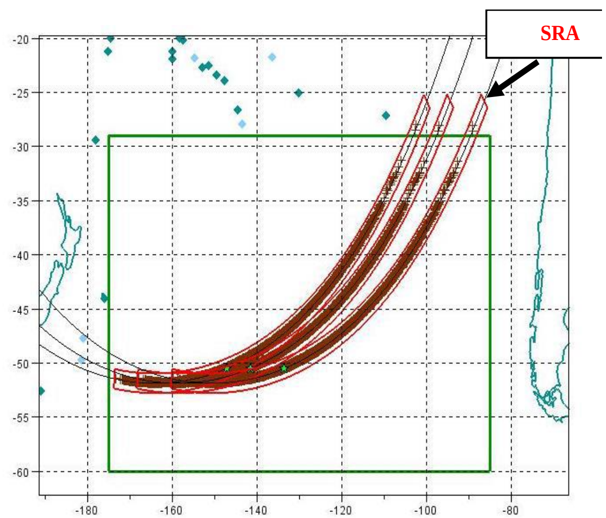
$$\ddot{x} = f(x, \dot{x}, p(t))$$

Numeric integration: RK6, DOPRI8 …



- Security Reentry Area

- Casualty risk

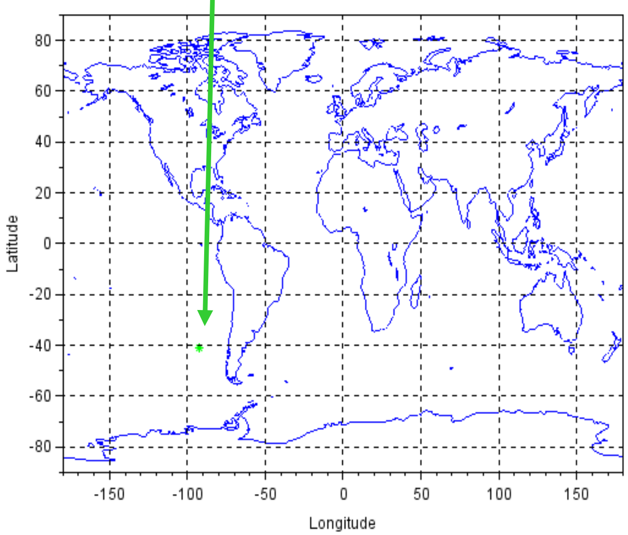


SWOT satellite test case

We retrieve the 4th order polynomials describing the position of the impact point:

$$\begin{cases} longitude = long_0 + \sum_i c_i(\varepsilon_1 - \varepsilon_{1,0})^{p_{i1}}(\varepsilon_2 - \varepsilon_{2,0})^{p_{i2}} \dots (\varepsilon_{12} - \varepsilon_{12,0})^{p_{i12}} \\ latitude = lat_0 + \sum_i c_i(\varepsilon_1 - \varepsilon_{1,0})^{p_{i1}}(\varepsilon_2 - \varepsilon_{2,0})^{p_{i2}} \dots (\varepsilon_{12} - \varepsilon_{12,0})^{p_{i12}} \\ altitude = 50 \end{cases}$$

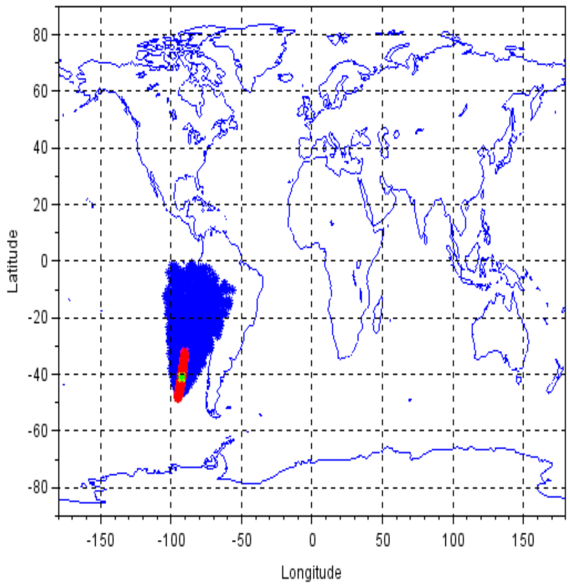
$Long_0$: Impact longitude without dispersion
Lat_0 : Impact latitude without dispersion
ε_k : dispersed parameters (12 of them)
$\varepsilon_{k,0}$: nominal value of dispersed parameters
ci: 1820 coefficients for longitude, same for latitude



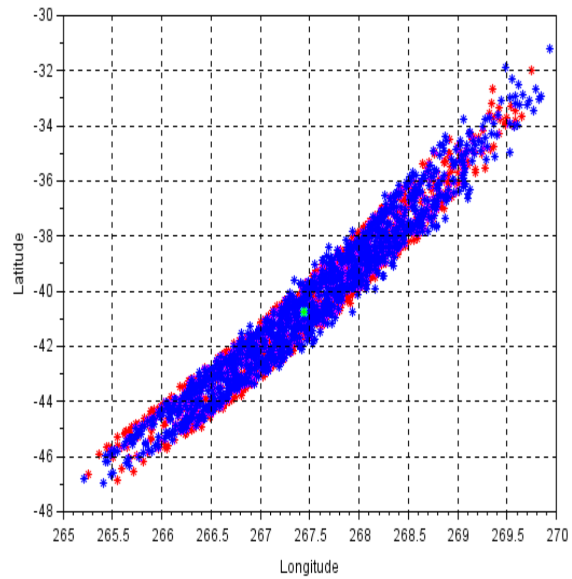
Dispersed parameters εk	Nominal values	Gaussian or Uniform law	Deviation
Initial Position of the falling object	200 – 980 km orbit	Gaussian	On position and velocity: σ*√3 with σ = 5 m and σ = 5 cm/s
De-orbit maneuver: thrust level	60 N	Uniform	± 8 %
De-orbit maneuver: thrust orientation	Opposite to velocity	Gaussian	3° in the direction and 3° transversal to commanded ΔV
Altitude fragmentation	75 km	Uniform	± 5 km
Drag factor	1	Uniform	± 50 %
Lift orientation (Roll)	0.	Uniform	± 180°

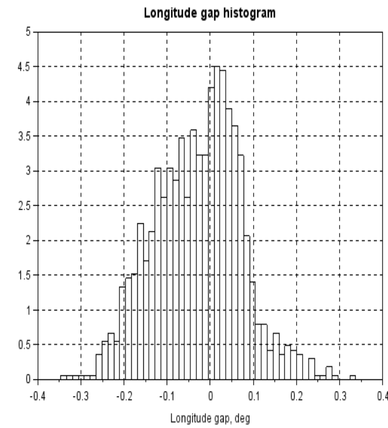
SWOT satellite test case : Roll dispersion

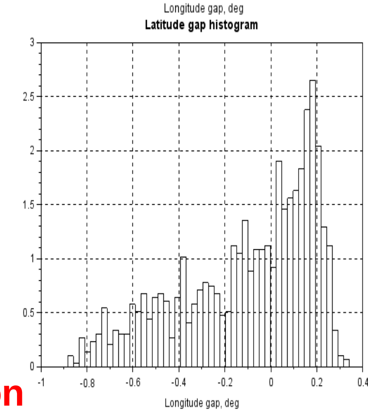
360° roll dispersion



Reduced roll dispersion (+/-45°)







- → **Computation of the polynomial 30 times faster than a 20000 Electra MC**
- → **Analysis of coefficients give "ANOVA-like" information**
- → **Singularity in the attitude law makes TDA not very well suited to this application (Further study could solve the issue)**
- → **At this time, TDA is being studied for "simpler" applications (Perturbed Lambert problem, covariance propagation…)**

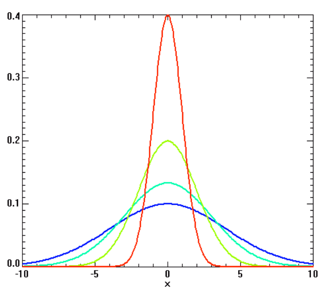
Polynomial Chaos Expansion (PCE)

- ❑ No “Chaos“ here
  - ❑ **Evolution of an initial domain**
  - ❑ **Problem is assumed to be deterministic (same inputs → same outputs)**
  - ❑ **Ouput is modeled as a multivariate polynomials**

Distribution de ξ	Polynôme Ψ	Support
Gaussien	Hermite	$(-\infty, +\infty)$
Gamma	Laguerre	$[0, \infty)$
Beta	Jacobi	$[a, b]$
Uniforme	Legendre	$[a, b]$
Poisson	Charlier	$\{0, 1, \dots\}$
Binomiale	Krawtchouk	$\{0, 1, \dots, n\}$
Binomiale Négative	Meixner	$\{0, 1, \dots\}$
Hypergéométrique	Hahn	$\{0, 1, \dots, n\}$

❑ Non intrusive method

Input



→

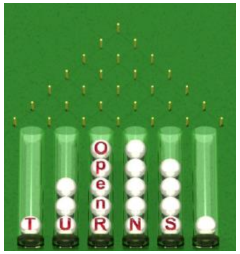
Orbit propagation
Collision risk estimation
Casualty area computation
…Today: DEBRISK

→

Output

$$res(x) = \sum_{k=1}^{N} c_k \Psi_k(x)$$

- ❑ **Global method (polynomial surrogate over a domain)**
- ❑ **Widely used in the industry (EDF, Total…)**
- ❑ **Strong mathematical basis**
- ❑ **Existing competences and toolbox**



Debrisk test cases

- ❑ **Monte Carlo simulation as reference (5000 samples)**
- ❑ **PCE computation (~ 400 samples, expansion order selected)**

	i [°]	Ω [°]	ω [°]	M [°]	h_{frag} [km]	C_D [−]	C_ρ [−]
Delta-II	$\mathcal{U}(95.6, 97.6)$	$\mathcal{U}(0, 360)$	$\mathcal{U}(0, 360)$	$\mathcal{U}(0, 360)$	$\mathcal{U}(73, 78)$	$\mathcal{U}(0.6, 1.4)$	-
Sobol Indexes Validation	$\mathcal{U}(0, 180)$	-	-	-	$\mathcal{U}(73, 78)$	$\mathcal{U}(0.6, 1.4)$	$\mathcal{U}(0.6, 1.4)$
Cas Satellite	$\mathcal{U}(44, 46)$	-	-	$\mathcal{U}(0, 360)$	$\mathcal{U}(75.5, 80.5)$	$\mathcal{U}(0.6, 1.4)$	$\mathcal{U}(0.6, 1.4)$

- ❑ **US 76 atmospheric model is used for DELTAII/Satellite (on going simulation with MSIS model)**

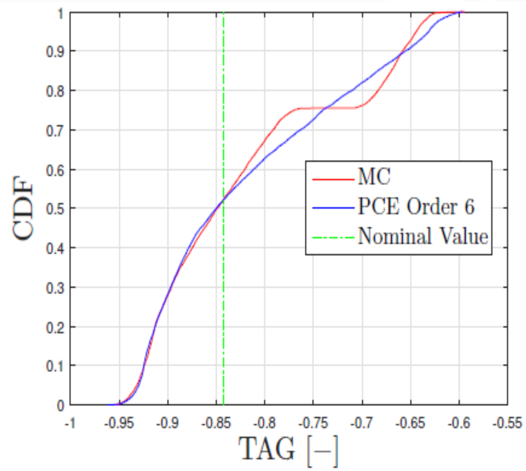
- ❑ **We are interested in:**
  - ❑ **Global ablation rate (TAG in French)**
  - ❑ **Total casualty area**
  - ❑ **Knowing what are the main contributors : ANOVA**

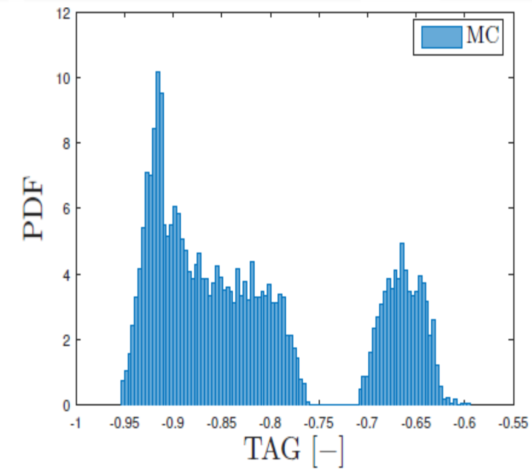
$$\mathrm{TAG} = \frac{\sum_{i=1}^{N} h_{fin_i}}{N\, h_{frag}} - \frac{\sum_{i=1}^{N} m_{g_i}}{\sum_{i=1}^{N} m_{b_i}}$$

$$\mathrm{CA} = \sum_{i=1}^{N} \mathrm{CA}_i$$

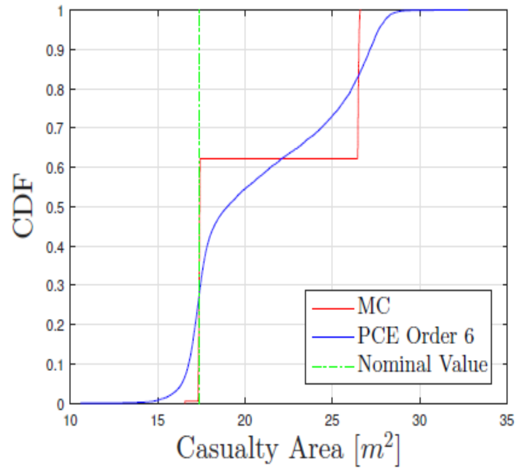
Delta II test case

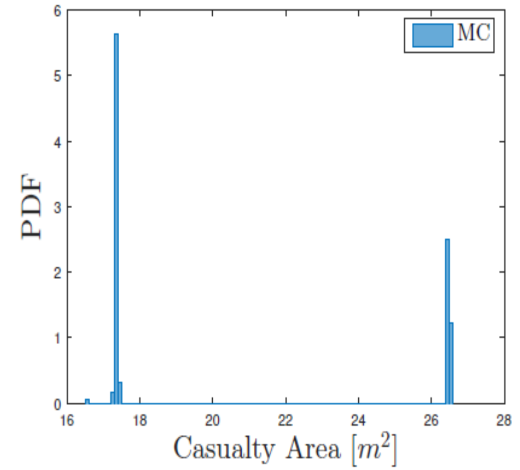
- **Global ablation rate**
  - Bi-modal distribution
  - Nice PCE approximation but limited due to the bi-modal distribution





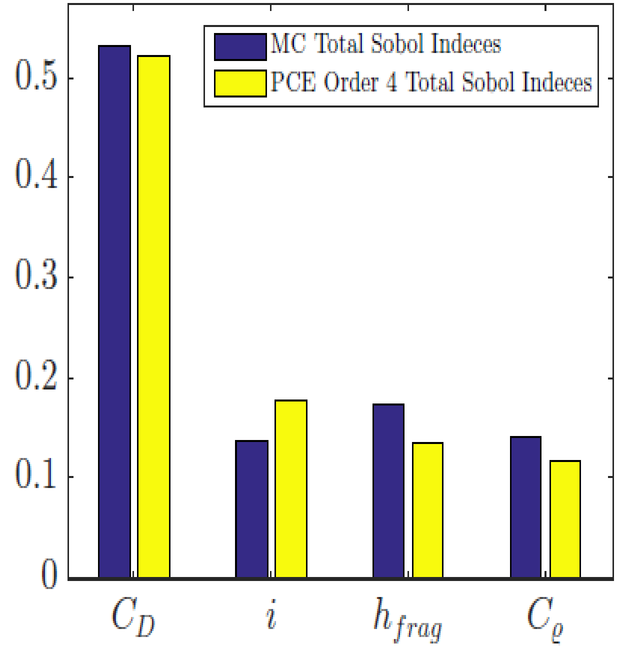
- **Casualty area**
  - Discrete results
  - PCE is not appropriate





Sobol indexes validation

- ❑Analyzed for global ablation rate
- ❑Order 4 PCE
- ❑Sobol index computation is “free” once the PCE has been computed
- ❑MSIS model used

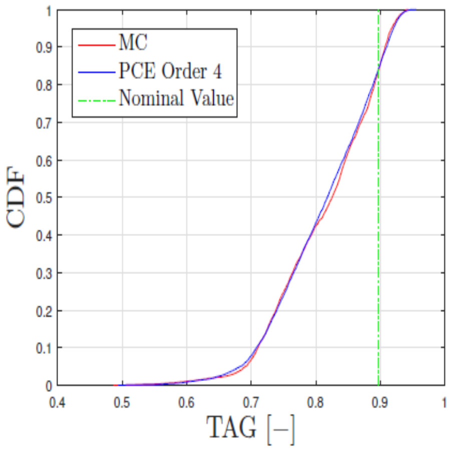


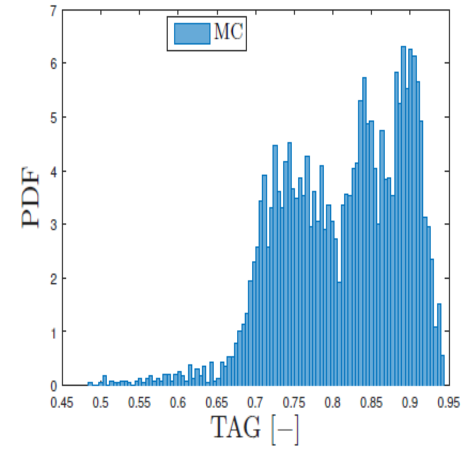
Polynomial Chaos Expansion allows efficient ANOVA

Small differences on the numerical values but the order of magnitude are obtained

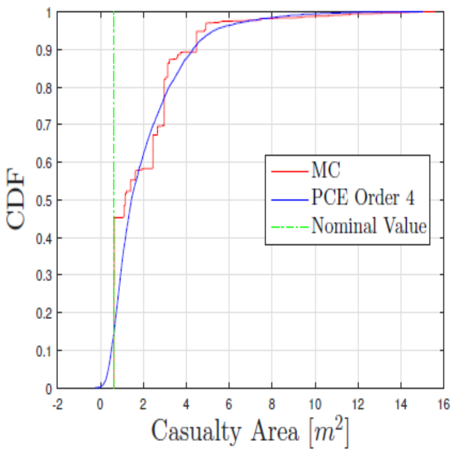
Satellite test case

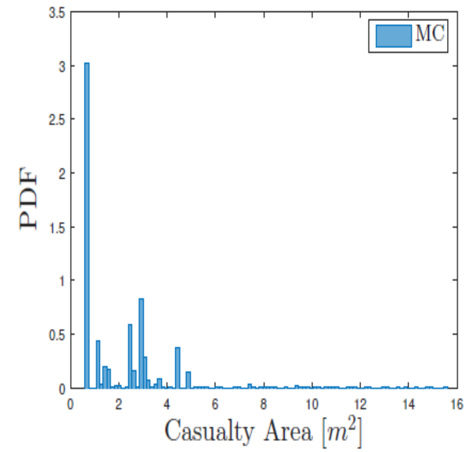
- ❑ **Global ablation rate**
  - ❑ Great PCE approximation





- ❑ **Casualty area**
  - ❑ PCE give a “smooth” distribution function



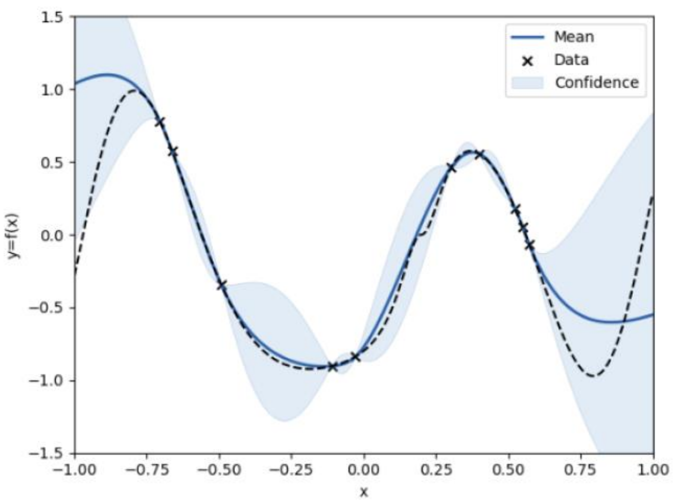


→ PCE is interesting to get the shape of the CDF faster than through Monte Carlo

Kriging Methods

- **On going action**
  - Interpolation method
  - Estimate of the error of the surrogate model
  - To be tested on ELECTRA and DEBRISK

Toblers 1st Law of Geography: *Everything is related to everything else, but near things are more related than distant things !*



Conclusions

- **Several uncertainty propagation methods are under evaluation**
  - TDA: Intrusive local method, maybe not the most suited one for complex atmospheric re-entry code
  - PCE : non-intrusive global method, suited for smooth non-discrete problems, very efficient for ANOVA Analysis
  - Kriging methods: to be evaluated, interpolation methods with an estimate of the error of the surrogate model

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