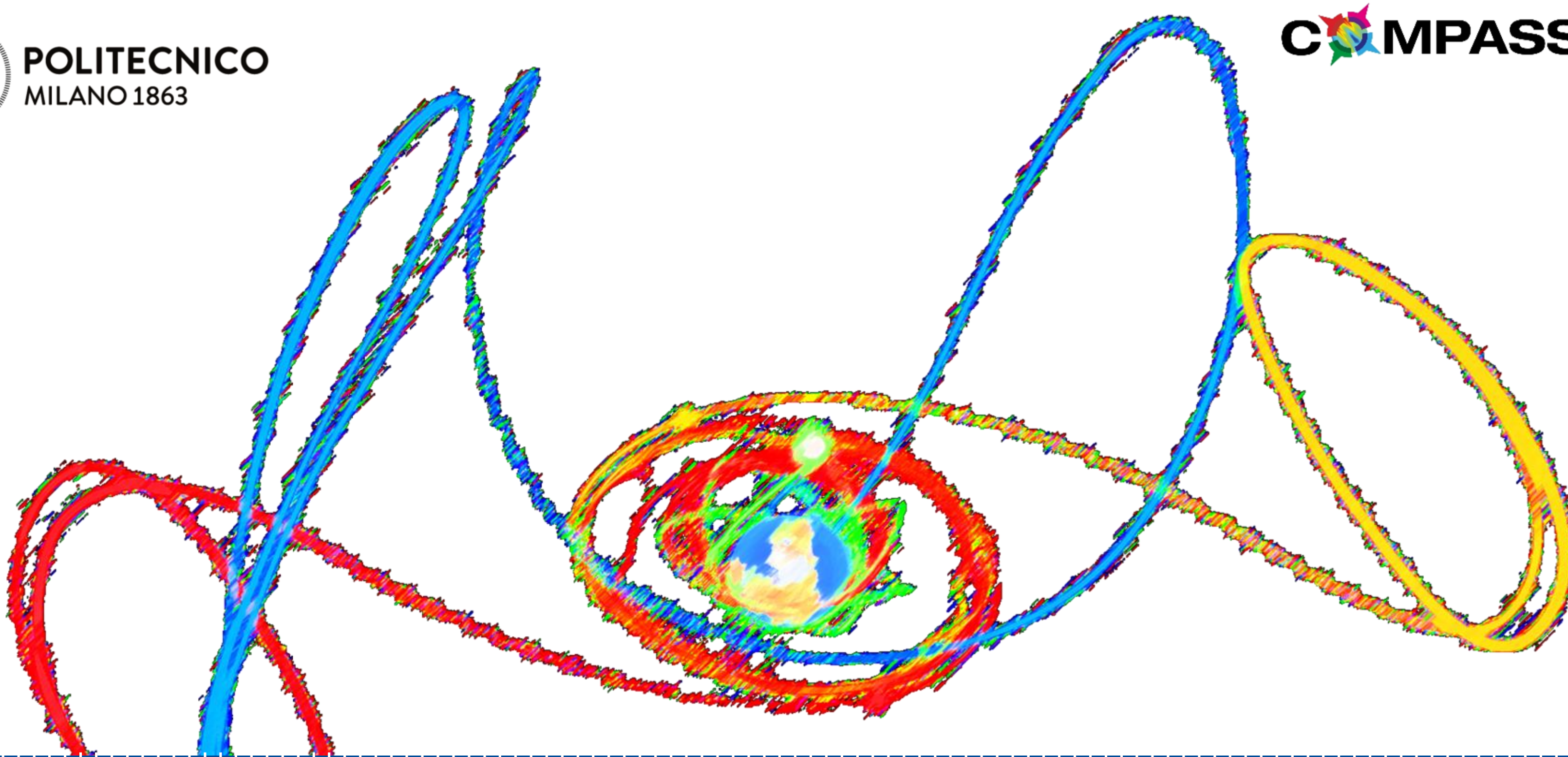


Extension of the King-Hele orbital contraction method and application to the geostationary transfer orbit re-entry prediction

Stefan Frey, Camilla Colombo, David Gondelach, Roberto Armellin

4th International Workshop on Space Debris Re-entry

ESOC, Darmstadt, 28 February 2018

INTRODUCTION

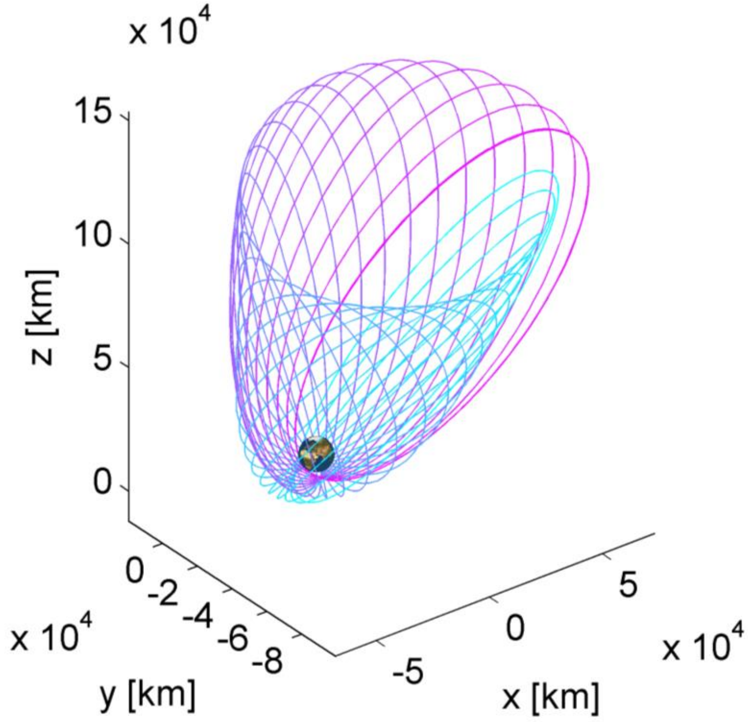
Introduction

Re-entry prediction and precise orbit propagation are a challenging task

- Complex dynamics of orbit perturbations
- Uncertainties related to spacecraft parameters and atmosphere

Semi-analytical techniques can be used:

- Reduce computational time
  - Sensitivity analysis (many initial conditions)
  - Zero-find algorithm for determination
  - Optimisation of disposal manoeuvres
  - Propagation of fragment clouds
- Give accuracy comparable with high fidelity dynamics if model is properly derived

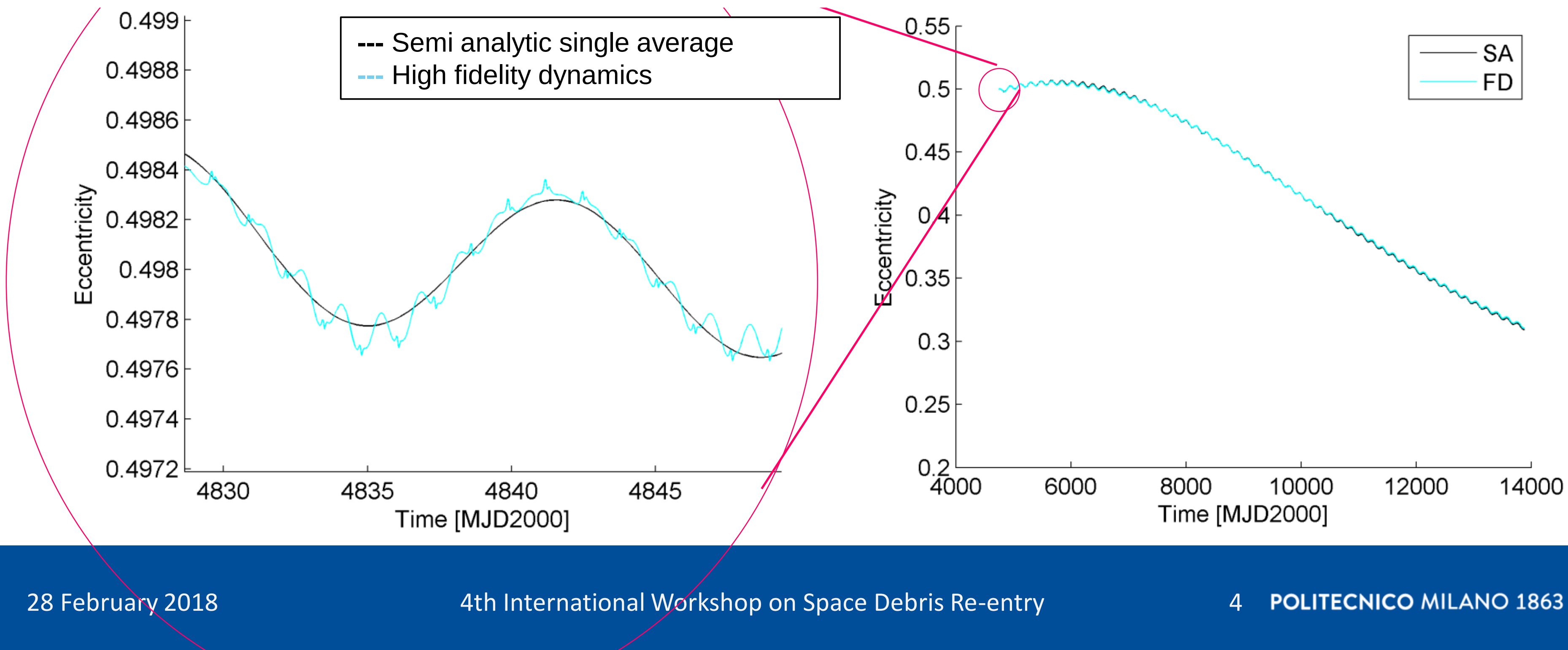


Planetary Orbital Dynamics

Why averaged dynamics

Average variation of orbital elements over one orbit revolution

- Filter high frequency oscillations
- Reduce stiffness of the problem
- Decrease computational time for long term integration



Planetary Orbital Dynamics

PlanODyn suite

Space Debris Evolution, Collision risk, and Mitigation
FP7/EU Marie Curie grant 302270

End-Of-Life Disposal Concepts for Lagrange-Point, Highly Elliptical Orbit missions, **ESA GSP**

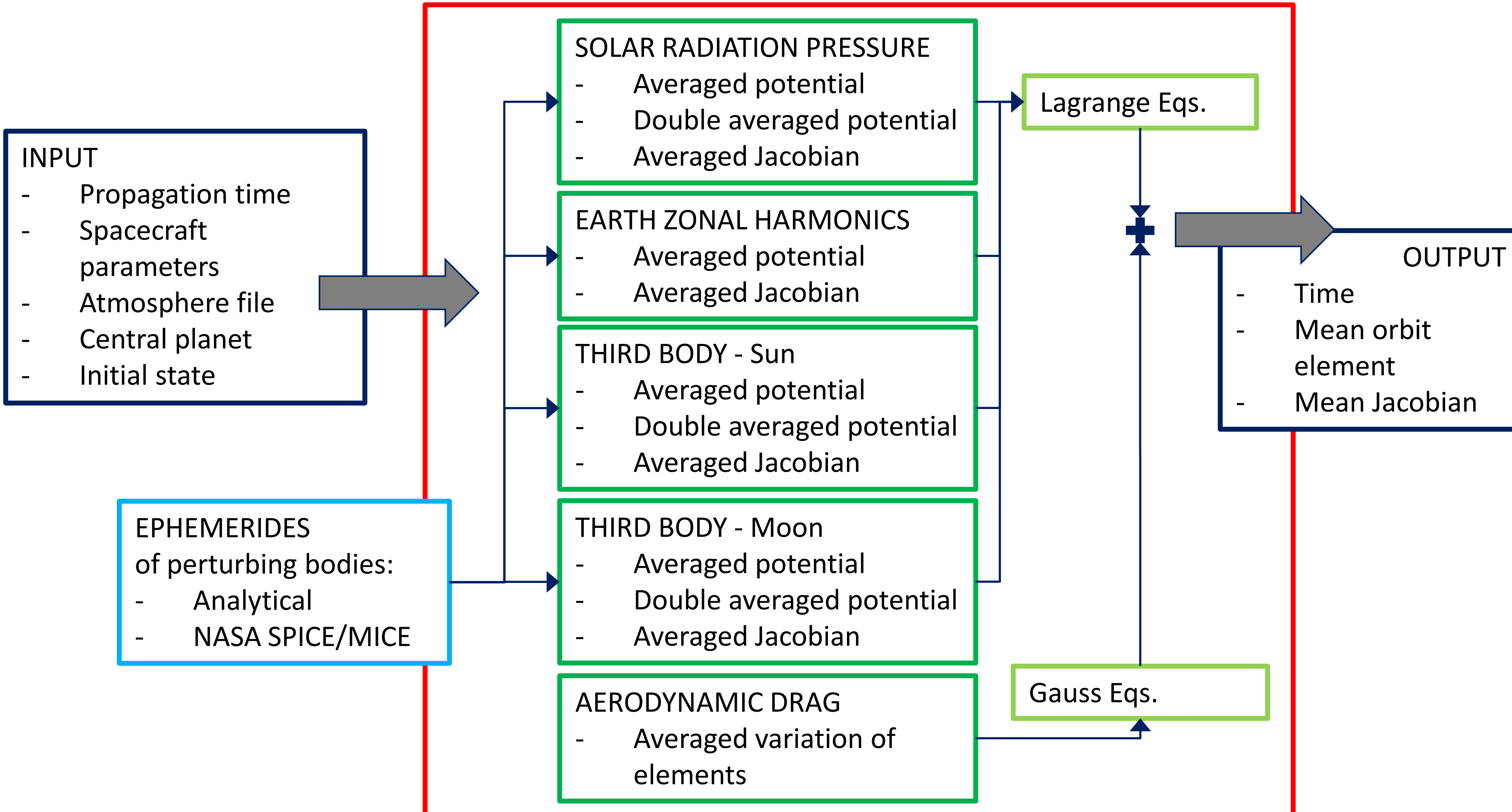
End-Of-Life Disposal Concepts Medium Earth Orbit missions, **ESA GSP**

EOL disposal in “Revolutionary Design of Spacecraft through Holistic Integration of Future Technologies” **ReDSHIFT, H2020**

COMPASS, ERC “Control for orbit manoeuvring through perturbations for supplication to space systems”

Planetary Orbital Dynamics

PlanODyn: Planetary Orbital Dynamics



► *Colombo C., "Planetary Orbital Dynamics Suite for Long Term Propagation in Perturbed Environment," ICATT, ESA/ESOC, 2016.*

Planetary Orbital Dynamics

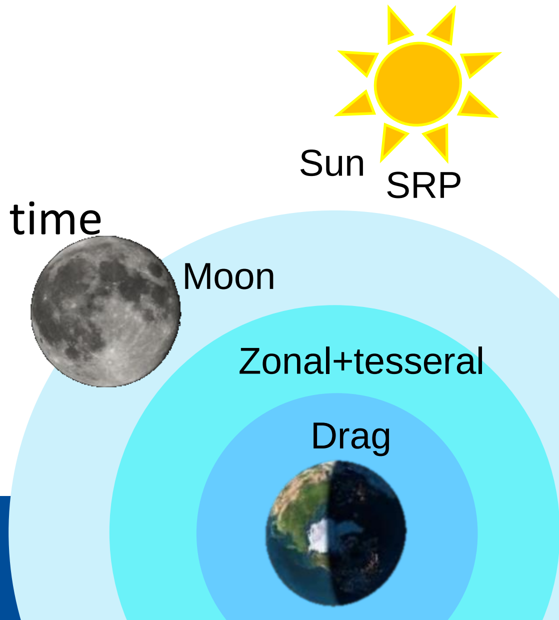
Perturbation in planet centred dynamics

- Atmospheric drag
  - Non-spherical smooth exponential model
  - J_2 short period coupling
- Earth gravity potential
  - Zonal up to order 6 with J_2^2 contribution
  - Tesseral resonant terms
- Solar radiation pressure with cannonball model
- Third body perturbation of the third body (Moon and Sun) up to order 5 in the parallax factor

Ephemerides options

- Analytical approximation based on polynomial expansion in time
- Numerical ephemerides through the NASA SPICE toolkit

Orbital elements in planet centred frame



Planetary Orbital Dynamics

Orbit propagation based on averaged dynamics

For conservative orbit perturbation effects

Disturbing potential function

$$R = R_{\text{SRP}} + R_{\text{zonal}} + R_{3-\text{Sun}} + R_{3-\text{Moon}}$$

Planetary equations in Lagrange form

$$\frac{d\boldsymbol{\alpha}}{dt} = f\left(\boldsymbol{\alpha}, \frac{\partial R}{\partial \boldsymbol{\alpha}}\right) \qquad \boldsymbol{\alpha} = \begin{bmatrix} a & e & i & \Omega & \omega & M \end{bmatrix}^T$$

<u>Average</u> over one orbit revolution of the spacecraft around the primary planet

$$\bar{R} = \bar{R}_{\text{SRP}} + \bar{R}_{\text{zonal}} + \bar{R}_{3-\text{Sun}} + \bar{R}_{3-\text{Moon}}$$

$$\frac{d\bar{\boldsymbol{\alpha}}}{dt} = f\left(\bar{\boldsymbol{\alpha}}, \frac{\partial \bar{R}}{\partial \bar{\boldsymbol{\alpha}}}\right)$$

Single average

<u>Average</u> over the revolution of the perturbing body around the primary planet

$$\bar{\bar{R}} = \bar{\bar{R}}_{\text{SRP}} + \bar{R}_{\text{zonal}} + \bar{\bar{R}}_{3-\text{Sun}} + \bar{\bar{R}}_{3-\text{Moon}}$$

$$\frac{d\bar{\bar{\boldsymbol{\alpha}}}}{dt} = f\left(\bar{\bar{\boldsymbol{\alpha}}}, \frac{\partial \bar{\bar{R}}}{\partial \bar{\bar{\boldsymbol{\alpha}}}}\right)$$

Double average

Dynamical model

Third body potential

- Series expansion of third body potential around $\delta = a/r' = 0$
- Expressed as function of orientation of orbit eccentricity vector and semi-latus rectum vector with respect to third body

$$R_{3B}(r, r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k F_k(A, B, e, E)$$

μ' gravitational coefficient third body

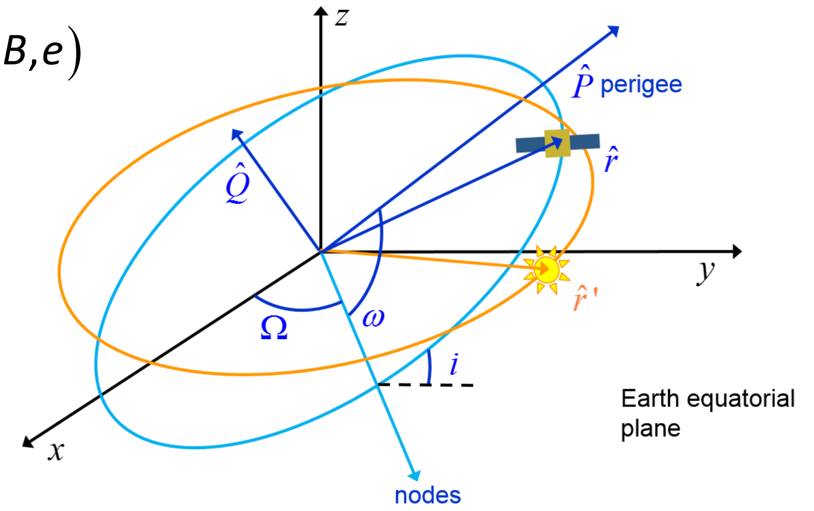
$\mathbf{r'}$ position vector of third body

E eccentric anomaly

- Average over one orbit revolution

$$\bar{R}_{3B}(r, r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k \bar{F}_k(A, B, e)$$

- Calculate partial derivatives for Lagrange equations

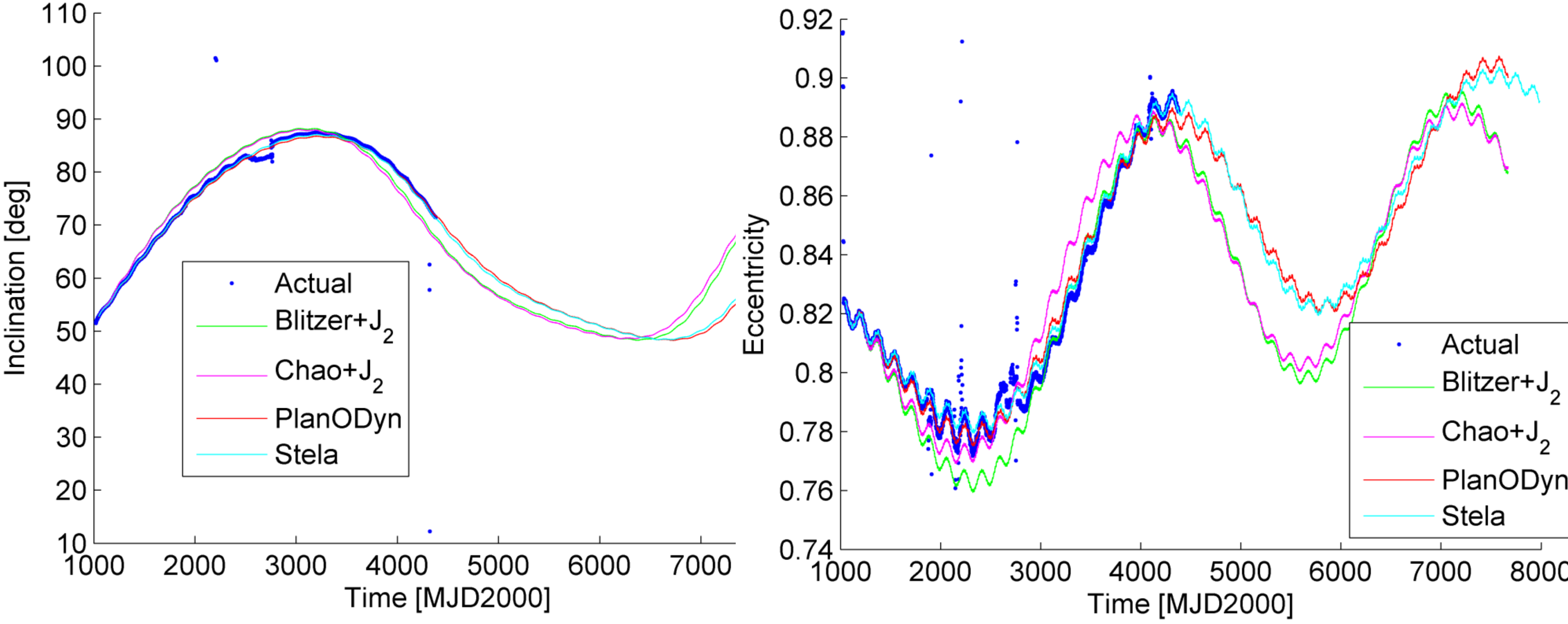


► *Kaufman and Dasenbrock, NASA report, 1979*

Dynamical model

Order of the luni-solar potential expansion

For HEO third-body perturbing potential of the Moon at least up to the fourth order of the power expansion



- *Blitzer L., Handbook of Orbital Perturbations, Astronautics, 1970*
- *Chao-Chun G. C., Applied Orbit Perturbation and Maintenance, 2005*

EXTENSION OF KING-HELE ORBITAL CONTRACTION METHOD

Orbital Contraction

Averaging

- Average out fast moving variable (f, E or M), assuming the other mean elements to be fixed

$$\bar{\dot{x}} = \frac{\Delta x}{P} = \frac{1}{P}\int_0^{2\pi} \frac{dx}{dE} dE \qquad x \in [a, e]$$

- The change $\frac{dx}{dE}$ is a function the Keplerian elements, $\boldsymbol{k}$, the density, ρ, at altitude, h, and the effective area-to-mass ratio, $\delta = c_D \frac{A}{m}$

$$\frac{dx}{dE} = f\left(\boldsymbol{k}, \rho\left(h(\boldsymbol{k})\right), \delta\right) \qquad \boldsymbol{k}^{\mathrm{T}} = (\mathrm{a}, \mathrm{e}, \mathrm{i}, \Omega, \omega, E)$$

$$h = h_m + \Delta h_\varepsilon + \Delta h_{J_2}$$

- Δh_{J_2} → Short periodic variation
- Δh_ε → Altitude above ellipsoid variation
- h_m → Mean altitude

Orbital Contraction

Averaging method

- The integrals can be approximated quickly numerically or analytically
  - E.g. *Gauss-Legendre* (GL) quadrature
    - \+ Flexible: can work with any drag model
    - \+ Valid for any eccentricity, i.e. series expansion avoided
    - − Multiple density evaluations (default $\mathrm{N} = 33$)
  - E.g. *King-Hele* (KH) method
    - − Requires exponentially decaying atmosphere model (next slide)
    - − Series expansion in eccentricity (solved for low and high eccentricities by KH)
    - \+ Only one density evaluation
    - \+ Analytical estimation of the Jacobian available
- Both are implemented in *PlanODyn*, with the (Superimposed) King-Hele method as default

➢ *Liu, J. J. F., Alford, R. L., An Introduction to Gauss-Legendre Quadrature, Northrop Services, Inc., 1973.*
➢ *King-Hele, D., Theory of Satellite Orbits in an Atmosphere, London Butterworths, 1964*

Orbital Contraction

Superimposed Atmosphere (ρ_S) and Superimposed King-Hele (SI-KH)

- KH requires atmosphere to decay exponentially
- Fit superimposed partial exponential atmospheres to any desired model

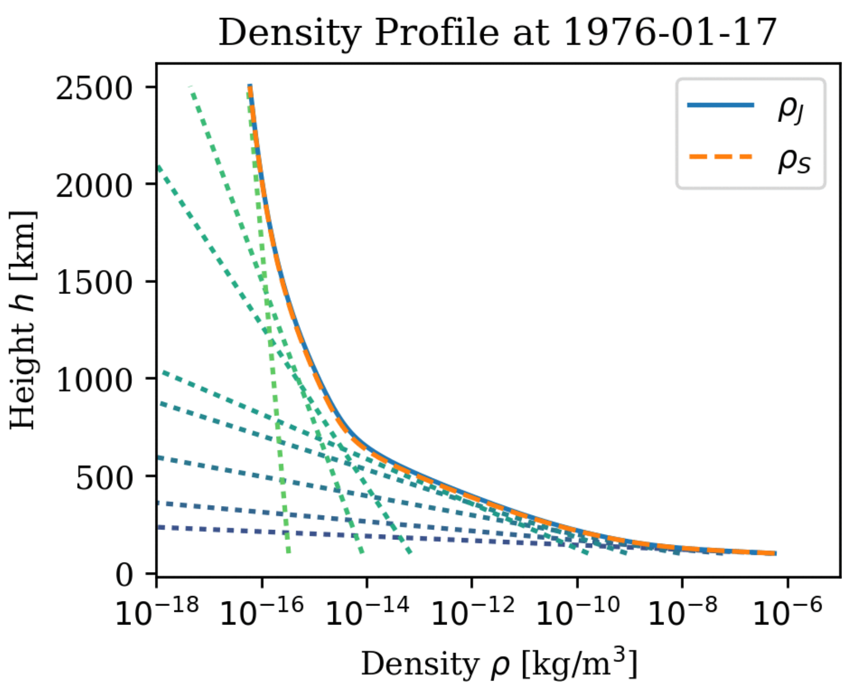
$$\rho_S(h) = \sum_p \rho_{0,p} \exp - \frac{h}{H_p}$$

- Then simply superimposed orbital contractions from KH

$$\Delta a = \sum_p \Delta a_p \qquad \Delta e = \sum_p \Delta e_p$$

- Can include temporal changes

E.g. fit to Jacchia-77, ρ_J

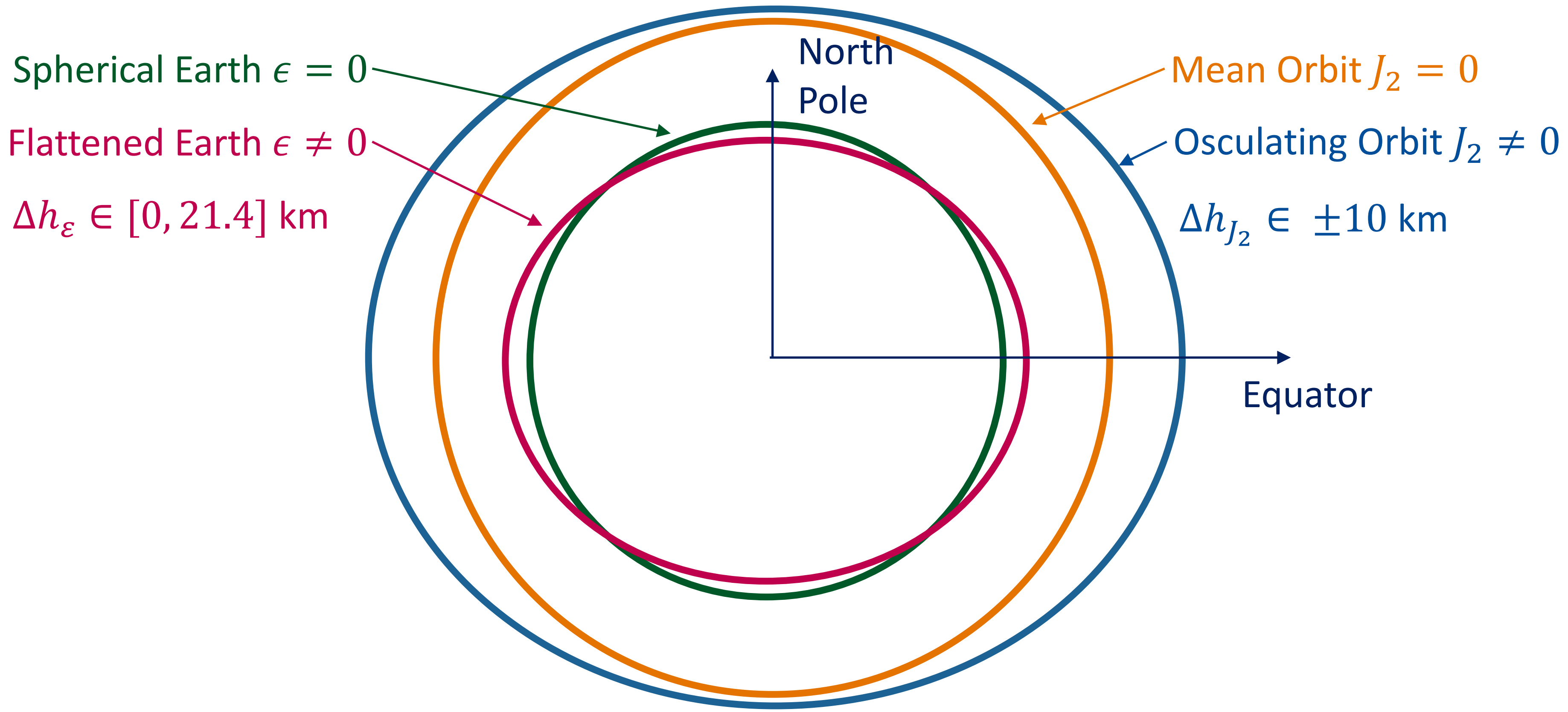


➢ *Jacchia, L. G., Thermospheric temperature, density, and composition: new models. SAO Special Report, 1977.*

Orbital Contraction

Non-Spherical Earth and Atmosphere

- Non-Spherical Atmosphere and coupling of Earth flattening and Drag



Orbital Contraction

Non-Spherical Earth and Atmosphere

- Mean Height

$$h_m = a(1 - e\ \cos E) - R_\oplus$$

- Height above non-spherical Earth surface ($\varepsilon \neq 0$)

$$\Delta h_\varepsilon \approx \varepsilon R_\oplus \sin^2 i \sin^2(\omega + f)$$

- Short periodic variation due to flattening ($J_2 \neq 0$)

$$\Delta h_{J_2} = \frac{J_2\, {R_\oplus}^2}{4a(1-e^2)}\left[\sin^2 i \cos(2(\omega + f)) + (3 \sin^2 i - 2)\left\{1 + \frac{e \cos f}{1+\sqrt{1-e^2}} + \frac{2\sqrt{1-e^2}}{1 + e \cos f}\right\}\right]$$

- During averaging, assume changes divided by scale height to be small

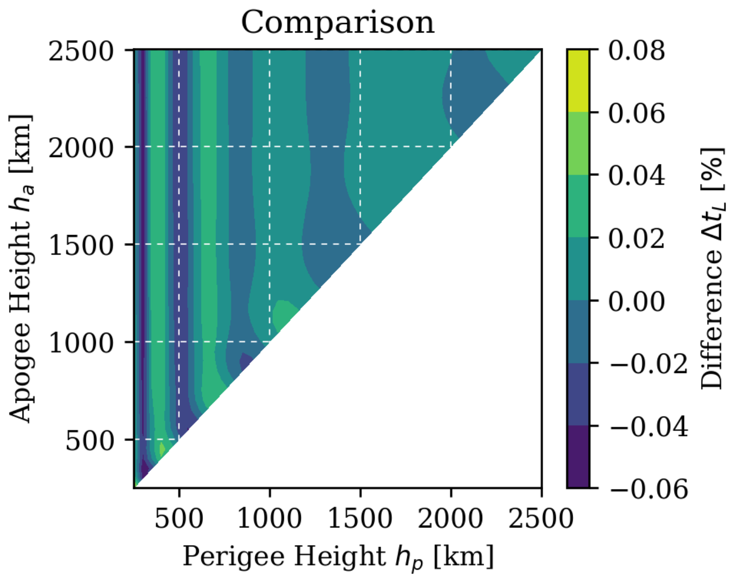
$$\exp\left(\frac{h_m + \Delta h}{H}\right) = \exp\left(\frac{h_m}{H}\right)\exp\left(\frac{\Delta h}{H}\right) \qquad \exp(x) \approx \left[1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots\right]$$

➢ *Liu, J.J.F, Alford, R.L., Semi analytic Theory for a Close-Earth Artificial Satellite. Journal of Guidance and Control, 1980.*

Orbital Contraction

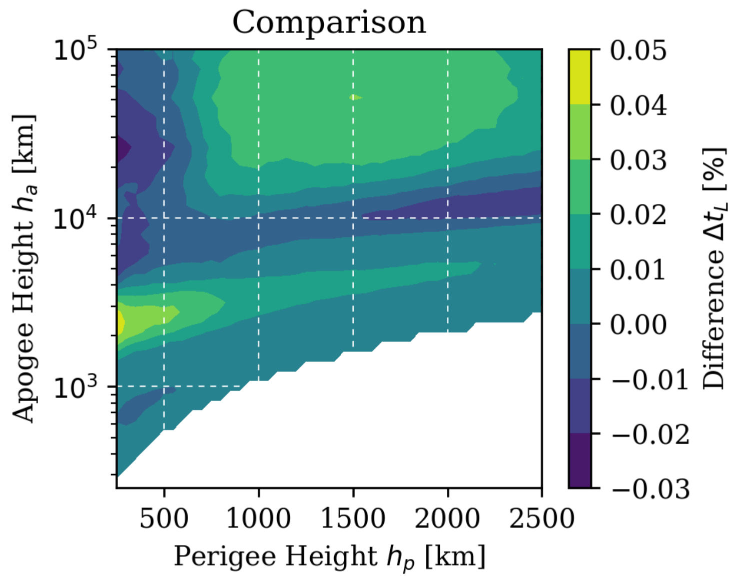
Validation: Spherical Earth, T_∞ fixed

- Comparing ρ_J with ρ_S
- Using GL quadrature
- Area-to-mass ratio $^A/_m = 1$ m²/kg



- Speed increase: 6.4x

- Comparing SI-KH with full numerical integration
- Using ρ_S
- Lifetime of ~1 year

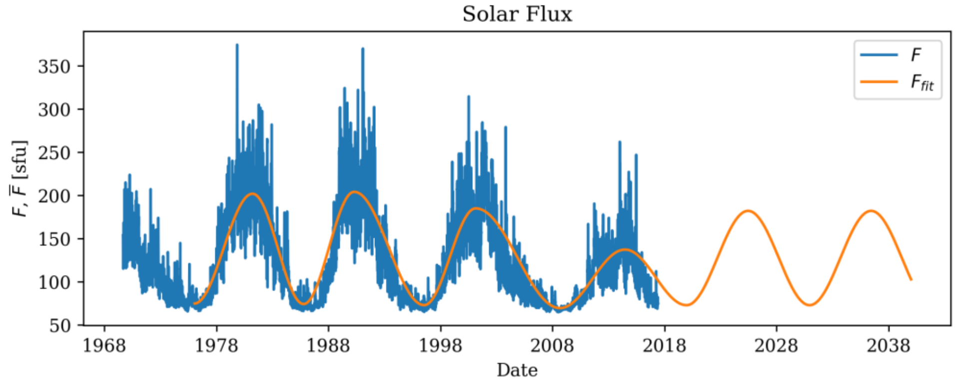


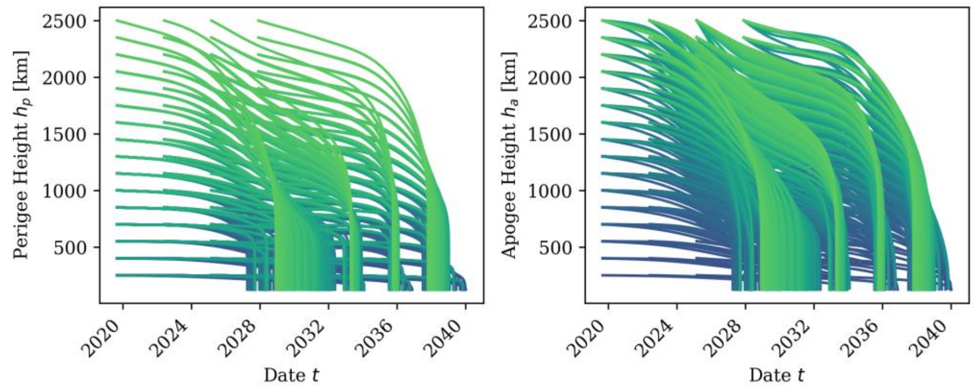
- Speed increase: 560x

Orbital Contraction

Validation: Spherical Earth, T_∞-dependence

- 544 initial conditions:
  - $h_p = 250 - 2500$ km
  - $h_a = 250 - 2500$ km
  - $t_0 = 0$, ¼, ½ and ¾ through predicted future solar cycle 2019-2030
- $^A/_m$ s. t. re-enters ~11 years
- ρ_S /SI-KH vs ρ_J/ GL
- Accuracy: $\frac{t_L(\rho_S/SI\text{−}KH)}{t_L(\rho_J/GL)}$



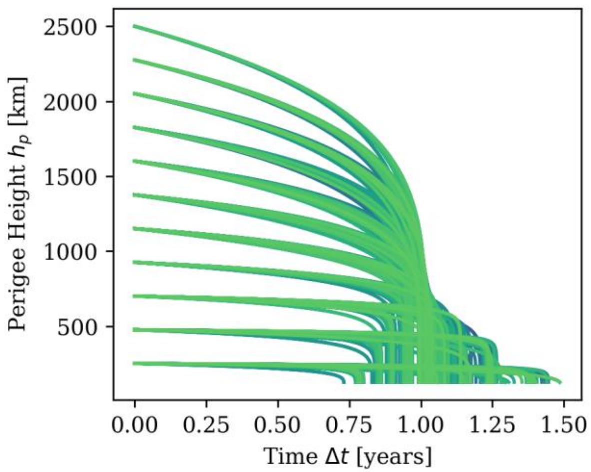


min	$q_{5\%}$	$q_{50\%}$	$q_{95\%}$	max	x CPU
0.9957	0.9987	0.9999	1.0005	1.0012	6.2

Orbital Contraction

Validation: J_2 and ε, T_∞ fixed

- 1092 initial conditions:
  - $h_p = 250 - 2500$ km
  - $h_a = 250 - 2500$ km
  - $i = 1, 45, 63.4, 90°$
  - $\omega = 0, 45, 90°$
- A/m s. t. re-enters ~1 year
- Using ρ_S
- SI-KH(ε, J_2) vs Full numerical



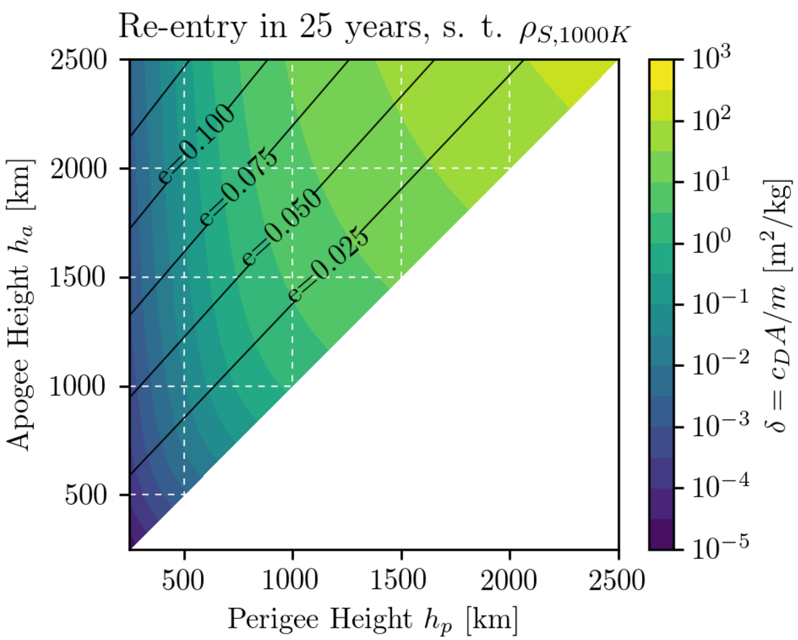
$$\frac{t_L(SI - KH)}{t_L(Full\ Num.)}$$

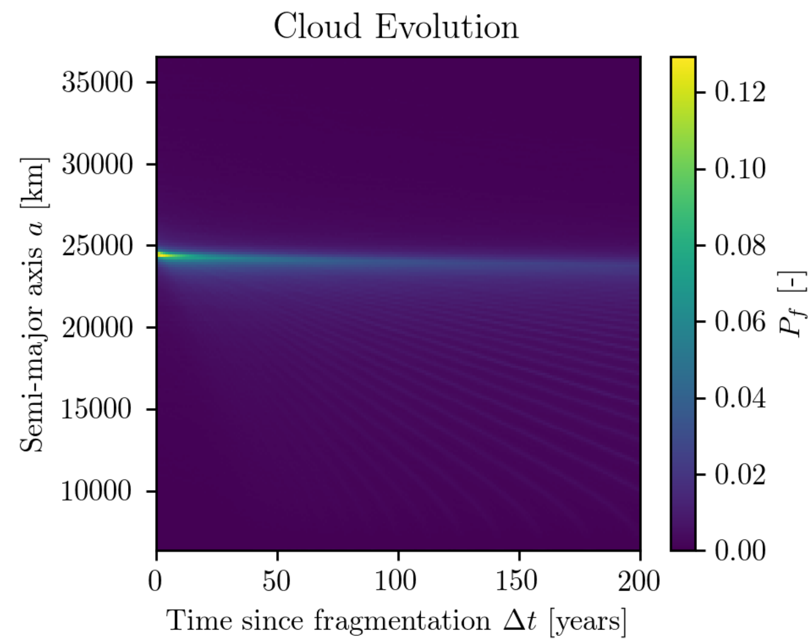
	min	$q_{5\%}$	$q_{50\%}$	$q_{95\%}$	max	x CPU
ε, J_2	0.739	0.858	1.035	1.380	1.531	323
ε, J_2	0.739	0.852	0.998	1.086	1.355	295
ε, J_2	0.996	0.999	1.031	1.256	1.414	320
ε, J_2	0.979	0.999	1.001	1.008	1.032	331

Applications

Drag induced re-entry: two examples

- Maps of effective area-to-mass ratio required for re-entry in x years (optimisation)
- Evolution of clouds of fragments (collision or explosion) or entire space debris population





- *Frey, S., Colombo, C., Lemmens, S., Krag H., Evolution of Fragmentation Cloud in Highly Eccentric Orbit using Representative, Proceedings of the 68th IAC, 2017*

GEOSTATIONARY TRANSFER ORBIT RE-ENTRY PREDICTION

TLE based re-entry prediction

Background

ESA study (DINAMICA, Uni of Southampton, CNRS)

Technology for improving re-entry prediction of European upper stages through dedicated observations, ESA-GSP study ITT 8155, 2015

- TLE-based parameter estimation
  - Develop BC estimation method
  - Develop BC and SRPC estimation method
- TLE based state estimation
  - OD state estimation method

TLE based re-entry prediction

Background

- Ballistic coefficient
  - Estimate depends on:
    - Initial state (perigee height)
    - Force model: Atmosphere model (density)
      Others forces (coupling)
  - B* parameter:

```
1 25496U 98057B   98284.49064516  .00114495 -27097-6  39493-2 0  9997
2 25496 024.9619 183.2533 7300304 180.6546 177.3823 02.29699499    34
```

    - Fitting parameter in TLE
    - Ballistic coefficient from B*: $BC = \frac{2B^*}{\rho_0 R_{\text{Earth}}} = 12.741621\ B^*$

TLE based re-entry prediction

BC estimation method

- BC estimation is based on comparing the change in semi-major axis from the TLE data to the change in semi-major axis computed from accurate orbit propagation between two epochs
  1. Compute the change in semi-major axis between two TLE epochs from the mean motion, n

     $$\Delta a_{TLE} = a_{TLE_2} - a_{TLE_1}$$

  2. Compute the change in semi-major axis between two TLE propagating the object trajectory

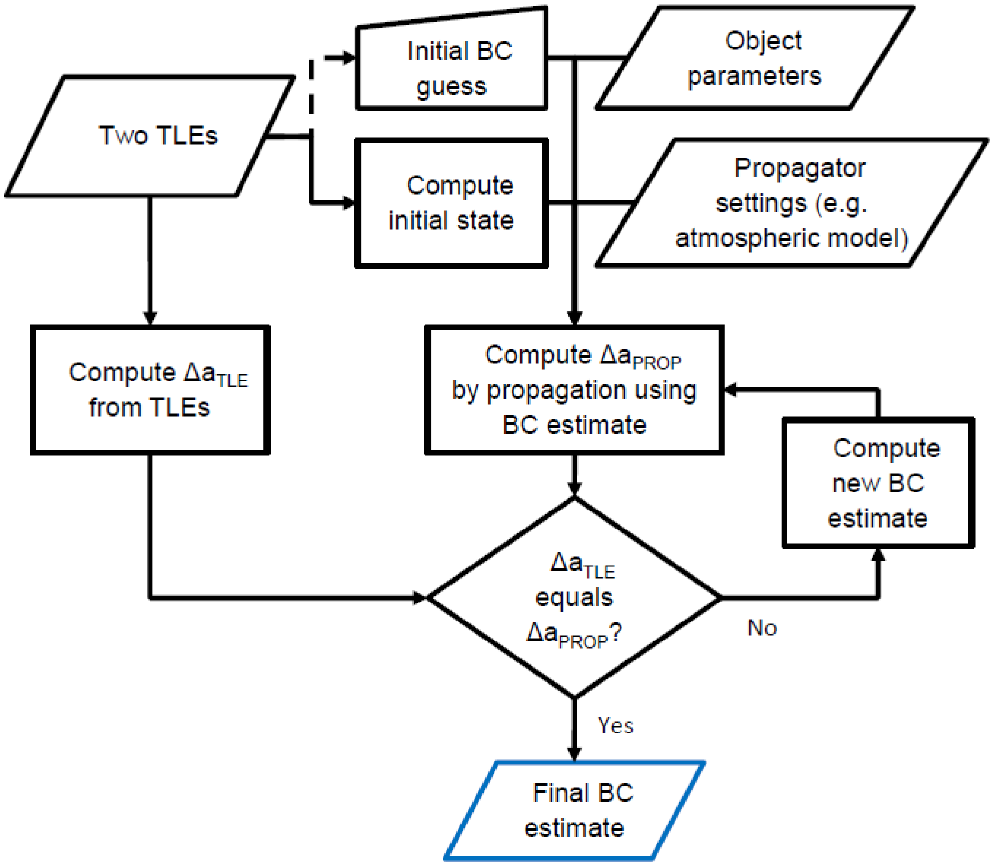
     $$\Delta a_{PROP} = \int_{TLE_1}^{TLE_2} \left(\frac{da}{dt}_{drag} \right) dt = f\left(BC_{guess}\right)$$

  3. Compute BC iteratively such that $\Delta a_{PROP} = \Delta a_{TLE}$
     - Δa_{PROP} is computed using the **average** semi-major axis because Δa_{TLE} is the change in **mean** semi-major axis
     - Δa_{PROP} can be computed by **backward** propagation to avoid re-entry during estimation

➢ *Saunders et al, 2012*

TLE based re-entry prediction

BC estimation method



- *D. J. Gondelach, R. Armellin, and A. A. Lidtke, Ballistic Coefficient Estimation for Re-entry Prediction of Rocket Bodies in Eccentric Orbits Based on TLE Data, Mathematical Problems in Engineering*

TLE based re-entry prediction

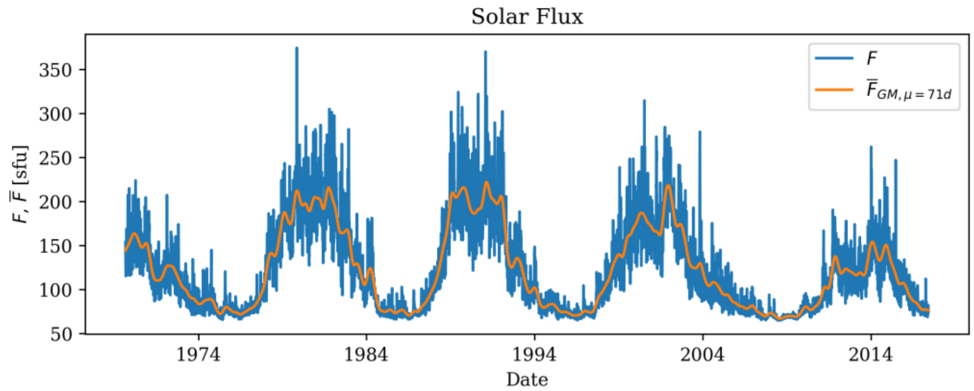
Propagation method: AIDA dynamics

- Geopotential acceleration:
  - EGM2008gravity model up to degree and order 10
- Atmosphere drag:
  - NRLMSISE-00 atmospheric model with updated weather files
  - Rotating atmosphere
- Solar radiation pressure:
  - Earth and/or Moon shadow
  - Cylindrical or biconical shadow
- Moon and Sun perturbations:
  - Moon and Sun ephemeris from NASA's SPICE kernels
  - SPICE toolkit used to time and reference frame transformations

TLE based re-entry prediction

Propagation method: PlanODyn dynamics

- Geopotential acceleration:
  - Zonal harmonics up to order 6
- Atmosphere drag:
  - T_∞-dependent smooth exponential atmosphere model, fit to Jacchia-77
  - Solar flux using Gaussian mean with standard deviation of 3 solar rotations
  - No atmospheric rotation
- Solar radiation pressure:
  - Cannonball model
  - No shadow considered
- Moon and Sun perturbations:
  - Moon and Sun ephemeris from NASA's SPICE kernels
  - Expansion of third body Legendre potential in a/a_3 up to order 5



TLE based re-entry prediction

Results

30 days and 180 days re-entry predictions of 83 and 92 objects to obtain a better understanding

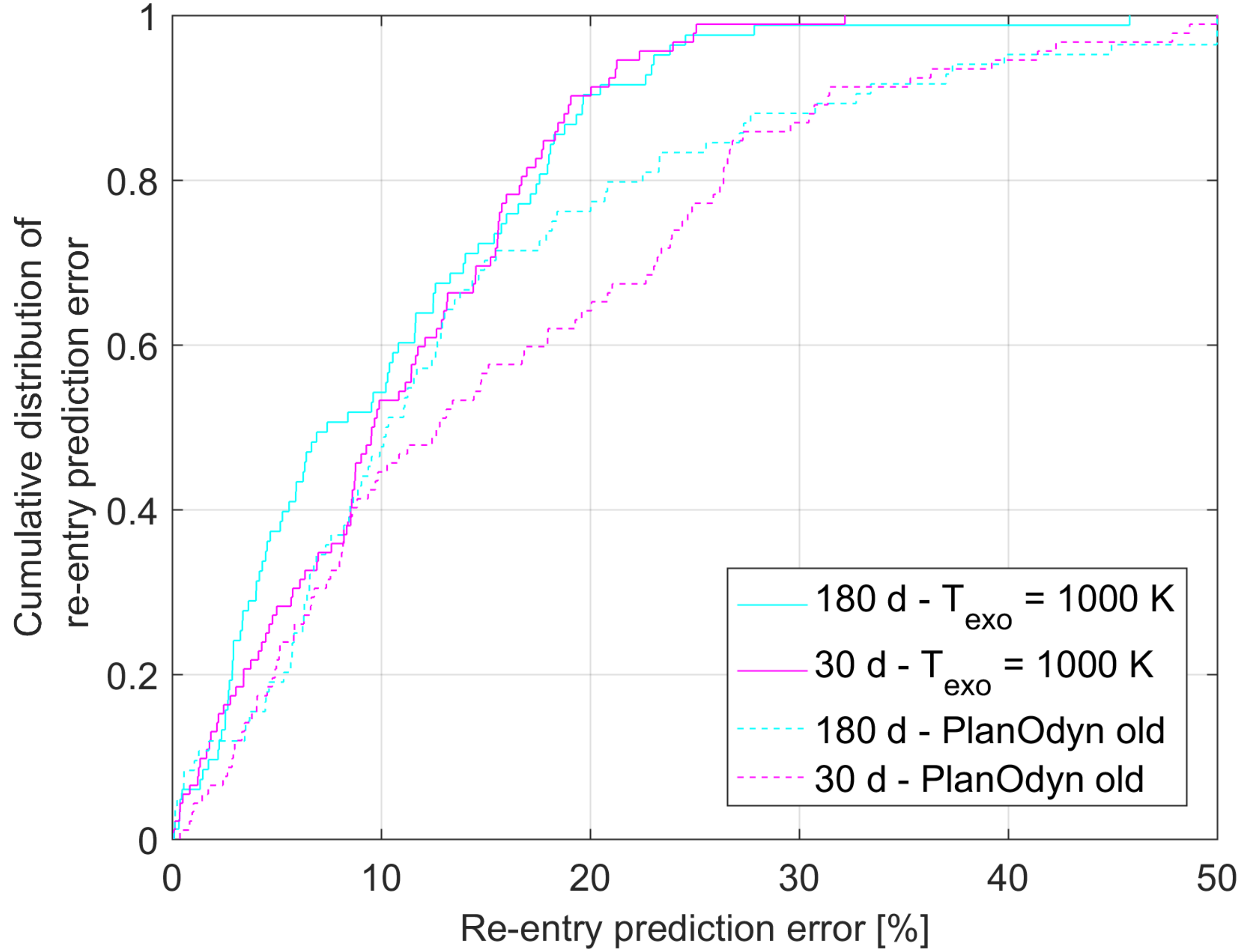
- Re-entry prediction accuracy
- Effect of dynamics

Error computation

$$error[\%] = \frac{t_{predicted} - t_{actual}}{t_{actual} - t_{lastUsedTLE}} \cdot 100$$

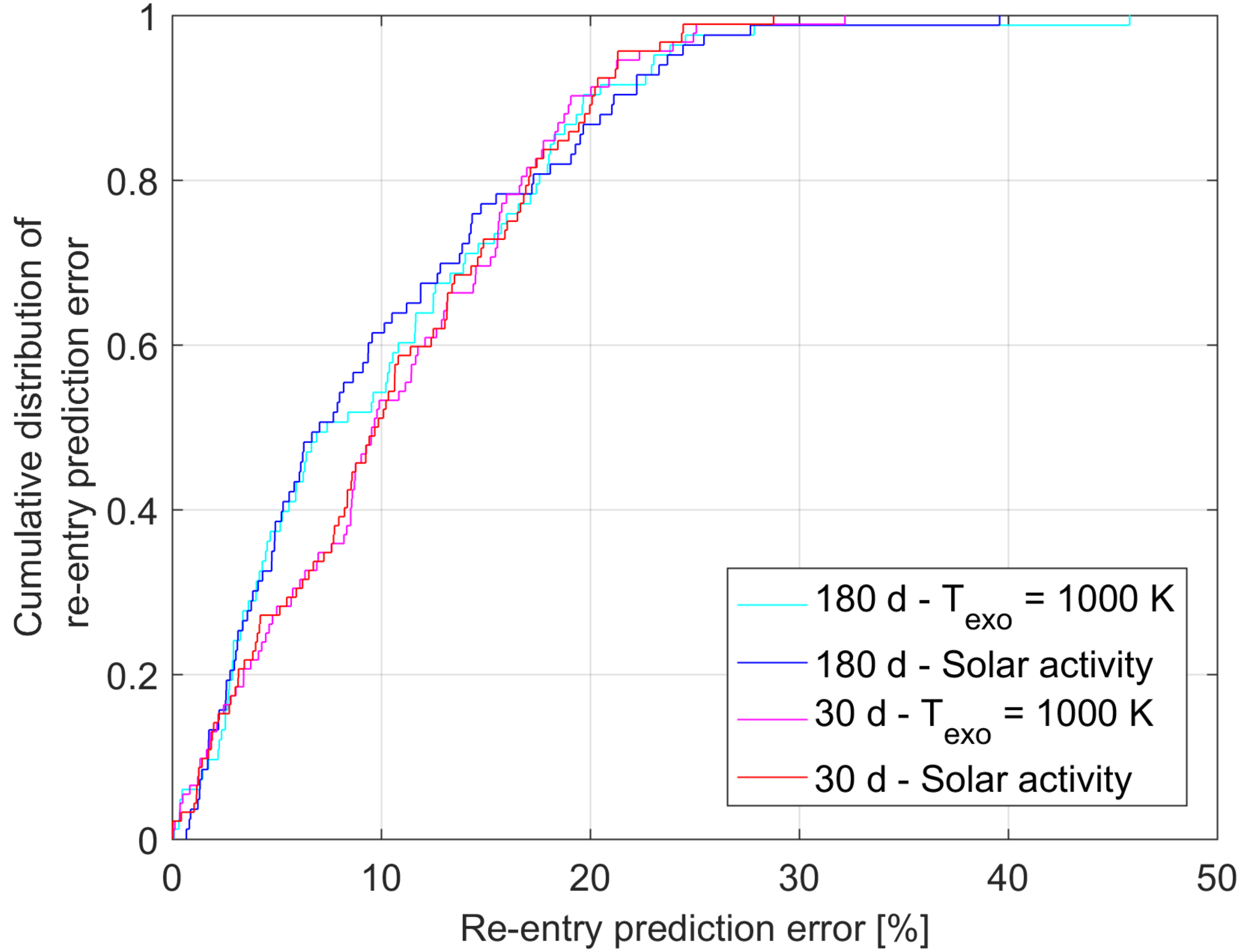
TLE based re-entry prediction

Improvement in the PlanODyn suite



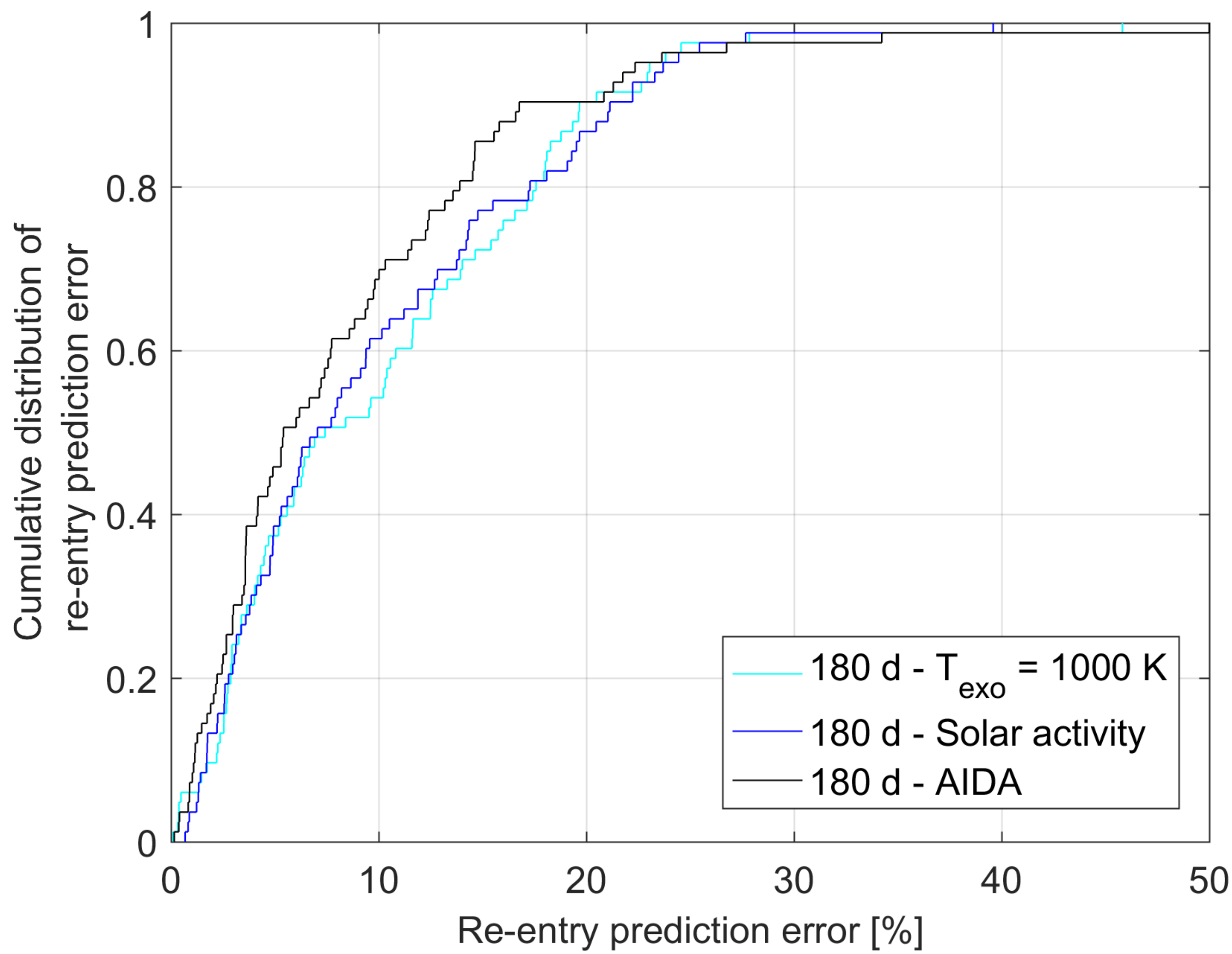
TLE based re-entry prediction

Effect of solar activity



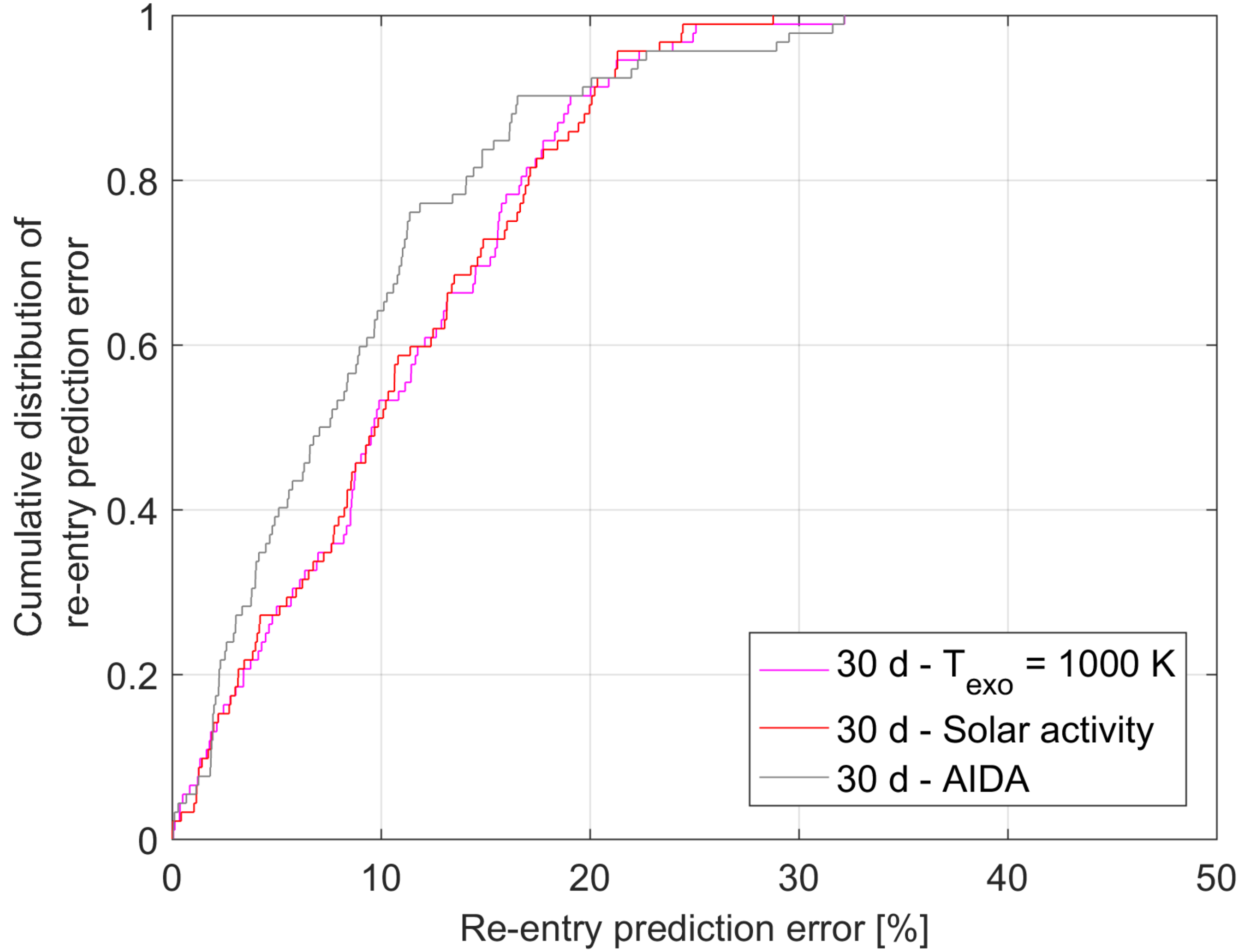
TLE based re-entry prediction

Semi-analytical (PlanODyn) versus high fidelity (AIDA)



TLE based re-entry prediction

Semi-analytical (PlanODyn) versus high fidelity (AIDA)



Conclusions

- Semi-analytical methods shows accuracy against numerical propagation
  - Especially for conservative forces
  - Also for drag induced forces up until shortly before re-entry

- Future work for improving re-entry prediction
  - Inclusion of tesseral terms
  - Inclusion of equator precession
  - Rotation of the atmosphere
  - Verify long-term re-entry prediction

- Possible applications
  - Disposal trajectory design
  - Re-entry modelling and orbit determination
  - Sensitivity analysis to spacecraft parameters and model uncertainties

POLITECNICO MILANO 1863

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Extension of the King-Hele orbital contraction method and application to the geostationary transfer orbit re-entry prediction

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