Forecasting indexes F 10.7 in the years of low solar activity

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Abstract

The results of the forecasting indices of the Sun radio emission (F10.7) in the years of low solar activity is presented.

The forecast is based on methods of the optimal forecasting Gaussian random process based on measurements at discrete time [1, 2]. The forecast of current and monthly average values is considered. The relevant autocorrelation functions are built.

It is shown, that application of the current index for forecasting the satellites reentry time gives the noticeable effect only when the intervals up to 4-5 days. To perform long-term forecasts, it is needed to be able to predict the average indexes values.

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1. Optimal forecasting Gaussian random process

We will consider the process *y(t)* with a zero a priori mean and the **autocorrelation function**

$$E\{y(t)\cdot y^T(\tau)\}=K_y(t,\tau)_0\,. \qquad (1)$$

The problem of determining the optimum estimate of the process $y(t)$, $t\geq t_k$ is solved based on measurements

$$z_i=y(t_i)+v(t_i),\quad i=1,\ldots,k\,. \qquad (2)$$

Here $v(t_i)$, $i=1,2,\ldots,k$ is the Gaussian random process with discrete time and independent values with a zero mean value and specified covariation matrix R_i (σ_z).

The processes *y(t)* и $v(t_i)$ are assumed to be mutually uncorrelated. The estimate, the errors of which have a minimum variance, is considered to be optimum.

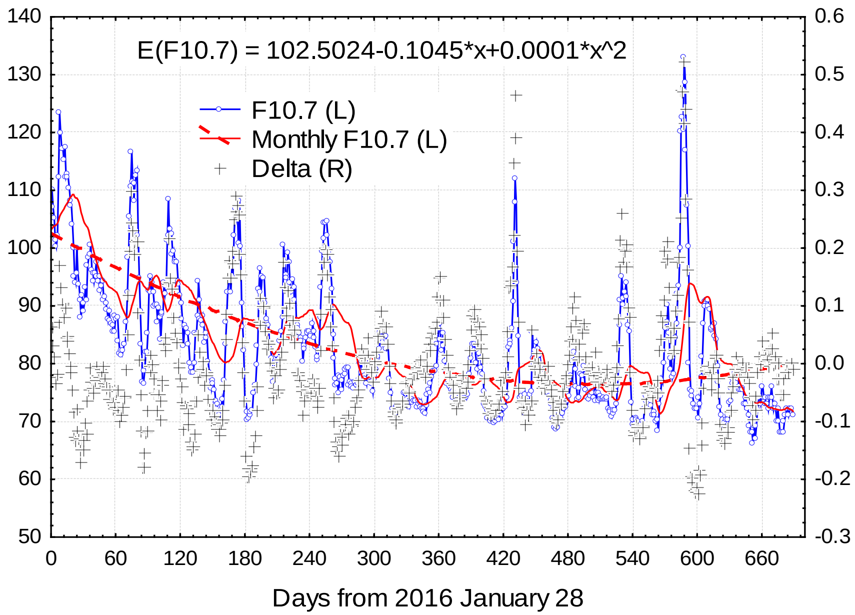
Problem solving methodology described in monograph [1]. It is based on **two functional recurrent relations (3) and (4).**

$$K_y(t,\tau)_i=K_y(t,\tau)_{i-1}-K_y(t,t_i)_{i-1}\cdot\left[K_y(t_i,t_i)_{i-1}+\sigma_z^2/\sigma_y^2\right]^{-1}\cdot K_y^T(\tau,t_i)_{i-1},\quad t,\tau\geq t_i \qquad (3)$$

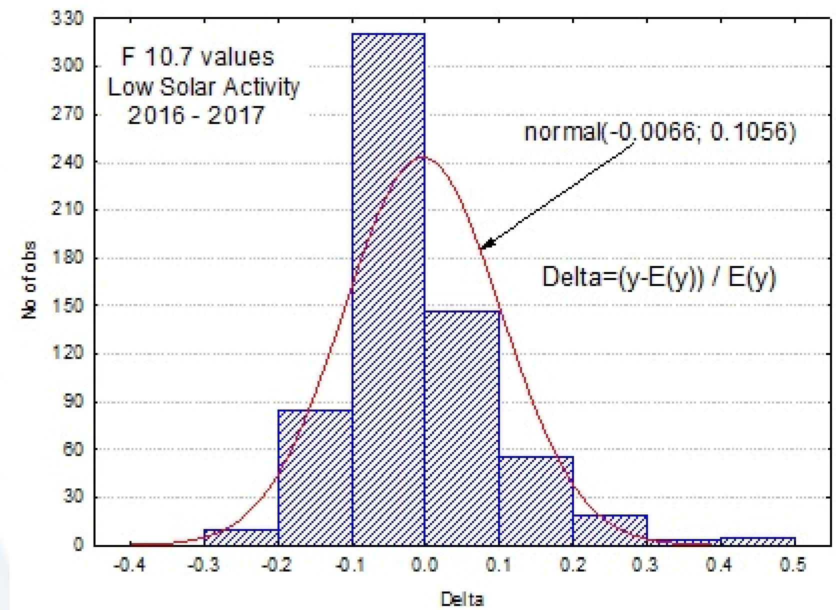
$$\hat{y}(t)_i=\hat{y}(t)_{i-1}+K_y(t,t_i)_{i-1}\cdot\left[K_y(t_i,t_i)_{i-1}+\sigma_z^2/\sigma_y^2\right]^{-1}\cdot\left[z_i-\hat{y}(t_i)_{i-1}\right],\quad t\geq t_i\,. \qquad (4)$$

The first one is used to build a matrix of correlation between the predicted values process in time *t* and *τ*, and the second - for the forecasting process. The constructed recurrence relations represent a basis of the filtering and forecasting algorithm for measurements in a discrete time. Unlike the recurrence relationships of the Kalman-Bucy filter, these relations are ***functional***. This feature made it possible to construct the filtering and forecasting algorithm for the Gaussian random process.

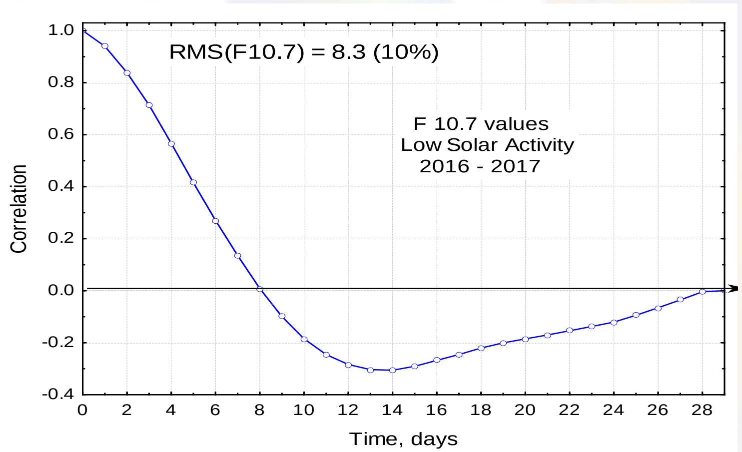
2. F10.7 values in 2016 - 2017



1. F10.7 (y) values in 2016 and 2017 [3]



2. Histogram of normalized deviations

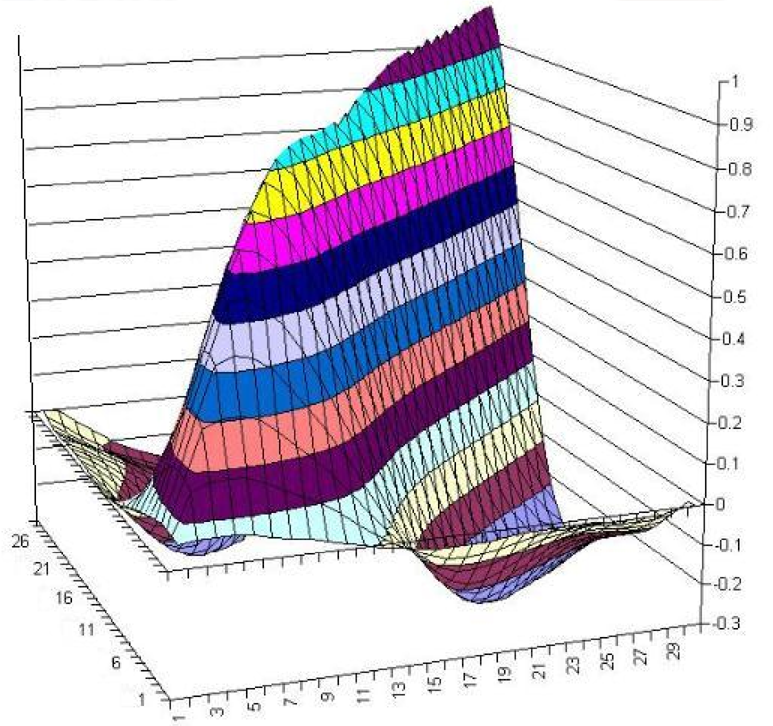


3. **Autocorrelation function of normalized deviations of index $F_{10.7}$ from average values**

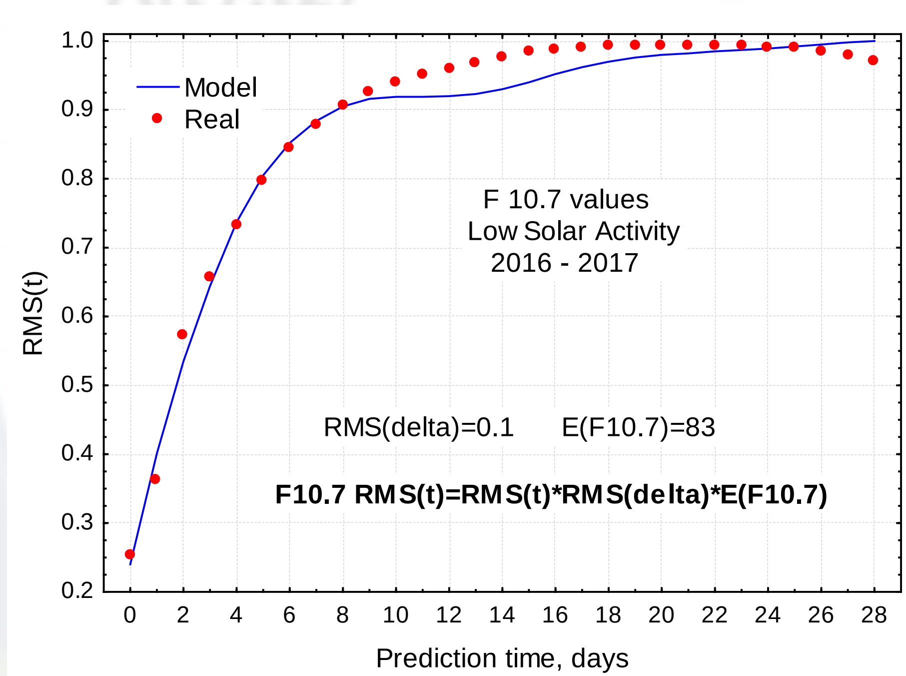
It can be seen from these data that the real distribution of normalized deviations is not too much different from a normal distribution with zero mean and RMS ≈ 0.1.

The posterior correlation matrix $K_y(t,\tau)$ was built using the autocorrelation function and the recurrence relation (3).

3. Monthly prediction of F10.7 values



4. Correlation matrix $K_y(t,\tau)$



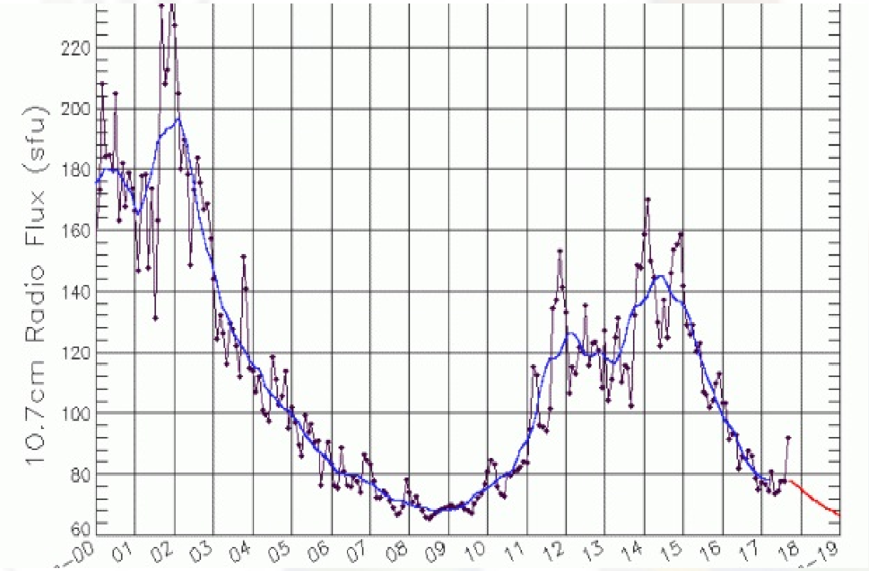
5. RMS of prediction errors $\sqrt{K_y(t,\tau)}$

Figure 5 demonstrate a good compliance of the calculated (model) and the real errors of the forecast. Based on this data, it is easy to calculate RMS deviations of the forecast values of the F10.7 index from the current average.

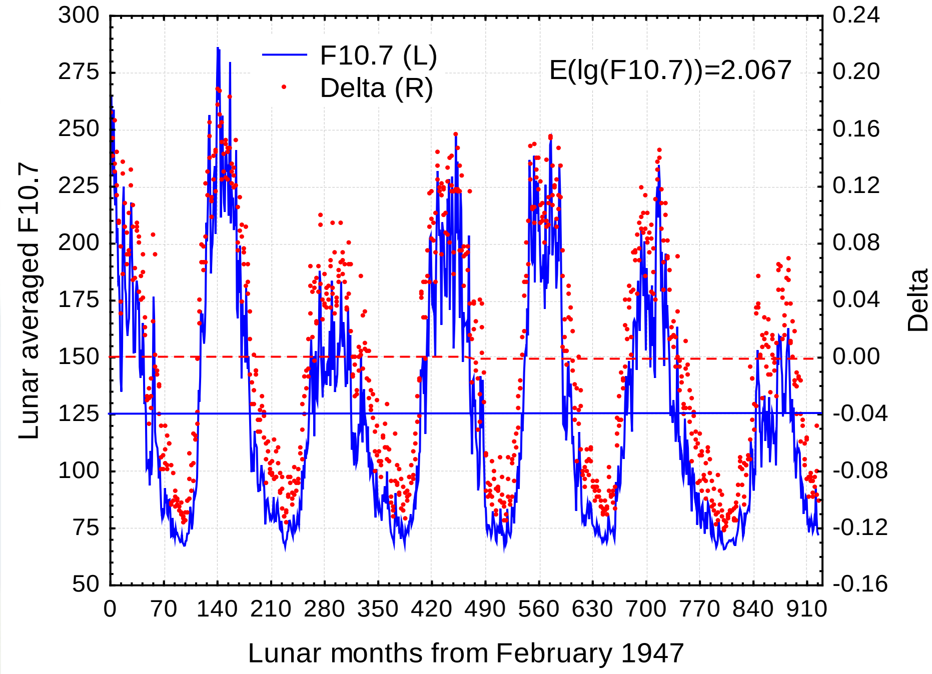
Thus, **the application of current estimates of the F10.7 index for prediction gives substantial effect only when the prediction intervals up to 4-5 days.**

To perform quantitative long-term forecasts we need to be able to predict the average F10.7 index.

4. Prediction of Lunar averaged F10.7 indexes



6. **NOAA** prediction data, years [4]



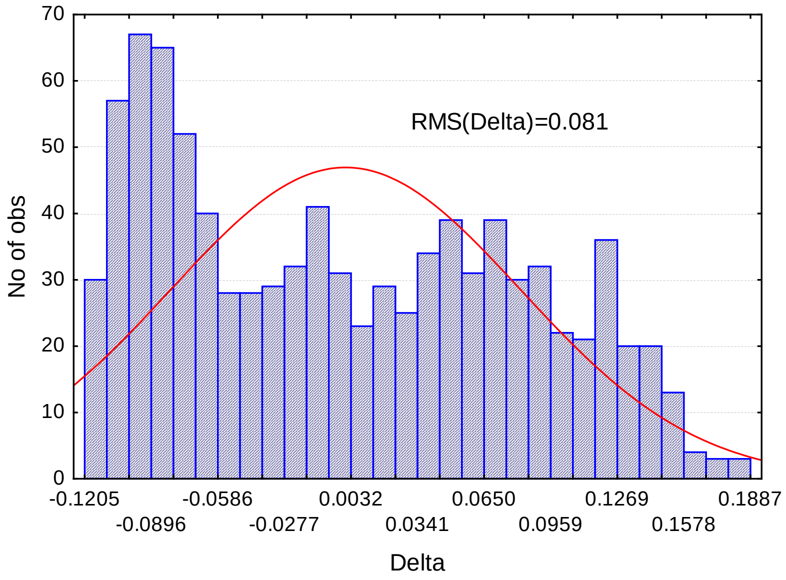
7. Lunar averaged (monthly) F10.7 indexes and normalized deviations (Delta)

You can see that **monotonous reduction of F10.7 index values to 66 units is expected on the considered time interval**.

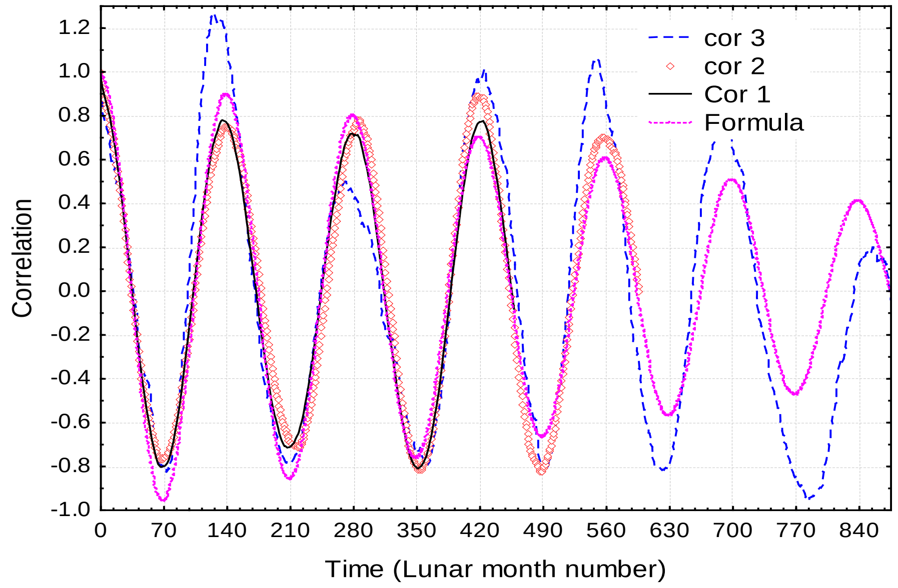
Therefore, the lifetime of the satellite Tiangong-1 will be increased.

Figure 7 shows the whole history of measurement index F10.7. It consist of **924 monthly cycles with a period of 28 days.**

4a. Monthly indexes (continuation)



8. Histogram of normalized deviations



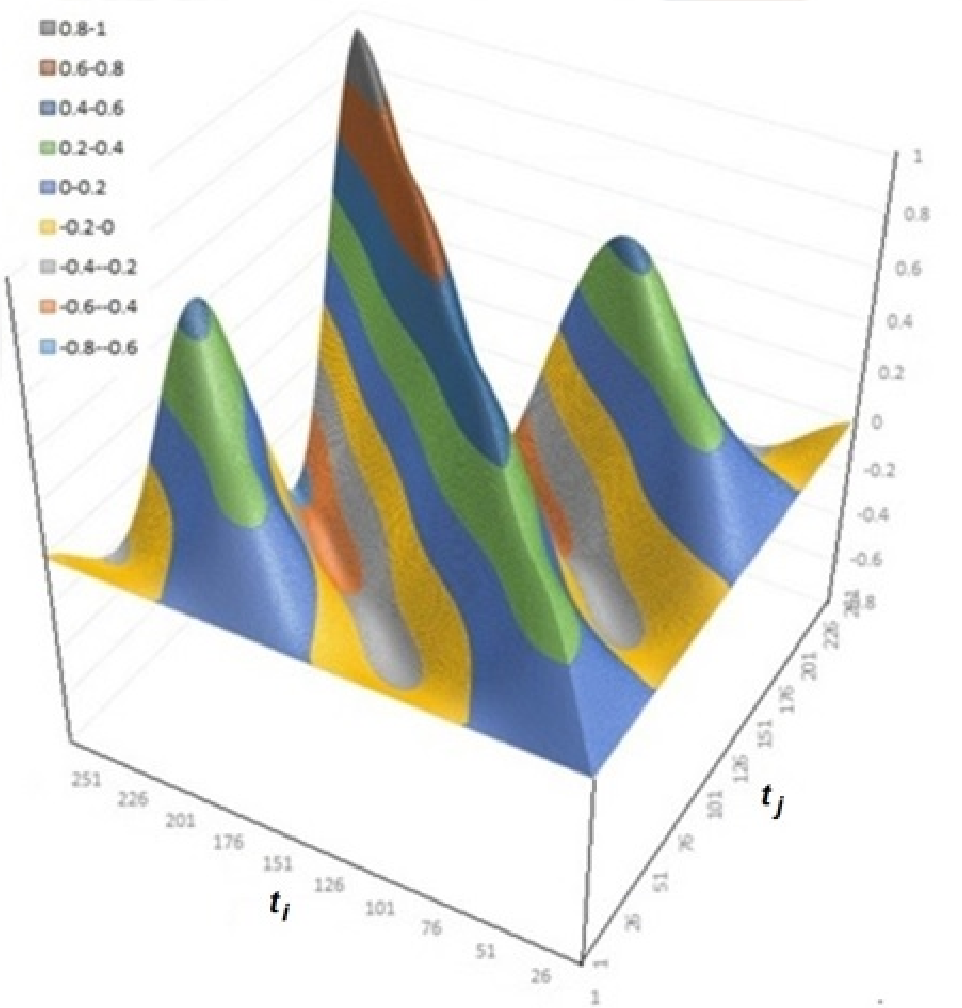
9. Autocorrelation functions of Delta deviations

Statistical distribution of Delta deviations is shown in Figure 8.

- Correlation function has a distinct period of 140 months;
- There is a smooth decrease in the maxima of correlation on the time interval, which exceeds the time interval of the initial measurements.

Proposed formula: $K(\tau) = \cos(2\pi \cdot \tau/T) \cdot (1 - \tau/\Delta), \quad \tau \le \Delta$.

4b. Monthly indexes (continuation)



10. **Correlation matrix** $K_x(t_i,t_j)$, $\Delta=315$

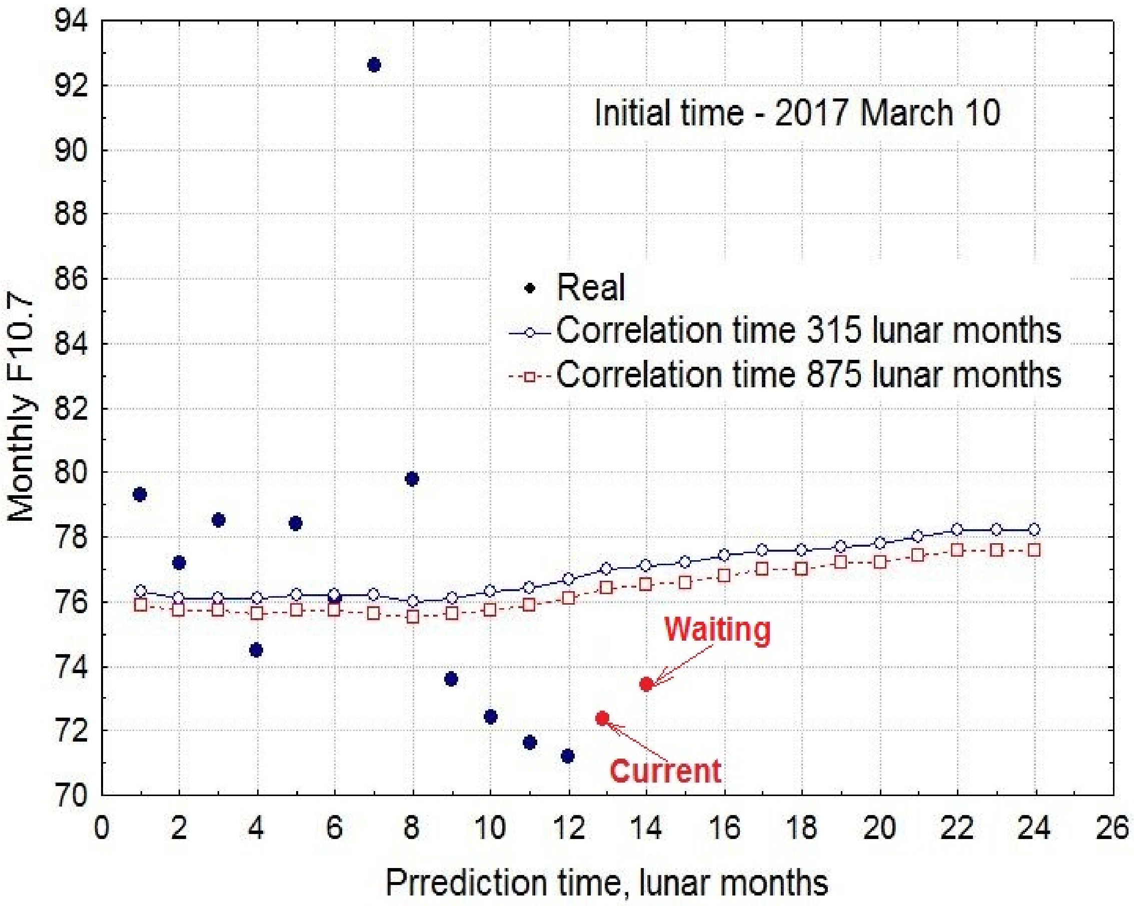
For stable filtering mode, the posteriori correlation matrixes of process Delta prediction errors was constructed on the basis of the relation (3) for two values of the correlation interval (Δ = 875 and Δ = 315 months). One of them is presented in Figure 10 (for Δ = 315).

Two factors characterize this matrix:

- With the increasing functional argument, the diagonal matrix members monotonically grows from 0.00080 to 1.0. These diagonal members make sense of dispersion of prediction errors.

- A clearly expressed periodic dependence with a period of 140 months takes place.

4c. Monthly indexes (ending)



11. Predicted and real values of monthly indexes

Figure data demonstrate the rather acceptable compliance of the predicted and real values of average monthly indices. The important conclusion follows from these data, that the **minimum value of monthly index was achieved in January 2018**.

Further, the slow growth of average monthly index values is expected during the period of the current minimum of solar activity in the 11-years cycle.

The results are applied in determining the reentry time of the Tiangong-1 spacecraft.

5. Conclusion

1. Application of current estimates of the F10.7 index for forecast gives a significant effect only when the prediction intervals up to 4-5 days. To perform quantitative long-term forecasts need to be able to predict the average index values.
2. The minimum value of monthly index was achieved in January 2018. Further, the slow growth of average monthly index values is expected during the period of the current minimum of solar activity in the 11-years cycle.

6. References

1. Nazarenko A.I. **Stochastic astrodynamics tasks. Mathematical methods and algorithms for solving**. Moscow, URSS, 2017, 352 (c).
2. Andrey Nazarenko. Solar Flux Forecast on Base of New Method of Optimum Measurements Filtering. International Journal of Advanced Computer Technology (IJACT), Volume-5 Issue-5: Publish On October 25, 2016, pp 7-13
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Thank you for attention