SALYUT-7 / KOSMOS-1686 ORBIT DETERMINATION FROM RADAR DATA

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ABSTRACT

During the final re-entry phase of the Salyut-7 orbital complex ESOC received weekly (in January) to daily (last seven days) transmissions of radar data from FGAN, the German Research Establishment for Applied Science. The radar data comprised measurements of slant range, range rate, azimuth and elevation. The data were processed at ESOC by an iterative least squares algorithm to derive the state vector and additionally the ballistic coefficient of the space station.

The algorithm is explained and critical areas where a straightforward convergence is hampered are highlighted. Methods to solve the problem of convergence are presented together with the solutions of the orbit determination.

1. INTRODUCTION

Following the recommendations issued at the workshop on re-entry of space debris in Darmstadt, FRG in 1985 a coordinated European effort was undertaken to predict the re-entry of the Salyut-7 / Kosmos 1686 complex in February 1991.

In order to make reliable predictions it is necessary to gain accurate information about the state vector and the ballistic coefficient of the Soviet Space Station. Already a single radar station is sufficient to provide near real-time orbital elements of excellent accuracy of a non-cooperative satellite[1]. In this paper the derivation of orbital elements by processing German radar data is described.

2. AVAILABLE DATA

The European Space Operations Centre (ESOC) received radar data from FGAN, the German Research Establishment for Applied Science. The radar station is located at Wachtberg-Werthhoven, Germany. Usually each day four passes of the Salyut-7 complex over the radar station were recorded and transmitted to ESOC per electronic mail. About one hour after the last passage the radar data were available at ESOC for processing.

The data were received in records of the following form:

- Epoch (day of year)
- Slant range (m)
- range rate (m/s)
- azimuth (rad)
- elevation (rad)

The radar data was already corrected for atmospheric refraction.

During one passage the observation period is about five minutes. FGAN selects up to about 100 records per passage where the signal-to-noise ratios are best and transmits them to ESOC. Thus on average there is 1 record every 3 seconds. However, their distribution is not equidistant in time due to the preselection.

Since Wachtberg-Werthhoven is located at 50.6°N (inclination of Salyut-7 is 51.6°) the observability was generally favourable. On most days there was at least one passage where the Space Station could even be seen in the North of the radar station. Fig. 1 illustrates the geometry of four typical passages as recorded on 21 January 1991. In a polar coordinate system azimuth is counted clockwise with zero degrees in the North. The elevation is zero degrees at the outer circle and 90 deg at the centre.

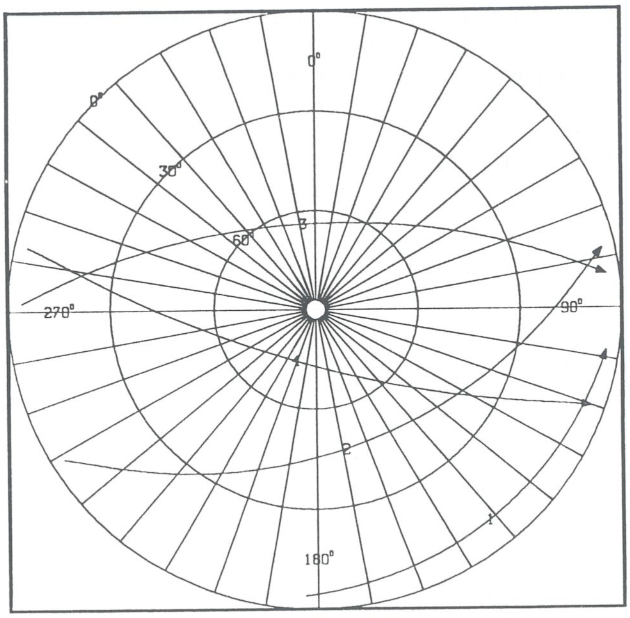

Figure 1. Salyut-7/Kosmos 1686 passes over Wachtberg-Werthhoven, Germany on 21 Jan 1991

The first pass starts in the South and culminates at very low elevation. During the second pass the station travels from East to West with a maximum elevation of 45°. The maximum elevation at pass 3 is more than 60° in the North, whereas the overall maximum elevation is reached on the fourth pass (75°, in the South).

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This geometry of passages was similar every day with minor changes. On February 1, the passages are slightly shifted to the North. In Fig. 2 it can be seen that pass 2 heads directly to the zenith.

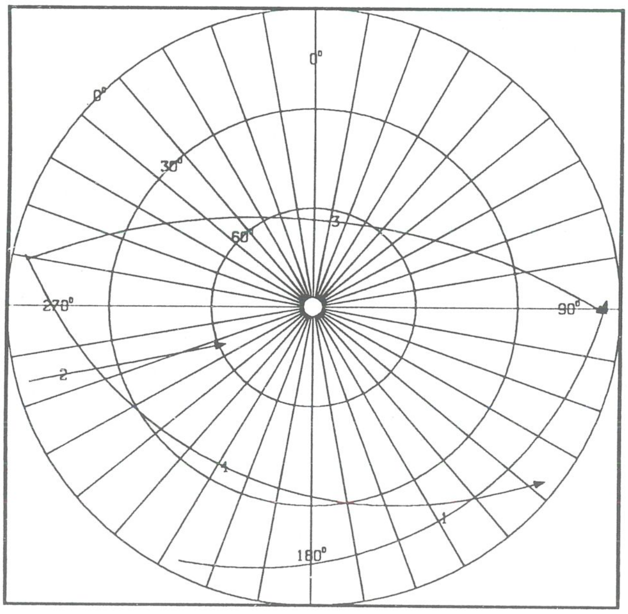

Figure 2. Salyut-7/Kosmos 1686 passes over Wachtberg-Werthhoven, Germany on 1 Feb 1991

Heavy antenna dishes preferably have a horizontal mounting (one axis perpendicular to the horizontal plane) rather than an equatorial mounting (one axis perpendicular to the equator plane). Therefore for passages close to the zenith they have to switch very rapidly from West to East and it may happen that the satellite gets lost on its fading path. Also on February 2, 4, 5 and 6, ESOC received only data of the first (incoming) part of the second pass. Fig. 3 illustrates the requirements radar dishes have to meet in order to follow the motion of a satellite.

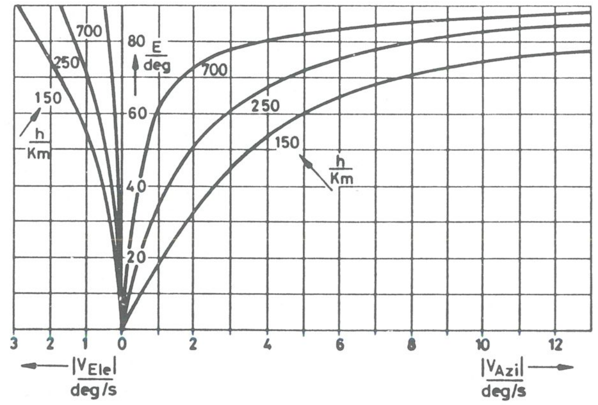

Figure 3. Maximum speed in azimuth and elevation axis as function of culmination angle E and satellite altitude h (Courtesy FGAN).

At a satellite altitude of 250 km the antenna dish has to rotate 5 degrees per second in azimuth when the satellite culminates 70° over the horizon. If the maximum elevation goes up to 80° the azimuth angle changes already 8 degrees per second.

One remark concerning the number of passages is important. On February 3 only two passages were observed (it was a Sunday). In Ref. 2, where the radar data of Kosmos 1402 Part A is analysed, it is shown that two passages are not sufficient to determine the ballistic coefficient of a satellite. Kosmos 1402 Part A re-entered on 23 January 1983, whereas Part C of the satellite decayed exactly 8 years before the Salyut-7 Space Station (7 February 1983). To derive reliable aerodynamic information at least three passes with noticable atmospheric drag are required.

3. DATA PROCESSING

The problem to be solved is to find a state vector at a reference epoch T_0 and a ballistic coefficient that provides the 'best fit' to the radar observations if you propagate the state vector to the epochs where observations are available. Best fit means the following loss function is minimised:

$$J = \sum (\vec{f}_{obs} - \vec{f}_{est})^T \, W \, (\vec{f}_{obs} - \vec{f}_{est}) \qquad (1)$$

where $\vec{f}_{obs}$ is the vector of observations (slant range, range rate, ...) and f_{est} is the vector of 'calculated observations' as derived from the estimated state vector. This vector is calculated by a numerical, multistep Adams-Bashforth propagator (Ref. 3) using the Jacchia-Lineberry atmospheric model.

Since range is internally stored in kilometres, range rate in kilometres per second and the angular directions in radians, a weighting matrix must be applied before adding the differences (= residuals) in observation and calculation. The values chosen for this paper are:

$$W = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Taking the units into account it is obvious that the range measurements are dominant for the minimisation of the loss function. However, it was tested that changing the weights had only little influence on the results.

In order to solve the minimisation problem a linear relation in the change of the solve-for-vector and in the residuals is supposed:

$$A \, \Delta\vec{x} = \Delta\vec{f} \qquad (2)$$

A contains the derivatives of the measurements with respect to the components of the solve-for-vector. Its number of rows is four times the number of records of observations:

$$A = \begin{pmatrix} \frac{\partial r_1}{\partial x} & \frac{\partial r_1}{\partial y} & \frac{\partial r_1}{\partial z} & \frac{\partial r_1}{\partial \dot{x}} & \frac{\partial r_1}{\partial \dot{y}} & \frac{\partial r_1}{\partial \dot{z}} & \frac{\partial r_1}{\partial BC} \\ \frac{\partial rr_1}{\partial x} & \frac{\partial rr_1}{\partial y} & \frac{\partial rr_1}{\partial z} & \frac{\partial rr_1}{\partial \dot{x}} & \frac{\partial rr_1}{\partial \dot{y}} & \frac{\partial rr_1}{\partial \dot{z}} & \frac{\partial rr_1}{\partial BC} \\ \frac{\partial az_1}{\partial x} & \frac{\partial az_1}{\partial y} & \frac{\partial az_1}{\partial z} & \frac{\partial az_1}{\partial \dot{x}} & \frac{\partial az_1}{\partial \dot{y}} & \frac{\partial az_1}{\partial \dot{z}} & \frac{\partial az_1}{\partial BC} \\ \frac{\partial el_1}{\partial x} & \frac{\partial el_1}{\partial y} & \frac{\partial el_1}{\partial z} & \frac{\partial el_1}{\partial \dot{x}} & \frac{\partial el_1}{\partial \dot{y}} & \frac{\partial el_1}{\partial \dot{z}} & \frac{\partial el_1}{\partial BC} \\ \frac{\partial r_2}{\partial x} & \frac{\partial r_2}{\partial y} & \frac{\partial r_2}{\partial z} & \frac{\partial r_2}{\partial \dot{x}} & \frac{\partial r_2}{\partial \dot{y}} & \frac{\partial r_2}{\partial \dot{z}} & \frac{\partial r_2}{\partial BC} \\ & & & \vdots & & & \end{pmatrix}$$

The solve-for-vector $\Delta\vec{x}$ consists of changes in the state vector in cartesian coordinates and of the change in the ballistic coefficient (BC). Eq. 2 is a overdetermined system of equations where residuals will be generated:

$$A\,\Delta\vec{x} - \Delta\vec{f} = \vec{v}$$

Demanding the squares of the residuals to be a minimum (here without weights)

$$\sum v^T v \rightarrow \min$$

the solution is given by the least squares method of Gauss:

$$A^T A\ \Delta\vec{x} = A^T \Delta\vec{f} \tag{3}$$

Eq. 3 are called the normal equations. $A^T A$ is called the information matrix. It is symmetric and nonnegative definite. If it is nonsingular the solution is:

$$\Delta\vec{x} = (A^T A)^{-1} A^T \Delta\vec{f} \tag{4}$$

This solution can iteratively be improved since the matrix A is only a linear approximation. Effectively, after adding $\Delta\vec{x}$ to the old estimate, A is newly determined at the new estimate and a second minimisation step follows. This procedure is repeated until the loss function has converged to its minimum.

4. CONVERGENCE PROBLEMS

When Salyut-7 orbited at high altitudes the solution was found after a few iterations. However, on January 21 when Salyut-7 reached an altitude of 250 km the loss function was no longer decreasing. The reason is the increasingly nonlinear relation between $\Delta\vec{x}$ and Δf where a linear relation A is assumed (Eq. 2). Here the application of Marquardt's algorithm is helpful. The algorithm is explained in the book by Wertz (Ref. 4). Basically, a multiple of the identity matrix is subtracted from the information matrix:

$$\Delta\vec{x} = (A^T A - \lambda I)^{-1} A^T \Delta\vec{f} \tag{5}$$

If λ tends to zero this has no influence and Eq. 5 is identical to the Gauss algorithm of least squares. If λ exceeds the diagonal values of the information matrix, $\Delta\vec{x}$ moves along the negative gradient of the loss function, i. e. in the direction of steepest descent of the loss function.

The idea of Marquardt's algorithm is to reduce λ after each iteration. This can be interpreted as a transition from the method of steepest descent to the least squares algorithm.

The algorithm was applied for orbit determination on January 21, 28 and 31. A disadvantage is the difficulty in finding appropriate values for λ. If it is too large the loss function decreases very slowly and it takes too many iterations to find the solution. But if λ decreases too fast the solution no longer converges.

An obvious possibility to reduce the number of iterations is to provide better initial estimates. Up to January 31 the first estimate was derived by the first two records of radar data. The first slant range, azimuth and elevation data were converted to a position vector and the differences between the first and second record were converted to a velocity vector. The ballistic coefficient was taken from the last orbit determination. Though this method renders useful position vectors, the accuracy in the velocity vector is poor.

An alternative to this approach is to propagate the solution of the last orbit determination to the epoch of new observations. If the propagation period is of the order of several days the along-track error becomes considerable. Since the solve-for-vector is given in cartesian coordinates the resulting position vector is very inaccurate.

But if the propagation period is only one day or less the propagated solution provides a better initial estimate than the estimate derived from two consecutive radar measurements. Therefore, in February when daily radar data were received, a far better initial estimate was available.

With this initial estimate the iteration process converged without applying Marquardt's algorithm until the Space Station reached an altitude of 200 km (on February 4). The problems that appeared at this stage were due to the numerical determination of matrix A. The derivatives in A are formed by adding small increments $d\vec{x}$ to the solve-for-vector. The phenomenon that could be observed was a increase or decrease of the loss function depending on the values of $d\vec{x}$. This can be explained by the fact that the propagation is increasingly sensitive to changes or errors in the state vector. The assumptions of a linear behaviour remain valid only in a extremely small region (see Fig. 4). But if $d\vec{x}$ is chosen small enough to stay within this region rounding errors become dominant.

To overcome these problems the direction of integration has to be reversed. Therefore, during the final phase of re-entry the estimate refers to the epoch of the last observation and is integrated backwards in time to the earlier observation epochs.

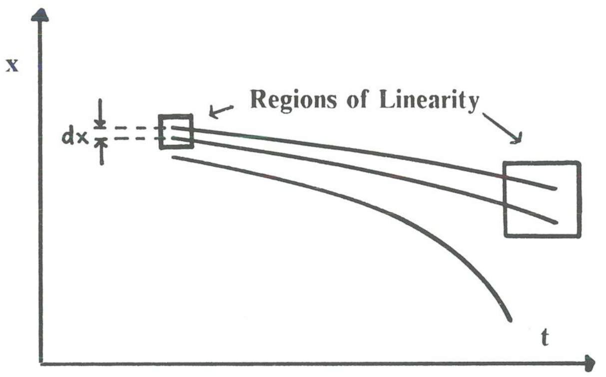

Figure 4. The size of the region of linearity depends on the direction of integration.

5. ORBIT DETERMINATION RESULTS

In Table 1 ballistic coefficients determined at 13 different epochs are presented together with the number of iterations. The ballistic coefficients before 28 January 1991 were calculated with a different air density model. So they should not be compared with the later results. These (later) results agree very well with the values given by Nazarenko[5] and by Anselmo[6].

The fourth column with the number of iterations represents the problems described in chapter 4. On January 21 and 28 the application of Marquardt's algorithm required some experiences and attempts. However, on January 31 with the improved initial estimate the solution was found within six iterations. Afterwards, due to the decreasing altitude the number of iterations increased (on February 3 no aerodynamic fit was made). Since February 4 backward integration was applied and the solution was always obtained after six steps.

In Table 2 the results of the orbit determination based on the German radar data are compared to independent results obtained in the United States and in the Soviet Union. All results had been converted to mean Liu-Alford elements and propagated to a common epoch to enable the comparison of data given in different elements and different epochs.

The deviations in semi-major axis are only some 30 metres (compared to the Two-Line Elements) and one hundred metres (compared to the Soviet data). Also eccentricity, inclination and ascending node agree very well.

6. REFERENCES

1. Mehrholz D 1985, Satellite observations with a single radar tracking station, Proc Workshop on Re-entry of Space Debris, Darmstadt, Germany, ESA SP-246, 19-24.

2. Jehn R 1989, ORBDET: a program for orbit determination, User's Manual, MAS Working Paper 306, ESA/ESOC, Darmstadt, Germany 38 p.

3. Janin G & Bello-Mora M 1990, A flexible tool for the calculation of orbits in the solar system, Adv Space Res, Vol. 10, No. 3-4, (3)327-(3)330.

4. Wertz J R 1978, Spacecraft attitude determination and control, Reidel Publ Comp, Dordrecht/ Boston/London 858 p.

Date	Altitude (km)	$BC = \frac{1}{2} c_D \frac{A}{m}$ (m^2/kg)	Number of Iterations
1990/11/08	320	0.0048	12
1991/01/03	280	******	6
1991/01/07	274	0.0042	9
1991/01/14	264	0.0032	12
1991/01/21	250	0.0021	21
1991/01/28	231	0.0028	18
1991/01/31	219	0.0031	6
1991/02/01	213	0.0028	6
1991/02/02	207	0.0031	9
1991/02/03	199	******	6
1991/02/04	191	0.0031	6
1991/02/05	178	0.0032	6
1991/02/06	159	0.0034	6

****** : no fitting (only two passes observed)

Table 1. Results and statistics of orbit determination for 13 sets of data between November 1990 and decay date.

	FGAN Radar Data	TLE	Soviet Data
A [*km*]	6611.024	6610.990 (-0.034)	6611.126 (+0.102)
E [–]	0.0005944	0.0005928 (-0.16E-5)	0.0006483 (+0.54E-4)
I [°]	51.5881	51.5876 (-0.0005)	51.5814 (-0.0067)
Ω [°]	219.3720	219.3683 (-0.0037)	219.3736 (+0.0016)

Common epoch: 1991/28/01 at 2:07:07.60

Table 2. Comparison of orbital elements derived from independent radar sources.

5. Nazarenko A I 1991, Determination and prediction of the satellites motion at the end of their lifetime, Proc Int Workshop on Salyut-7/Kosmos 1686 Re-entry, 9 April 1991, Darmstadt, Germany.

6. Anselmo L 1991, Salyut-7/Cosmos-1686 re-entry predictions for the Italian Civil Defense Authority, Proc Int Workshop on Salyut-7/Kosmos 1686 Re-entry, 9 April 1991, Darmstadt, Germany.