DEBRIS RE-ENTRY PREDICTION: SALYUT 7/ KOSMOS 1686

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ABSTRACT

The problems encountered when attempting to predict the re-entry of space debris objects are considered. The King-Hele rate of change of mean motion lifetime prediction method is applied to the re-entry estimation of the Salyut 7 / Kosmos 1686 complex. Deficiencies in the current status of the method are identified and new data computed to account for these deficiencies. A method is then proposed which can provide aerodynamic data on re-entering objects from observation of their orbital evolution. Such data will enable the active de-orbiting of objects using aerodynamic techniques as opposed to propulsive strategies which make great demands on the propellant budget.

Keywords: Space Debris, Satellite Lifetime, Satellite Aeodynamics, Active De-orbiting.

Nomenclature

a	semi-major axis of orbit
C_D	drag coefficient of body
e	eccentricity of orbit
E	eccentric anomaly
F	atmospheric rotation factor
$F_{10.7}$	index of solar activity
H	density scale height
I_n	bessel function of order n
i	inclination of orbit
J	function of e and z
l	direction cosine
L	orbital lifetime
m	mass of body
n	mean motion of orbit
$\dot{n}$	rate of change of mean motion
Q	function given by: $e \,.\, n \,.\, F(e)$
r	orbital altitude
S	profile area of body
v	inertial velocity of body
z	function given by: $a \,.\, e \,/\, H$
δ	ballistic coefficient of body
η	density scale height gradient
ρ	atmospheric density
σ	momentum accommodation coefficient
Ω	right ascension of ascending node
μ	gravitational constant of Earth
ν	true anomaly
Ω	right ascension of ascending node
ω	argument of perigee
$\oint$	integral around an orbit

1. INTRODUCTION

It is no coincidence that the two most publicised re-entries of large space debris objects have occurred close to the recorded peaks of the 11 year sunspot activity cycle; Skylab returned to Earth on 11 July 1979 (a more accurate description of the phenomenom than re-entry which implies that the debris has entered the atmosphere before); the Salyut 7/ Kosmos 1686 complex returned to Earth on 7 February 1991. The Solar Max or so-called *harvest time*[1] for satellites refers to the period when the number of observed sunspots is at its maximum. This period is important when attempting to predict the lifetimes of satellites as there is found to be a strong correlation between the variation of the neutral atmospheric density and the sunspot cycle. Depending on the altitude, neutral density can vary by a factor of up to 50 between sunspot minimum and maximum.

For debris objects in orbits of moderate eccentricity (e<0.4) the orbital lifetime is determined by the size and shape of the orbit, represented by the semi-major axis and eccentricity. The rate of change of these orbital elements is found from development of the Lagrange Planetary equations[1] to depend on the atmospheric density and the ballistic coefficient of the object in the following manner:

$$\dot{a} = - \frac{a^2 \rho \; v^3 \delta}{\mu} \qquad (1)$$

$$\dot{e} = - \rho \, v \, \delta \, (e + \cos \nu) \qquad (2)$$

where a is the semi-major axis of the orbit, e is the eccentricity of the orbit, ρ is the atmospheric density around the orbit, v is the inertial velocity of the object, ν is the true anomaly and μ is the gravitational constant of the Earth. The parameter δ is termed the ballistic coefficient and is given by an expression of the form :

$$\delta = \frac{F \, S \, C_D}{m} \qquad (3)$$

F accounts for the fact that the neutral atmosphere is not inertially fixed but rotates with the Earth, S is the profile area of the body that is presented to the oncoming flow, C_D is the drag coefficient and m is the mass of the body.

It is apparent that for a particular orbit, it is the neutral density ρ and the ballistic coefficient δ which dictate the rate of change of a and e and therefore the orbital lifetime. If the value assigned to the density or the ballistic coefficient is in error, then this error will be passed on to the estimate of orbital lifetime.

In addition to varying with the level of solar activity, atmospheric density varies with altitude, with the local solar time and on a periodic cycle such as the semi-annual. In all the macroscopic behaviour of the atmosphere leads to a variability in neutral density of at least 10% that cannot be predicted.

Assigning a value to the ballistic coefficient implies a knowledge of the size, shape, mass, attitude and aerodynamic behaviour of the debris object. In many cases the values of these parameters can only be estimated with an uncertainty of greater than 10%.

These apparently insoluble problems lead King-Hele and Walker[2] to develop a technique for the estimation of the lifetime of a satellite which is effectively independent of both *density* and ballistic information. Instead the method uses a measure of the size of the orbit (the mean motion n) and the rate of the contraction of the orbit in order to derive a lifetime estimate.

This technique and its application to the case of the Salyut 7 re-entry prediction campaign is the subject of the present paper. The record levels of solar activity observed during the period leading up to the re-entry of the complex had important implications for the lifetime predictions derived using the method even though the method is apparently independent of density. The exact nature of these implications will be discussed.

Finally an extension of the technique developed by Crowther and Stark[3] will be presented as a possible means of determining the aerodynamic coefficients of such a complex vehicle such as Salyut 7 from direct observation of the orbital evolution. Such data will enable operators to carry out active de-orbiting of satellites using aerodynamic techniques as opposed to propulsive strategies.

2. ATMOSPHERIC DENSITY

In addition to varying with the level of solar activity, the long term variations corresponding to the level of solar radiation energy at a wavelength of 10.7 centimetres, the shorter term variations showing good correlation with the degree of disturbance of the Earth's magnetic field, atmospheric density varies with ; altitude in an approximately exponential manner; with the local solar time, the so-called diurnal variation with a minimum at about 4 a.m. and a maximum at about 2 p.m.; a semi-annual variation with maxima during April and October and minima during January and July. In all the large scale variations result in a variability in atmospheric density of at least 10% that cannot be predicted. There are a large number of atmospheric models which provide a basis for understanding the general behaviour of the atmosphere using the parameters above as inputs. However the agreement between the models for both density parameter prediction[4,5] and orbit prediction[6,7] purposes still shows a variation of the order of 10%.

3. AERODYNAMICS IN LOW EARTH ORBIT

There is a dearth of information on the aerodynamic environment in Low Earth Orbit (LEO). This is due mainly to the difficulty in obtaining direct measurements[8] of the aerodynamic forces acting on an object as its transits the upper atmosphere of the Earth. Attempts to reproduce the environment in the laboratory have achieved only limited success as the major constituent of the atmosphere at orbital altitudes is oxygen, a very reactive monatomic species. It is found that in practice it is very difficult to produce pure neutral beams of atomic oxygen with the characteristic energy of 5 eV encountered in LEO. The failure to characterise the so-called gas-surface interaction between atmospheric molecules and the spacecraft surface is one of the major stumbling blocks when attempting to calculate the aerodynamic forces acting on a vehicle. In addition to this there are found to be a number of other problems:

i) Depending on the size of the vehicle and the atmospheric conditions, the aerodynamic regime encountered by the vehicle as its orbit decays changes from that of Free Molecular Flow to Transitional Flow. In the case of Free Molecular Flow the atmospheric molecules are assumed to move independently of each other whereas in Transitional Flow the density of the atmosphere and therefore the mean free path between the atmospheric molecules dictates that inter-molecule collisions be accounted for. The aerodynamic codes for the different regimes must therefore be applied at the appropriate points on the trajectory. The dependency of the aerodynamics on an ill-defined parameter such as neutral density and the problem caused therein was illustrated by the work of Blanchard et al[9] who when attempting to derive Free Molecular aerodynamic coefficients from flight data for the re-entering Shuttle found that due to high levels of solar activity, the vehicle was in fact well within the Transition regime when they took their measurements.

ii) The drag coefficient of an object is found to be very sensitive to the orientation of the body to the oncoming flow. In many cases an object nearing the end of its orbital lifetime will be uncontrolled and its attitude no longer monitored so that the orientation of the body will need to be determined indirectly from such techniques as visual observation or radar imaging[10].

iii) For the case of large complex vehicles such as the Salyut 7 / Kosmos 1686 complex, there is a further complication introduced in the form of multiple reflections and aerodynamic shielding. These phenomena are illustrated in figure 1 and represent a variance in the number of molecules striking a surface as compared to the *freestream* value. These effects must be accounted for in the calculations and accordingly also make the results very dependent upon the representation of the vehicle used.

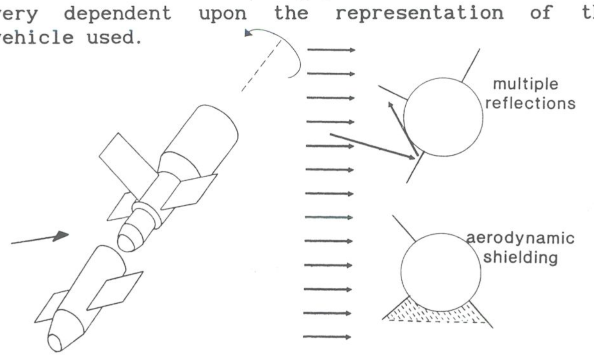

figure 1 flow phenomena encountered in LEO

iv) Finally when all of these phenomenon are accounted for, the practical problem of averaging these effects over one spin cycle of the body rotation must be solved and the result integrated into an ephemeris program for orbit prediction.

4. THE KING-HELE METHOD FOR LIFETIME ESTIMATION

A method has been developed by King-Hele and his co-workers which uses the NORAD 2-line elements as inputs. In these elements the mean motion of the orbit, n is given in units of orbital revolutions/day. Given two sets of elements it is possible to derive the rate of change of mean motion (a preferred course of action to taking the instantaneous value given by NASA). The strength of the technique is that no *a priori* knowledge of the satellite ballistic coefficient or density values are required. Instead King-Hele has used the expression:

$$\rho = \rho_p \exp\left((r_p - r) / H \right) \qquad (4)$$

in order to represent the density variation around the orbit where ρ_p is the density at perigee, H is the density scale height, r is the altitude around the orbit and r_p is the perigee altitude. If this is substituted into the expressions (1) and (2) and integrated around the orbit, it can be shown[1] that the lifetime or the orbit, L is given by an expression of the form:

$$L = \frac{e\ n}{\dot{n}} F(e) \qquad (5)$$

where $\dot{n}$ is the rate of change of mean motion and $F(e)$ is a complex function of eccentricity, scale height H and scale height gradient η or dH/dr. Both H and η are dependent on the altitude and level of solar activity prevailing at the time. This influence has been derived from the CIRA 1972 atmospheric model and is shown in figures 2 and 3 for levels of solar activity denoted by the $F_{10.7}$ index between 50 and 250 $\times 10^4$Jy and altitudes between 100 and 400 km, the altitudes most relevant to the re-entry problem.

King-Hele has been able to derive analytic expressions for $F(e)$ depending on the phase of the orbit and these are given by:

for phase 1 of the re-entry, (i.e., e < 0.2, z > 3) (z = a e / H)

$$F(e) = \frac{3}{4}\left\{ 1 + \frac{7e}{6} + \frac{1}{2z}\left(1 + \frac{3}{4z}\right) - \eta\left(\frac{1}{4} - \frac{1}{2z}\right) + O\left(\frac{5e}{16}, \eta, \frac{3}{8z^3}\right)\right\} \qquad (6)$$

for phase 2 of the reentry, (i.e., e < 0.2, z < 3)

$$F(e) = \frac{3}{4} I(z) \left(1 + \frac{2e}{I(z)} - \frac{9ez}{40} + O(0.008)\right)(1 - \eta J)$$

(Equation 7)

where $I(z) = I_0(z) / I_1(z)$ and

$$J = \left(\frac{1}{4} - \frac{1}{2z}\right) \Big/ \left(1 + \frac{7e}{6} + \frac{1}{2z}\right) \quad \text{(from ref 1)}$$

for the final phase of the reentry when the orbit is circular,

$$F(e) = \frac{3}{2}\frac{\eta H_H}{a e} \qquad (8)$$

where H_H is the value of H at H km below perigee altitude.

Equation (5) is normally expressed in the form :

$$L = \frac{Q}{\dot{n}} \qquad (9)$$

where: $Q = e\ n\ F(e)$

The function Q has been calculated by King-Hele for different values of eccentricity and mean motion. As the expressions for $F(e)$ are also dependent upon the values of H and η, the Q values are determined relating to two levels of solar activity , low level activity represented by $F_{10.7} = 80\times10^4$Jy and high level activity represented by $F_{10.7} = 150\ \times10^4$Jy. These Q values are reproduced from reference 1 and shown as figure 4.

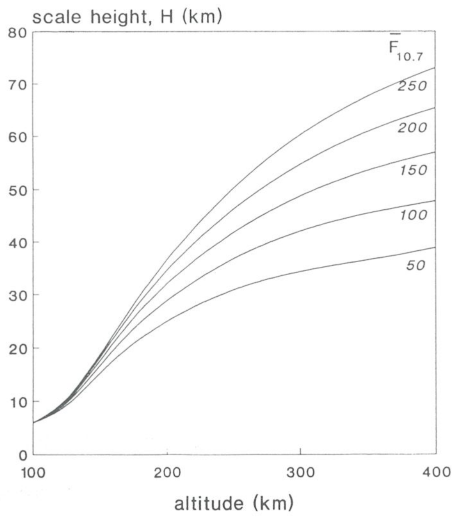

figure 2 variation of scale height H

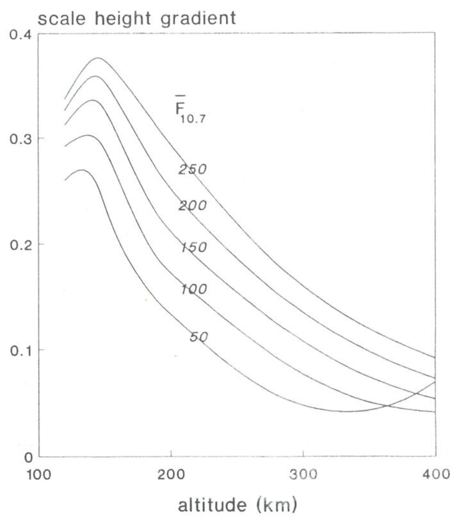

figure 3 variation of scale height gradient η

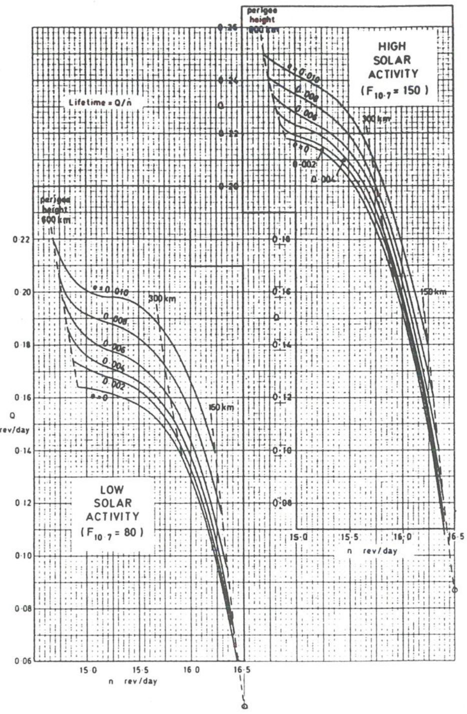

figure 4 Q variation taken from reference 1

Therefore given a value for n, e and $F_{10.7}$, it is possible to read a value of Q from the relevant graph of figure 4. The initial lifetime estimate is then given by Q / ṅ where ṅ is the change in n between two epochs divided by the time difference (in days) between the epochs. This lifetime estimate is then corrected for a number of naturally occuring phenomena which are not accounted for in the theory. These are:

i) Due to the influence of the odd zonal harmonics of the gravitational field of the Earth, the perigee distance varies sinusoidally with argument of perigee ω. Using graphical techniques this effect is accounted for by use of a correction factor applied to the initial lifetime esimate (see reference 1).

ii) It was assumed in the development of the theory that the atmosphere was spherically symmetric. In reality it resembles the oblate form of the Earth. Again the influence on the lifetime estimate is a function of the argument of perigee and is determined graphically (see reference 1).

iii) If the lifetime estimate is greater than 20 days it is necessary to account for the effect of semi-annual variation in atmospheric density on the lifetime prediction.[1]

iv) Further corrections are carried out to account for the solar cycle variation in atmospheric density and the day to night variation. Again the theory is considered to be outside the scope of the present paper and the interested reader is referred to reference 1 for greater detail.

So finally the initial estimate for the orbital lifetime is corrected for the effects mentioned above by an expression of the form:

$$L = L * f(oz) * f(ao) * f(sa) * f(sc) * f(dn)$$

to give the value of lifetime used for a re-entry estimate.
A schematic of the technique is shown as figure 5.

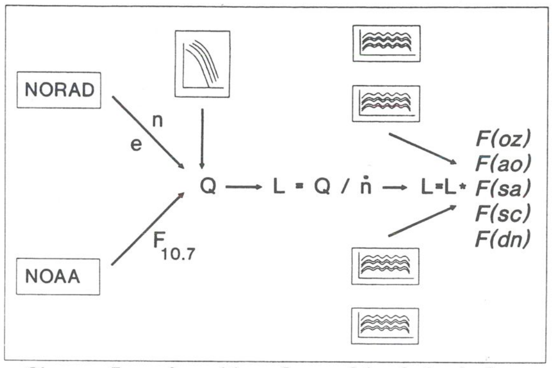

figure 5 schematic of graphical technique

5. APPLICATION TO SALYUT 7

This analysis will concentrate on the last 40 days of the Salyut 7/ Kosmos 1686 re-entry campaign. The mean motion and the rate of change of mean motion calculated from the NORAD two line elements for this period are plotted against time in figures 6 and 7 respectively. Also plotted on the graphs are the daily values of the $F_{10.7}$ index and the 90 day average values supplied by NOAA. The first thing that is apparent is the large increase in the instantaneous $F_{10.7}$ index as the date of re-entry is approached. This is matched by a corresponding but not so pronounced increase in the 90 day average value. It is the 90 day average value of $F_{10.7}$ which is used as an input to the King-Hele theory and it may be argued that in its present form, the theory cannot account for large variations in the instantaneous value of $F_{10.7}$ and therefore its effect on the orbital decay. However if we look closely at the plots of n and ṅ it is apparent that although n is not sensitive to the instantaneous values of $F_{10.7}$ the sudden increases in ṅ show very good correlation with the sudden sharp increases in daily $F_{10.7}$ values. Therefore it might be argued that the technique can account for these sudden increases in solar activity but indirectly through the ṅ input.

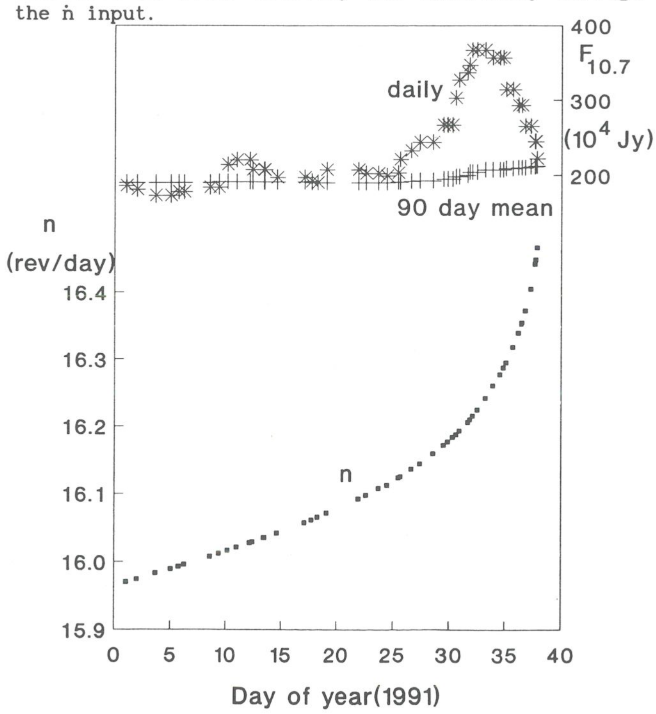

figure 6 variation of mean motion

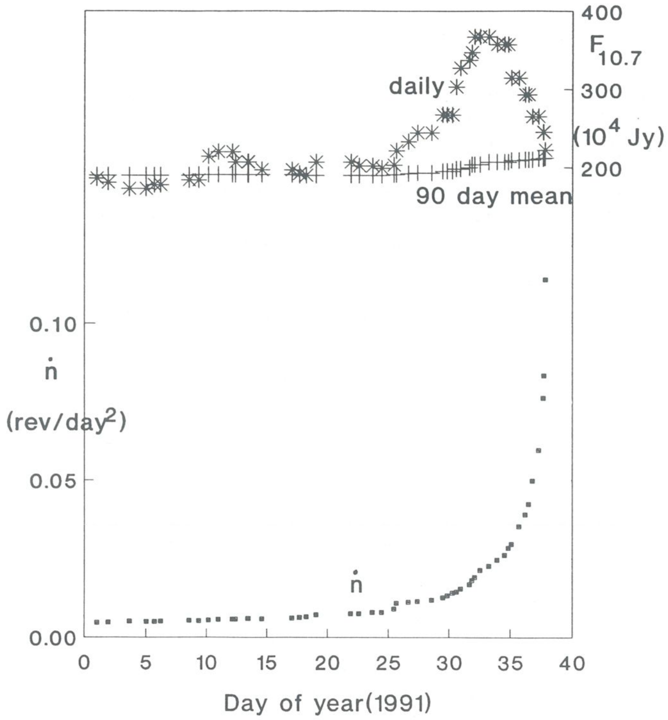

figure 7 variation of rate of change of n

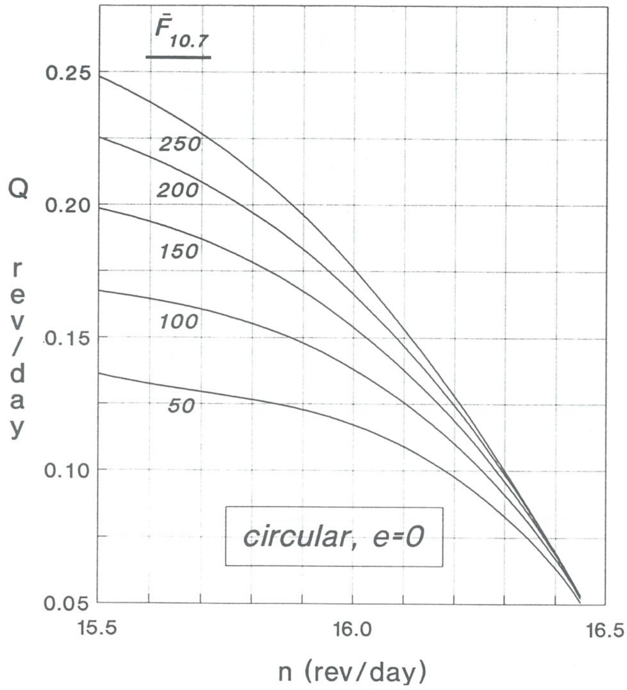

figure 8 variation of Q with solar activity

The argument that the graphs of Q against e for solar activity levels of $F_{10.7}$=150 x 10^4Jy should not be used when the 90 day average is of the order of 200 x 10^4Jy cannot be dismissed so easily. A higher level of solar activity suggests a larger value of scale height which will lead to a higher value of Q. Therefore if $F_{10.7}$ is underestimated, the Q value is underestimated and from equation (9) we see that the lifetime prediction is underestimated. In other words if we assume a lower value of solar activity than that which actually exists, our lifetime prediction is less than the *true* prediction which is the opposite to what we would intuitively expect. It was considered appropriate to recalculate the Q values for a number of limited cases relevant to the Salyut 7 / Kosmos 1686 re-entry. These new values of Q are plotted against mean motion n in figure 8. Ninety day average solar activity levels ranging from $F_{10.7}$= 50 to 250 x 10^4 Jy were chosen and a circular orbit was assumed.

The biggest test of the robustness of any theory to cope with large fluctuations in solar activity must be the 10 day period leading up to re-entry of Salyut 7. The predicted re-entry date is plotted against the date on which the prediction is carried out in figure 9. Three plots of predictions are shown, one set of data was calculated using the 90 day mean value of $F_{10.7}$= 150 x10^4Jy plot of Q against n of figure 4, another set using the recomputed values of Q against n assuming 90 day mean value of $F_{10.7}$= 200 x10^4Jy from figure 8 and a further set of data which attempts to account for the instantaneous changes in solar activity by computing Q using scale height gradient and scale height values calculated from the CIRA 72 atmosphere model accounting for the instantaneous value of $F_{10.7}$ as well as the 90 day mean value of $F_{10.7}$. It can be seen that the *$F_{10.7}$= 200* gives a better estimate of lifetime than the *$F_{10.7}$= 150* curve but that by far the most accurate results are given by calculating Q from the daily and 90 day mean values of $F_{10.7}$. The flexibility required in such an approach implies a move away from a graphical approach to the problem. It should also be noted however that even with this greater flexibilty introduced, the predictions do still suffer during the peak activity period and it could be argued that the relatively small increase in accuracy achieved in moving away from the graphical technique cannot be justified. This will remain the subject of some debate.

It should be noted that any prediction method will fall victim to these sudden increases in solar activity and it is evident from the graph that the King-Hele method still shows remarkable robustness.

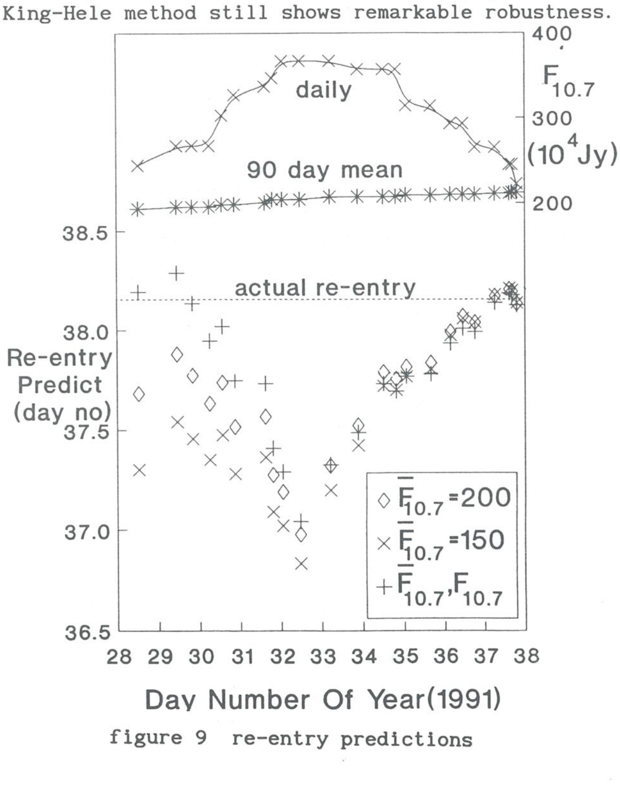

figure 9 re-entry predictions

However it must be accepted that there is a need to improve the current prediction models to try and account for such extreme fluctuations. It is felt that the King-Hele method, being by far the most powerful tool available for lifetime predictions, should be the candidate for further attention.

6. EXTENSION OF THEORY

Both Skylab and Salyut 7 / Kosmos 1686 were actively de-orbited at the end of their lifetimes and aerodynamic means used to try and control the point of re-entry. The somewhat limited success of both attempts suggests that there is a need to provide frequently updated values for the satellite ballistic coefficient as the orbit decays. A method is therefore proposed for providing such data. It relies on the fact that given a series of accurately determined orbits, it is possible to relate the rate of change of the orbital elements to the aerodynamic forces acting on the satellite.

It can be shown[3] that if the aerodynamic coefficients of a complex vehicle are represented using a combination of *m* primitive shapes such as cones, cylinders and flat plates (see figure 1) and the surfaces of these shapes are divided up into *n* surface elements to account for aerodynamic shielding and multiple reflection effects, then it is possible to derive an expression linking the change in orbital inclination and semi-major axis to the aerodynamic coeffients of these surface elements by an expression which is independent of perigee density of the form:

$$\frac{\Delta i}{\Delta a} = \frac{1}{2\, a \sqrt{1-e^2}} \; \frac{\mathcal{S}_1 + \mathcal{S}_2}{\mathcal{S}_3} \qquad (10)$$

where :

$$\mathcal{S}_1 = \int_0^{2\pi} v^2 f_1(E, e, \omega, i, \Omega, H, \eta) \sum_{j=1}^{m} \sum_{k=1}^{n} (C_D l_{WD})_j \, dE$$

$$\mathcal{S}_2 = \int_0^{2\pi} v^2 f_1(E, e, \omega, i, \Omega, H, \eta) \sum_{j=1}^{m} \sum_{k=1}^{n} (C_L l_{WL})_j \, dE$$

$$\mathcal{S}_3 = \int_0^{2\pi} v^2 f_2(E, e, \omega, i, \Omega, H, \eta) \sum_{j=1}^{m} \sum_{k=1}^{n} (C_D l_{TD})_j \, dE$$

where Δi and Δa represent the changes in the orbital elements inclination and semi-major axis due to the influence of aerodynamic forces, f_1, $f_2(E, e, \omega, i, \Omega, H, \eta)$ represent complex functions of these parameters, C_D and C_L represent the drag and lift coefficients of the surface elements and l represents the direction cosines between the drag, *D* and lift, *L* vectors and the vectors normal to the orbit plane, *W* and tangential to the orbit, *T*. If the drag and lift coefficients are represented by a set of momentum accommodation coefficients, either the Schaaf and Chambre[12] surface-based set or the Crowther and Stark[13] aerodynamic-based set, it is possible to determine the form of the gas-surface interaction which gives a best fit to the orbital data. The data can then be used as an input to models of the vehicle motion about its centre of mass and of its centre of mass. Accordingly active de-orbiting strategies can be investigated using aerodynamic techniques which are preferred to propulsive solutions as they make much smaller demands on the propellant budget.

7. CONCLUSIONS

The rate of change of mean motion method which was developed by King-Hele has shown itself again to be a very powerful tool for predicting satellite lifetimes. Its strength lies in the fact that no aerodynamic or atmospheric data need be be available to make the lifetime predictions. The use of the observed rate of change of the size of the orbit to predict re-entry makes the method very robust and only during periods of extreme solar activity does there appear to be any deviation from predicted re-entry epochs and the actual re-entry epoch.

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