DETERMINATION AND PREDICTION OF SATELLITE MOTION AT THE END OF THE LIFETIME

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ABSTRACT

New methods and technical aspects of improving accuracy and reliability of predicting of reentry region and the satellite falling time are discussed with examples of their recent application to trajectory information from orbital complex "Salut7-Cosmos-1686".

Keywords: Upper Atmosphere, Ballistic Coefficient, Model of Satellite Motion, Determing Initial Conditions, Lifetime Prediction, Solar and Geomagnetic Activity, Errors of Prediction.

1. INTRODUCTION

Various organisations in the USSR and abroad took part in works concerning the determination of the time and the area of the space complex "Salute 7 - Cosmos -1686" re-entry. The analysis of obtained results showed that the most exact data had been regularly and operatively presented by the Space Control System (SCS) of the USSR.
Rather high quality of data, given by SCS on the time of the satellite re-entry, is explained by fact that the great attention has been given to the problem of the determination and the prediction of the satellite motion in the atmosphere: for the solution of this problem they have been conducting the cycle of theoretical and experimental studies during 20 last years.
The determination and the prediction of orbits of the satellites which are almost burned out, are based on classic methods of celestial mechanics and astrodynamics (Refs.1,2,3). The particular realization of these methods takes into account specific features of motion of considered satellites. The main feature is a great influence of atmospheric drag, which exceeds perturbations from the second zonal geopotential harmonic, which is dominant among perturbation factors for satellites flying on higher orbits. As is known satellite acceleration caused by atmospheric drag equals

$$w = \rho \cdot k \cdot V^2, \qquad k = \frac{C_d \cdot A}{2 \cdot m}, \qquad (1)$$

where ρ-athmospheric density, m-mass, C_d-dimensionless coefficient of aerodynamic drag, A - cross-section, k - ballistic coefficient, V-velocity of the main gas flow. The important distinction of atmospheric drag from other perturbation factors is a high level of errors in acceleration calculation by formula (1), which is the consequence of unsufficient exactness of data concerning the atmospheric density ρ and aerodynamic characteritics of a satellite (k). The total relative error of known acceleration values is a value of order 10%. The influence of this error is by two--three orders greater than the influence of gravitational potential errors. It is this circumstance which makes known methods of determination and prediction of satellite motion be modified and improved with taking into account features of atmospheric drag more fully.
The advance in determination and prediction of satellite re-entry trajectory is connected with research and solution of following problems.

1. The model of upper atmosphere density
$$\rho_m = f(t,h,\dots). \qquad (2)$$
2. The model of variability of the satellite ballistic coefficient
$$k(t) = k(t_o) + \delta k(t). \qquad (3)$$
3. The model of satellite motion (with the state vector x)
$$x(t) = f[x(t_o),t,\dots]. \qquad (4)$$
4. The method of determining initial conditions $x(t_o)$ and other data $(k(t_o),\dots)$ by trajectory measurements.
5. The methodology of applying the current data on solar and geomagnetic activity is necessary for increasing the precision of determining the density by the model (2) during the measurement data processing and for increasing the exactness of predicting satellite motion.
6. The control of variability of the satellite ballistic coefficient and errors of the atmosphere model. This procedure turns out to be feasible by using data on multiple satellite tracking in the atmosphere. It gives the possibility to separate the influence of variability δk and atmosphere density variations $(\delta\rho)$ not considered in the model. The estimate of the real atmosphere density is calculated by using a formula
$$\rho(t) = \rho_m(t) \cdot [1 + \frac{\delta\rho}{\rho}(t)]. \qquad (5)$$
7. The determination of the time and the area of the satellite re-entry on the basis of using the models and the methods mentioned above, and also on the basis of accumulated experience.
8. A posteriori building of the model of error in determination and prediction of satellite orbits in the atmosphere.
9. A posteriori analysis of results obtained in determining the time of the space complex re-entry.

New theoretical and practical results over each of the enumerated directions have been obtained. Consider the enumerated questions in detail and illustate them with the help of data on the determination of the time and the range of the space complex "Salute - 7" - "Cosmos - 1686" falling.

2. MODEL OF THE UPPER ATMOSPHERE DENSITY

A qualitative solution of ballistic problems concerning low-altitude satellites is impossible without knowledge of regularities in the upper atmosphere density variation with time. It is known that the upper atmosphere is very dynamic, exposed to the effect of short-wave corpuscular solar radiation and other factors. In addition to the dominant cyclic component of time dependence, variations of the upper atmosphere parameters also have random character. Current models of the atmosphere take into account the most essential variations of density and give the possibility to determine its values at the near space points for a given time moment if the index valuesof solar activity $F_{10,7}$ and geomagnetic disturbance k_p are known. The well-known foreign models are: CIRA -72, MSIS, Jacchia, Barlier (Refs.4,5,6,7) and others.

In the USSR the most widely used models are empirical ones, which are built on the basis of data on satellite drag (Refs.8,9). They are rather simple and econmical when being used on the computer and at the same time they provide the precision which is not inferior to the precision of other more complex models.

So in the model ГОСТ 25645.115-84 (ref.9) the atmosphere density is calculated as

$$\rho_m = a_0 \cdot \exp\left[a_1 - a_2 \cdot \sqrt{h - a_3} \right] K_0 \cdot K_1 \cdot K_2 \cdot K_3 \cdot K_4, \quad (6)$$

where $a_0 = 9.80665$ kg/m^2, h-satellite height above the earth surface. The coefficients a_1, a_2, a_3 and other ones, forming functions $K_i (i=\overline{0,4})$, are given in the model for six levels of solar activity F_0 and three height ranges.

The results of the calculations over a great number of real data indicate that at the heights 200-500 km root mean-square relative errors of determining density according to the model ГОСТ 25645.115-84 don't exceed 10 % in *quiet* periods and can exceed 30% during strong geomagnetic storms (Ref.10).

3. MODEL OF THE VARIABILITY OF THE BALLISTIC COEFFICIENT

It follows from the expression (1) that the value of the ballistic coefficient depends on the cross-section A, the coefficient C_d and the mass m. The variability of the ballistic coefficient is usually caused by the variability of the area A, connected with inconstancy of the orientation of the satellite of complex form relative to the velocity vector, and also with the dependency of the coefficient C_d on the air composition, the molecule concentration, satellite surface material and other factors. The function (3) can be built on the basis of satellite test data and its model in aerodinamic tubes and also on the basis of special calculations. The consideration of the variability of the cross-section A is connected with the necessity for knowledge of the satellite orientation at each previous and future time moments and in general case it can be realized on the basis of the integration of equations of motions relative to the center of mass.

The modeling of the rotational motion of the space complex relative to the center of mass on the basis of initial conditions, corresponding to the mode of gravitational stabilization, was carried out.

The analysis of the calculation results showed, that the effect of aerodynamic forces led to the disturbance of the gravitational stabilization of the complex. However, it was impossible to get data on the particular orientation ot the given time.

Therefore on the interval of the measurement data processing the ballistic coefficient was admitted to be constant. Its values a priori were considered to be unknown and in each operation results of the measurement data processing were specified a new. It gave the possibility to build the function k(t) on the basis of many operations. These results are given in chapter 7.

4. MODEL OF THE MOTION.

The completeness of satisfying requirements concerning precision and operativeness of determining the time and the area of low-orbit satellite re-entry depends to a great extent on the choice of the model and on the methods of prediction of its motion.The analysis of forces disturbing the motion ,showed that the most essential forces were atmospheric drag and forces conditioned by the distinction of the earth gravitational field from the central field. Most of the current gravitational field models contain the expansion of geopotentional in spherical functions up to 36 and higher orders. It is natural that the using of these models in a full scale for the prediction of low-orbit satellite motion seems not to be worthwhile because of too high increas of the required computer time and a negligible effect of higher terms .

The value of the gravitational disturbances changes with decrease of the satellite height insignificantly, but atmosphere resistance force increases by some orders. Therefore at low altitudes two-three days before the decay of the satellite the using of the simplified geopotentional model is guite suitable. Such a model includes zonal harmonics up to the 6-th ordere and the second sectorial harmonic. With the greater satellite lifetime the disturbances from all harmonics are taken into account up to the 8-th degree and order inclusively.

For prediction of the low-orbit satellite motion numerical and semi-analytical methods of integration of motion equations are mainly applied. On the great intervals semi-analytical methods, which are more economical in the sense of required computer time and which are based on the asymptotic methods of solution of a system of differential equations (Krilov-Bogoljubov,Zeipel-Brouwer averaging methods) (Refs.1,2) are applied.

It is nesessary to note that in practice some variants of numerical and semi-analytical methods are realized and applied. The principles of developing a part of them are presented in (Refs.3,11,12).

The specialized fast-acting semi-analytical algorithms of the prediction of the low-altitude orbit of satellite with counting time of order 0.01 sec. for one day prediction (Ref.12) and universal algorithm, which has slightly less speed but provides the possibility of predicting satellites motion in any orbits to a high precision (Ref.11) are also applied.

Methodical errors of the prediction of the low-altitude satellite motion on the base of using the universal semianalytical algorithm, which were obtained by the comparison with numerical integration results by means of Everchart method are presented in figures 1,2. The variants of initial

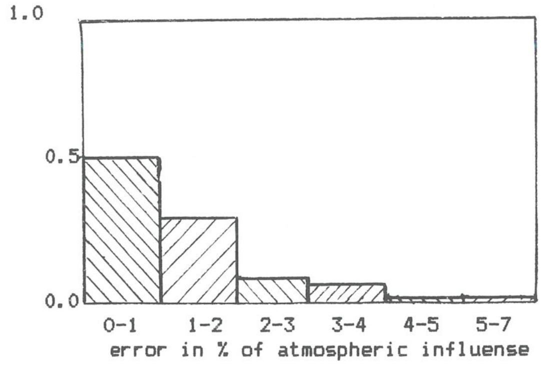

Figure 1.Distribution of errors

conditions differed in perigee altitude, eccentricity, inclination, the perigee position relative to "the atmospheric swell" of the atmosphere and in the value of the ballistic coefficient. 230 variants were compared on the interval of prediction, which equals to 50 revolutions. The histogram of the distribution of methodical errors of prediction in binormal and along the trajectory is presented in figures 1 and 2. The last errors are expressed in percents from the value of the atmospheric drag. It is obvious that the methodical error doesn't exceed some percents along the orbit and 80 meters in binormal. The obtained precision is rather satisfactory for a current level of errors of the atmospheric density model.

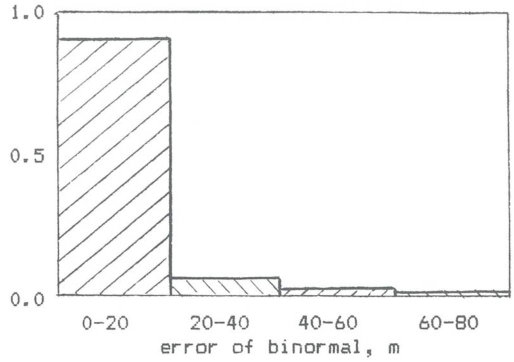

Figure 2. Distribution of errors

This circumstance and the fact, that the applying of semi-analytical methods instead of the numerical ones reduces the required computer time by a factor of 50-100, are a serious argument for their widly development recently.

5. ORBIT DETERMINATION OF THE LOW-ALTITUDE SATELLITE

The initial conditions for the prediction of the low-orbit satellite motion are determined as a result of specifying motion parameters on the basis of measurement data. For this purpose various methods of statistical estimation are applied. In practice the algorithms based on the least squares methods (Refs. 14,15) and on the various modifications of Kalman-Buicy filter (Refs.16,17,18), became widely used.

The low-orbit satellite motion models have considerable errors, caused by the destinction between real and calculated values of atmospheric density and by variability of aerodynamic characteristics. Statistical characteristics of motion model errors vary depending on the geophysical environment and features of the particular satellite motion. This circumstance must be taken into account in the processing of measurements, spaced in time.

The least-squares estimators of the algorithm can be found by means of the minimizing of a functional:

$$J(x)=\sum_i [z_i-f(x,t_i)]^T\cdot W_i\cdot [z_i-f(x,t_i)] + (x_a-x)^T\cdot P_a\cdot (x_a-x), \qquad (7)$$

where z_i is a vector of i-th measurement, x-required estimate vector, W_i weight matrix of i-th measurement; x_a, P_a - apriori value of a vector x and covariance matrix of its errors.

Criterion (7) is based on the assumption of the independence of differences $[z_i - f(x, t_i)]$, which is not kept in the processing of measurements, spaced in time. However, one has to admit this simplification to avoid calculation difficulties, connected with nondiagonal matrix inversion of a large dimension.

The motion model erros are taken into account in assigning measurement weights W_i. In calculating the satellite lifetime one usually applies the averaged ballistic coefficient. The interval of averaging depends on the rest of the satellite lifetime. It gives the possibility to reduce the effect of short-period pertubations on calculation results. In chapter 10 this problem is considered in detail.

The recursive algorithm of specifying the low-orbit satellite motion parameters generalizes Kalman-Buicy discrete filter for the case of"colour" noise of motion equations (Ref.18)]. For this case atmospheric noise is presented bythe difference between the real and the calculated satellite drag. The main feature of the algorithm lies in the fact that it enables to take into account the correlation of values of atmospheric noise in time,which is caused by inertiality of the upper atmosphere parameters, and is essential on the time interval, equal to a day (Ref.3).

The correlation function of atmospheric noise is given in the table in some array of argument values, which enables to change it without improving the algorithm itself. The atmospheric noise dispersion is speciified during satellite tracking, which gives the possibility to react operatively to variations of the level of the motion model errors.

The algorithms of orbit determination, considered above, are used in the processing of measurements in real time. For increasing the precision of estimates and the detailed analysis of aerodynamic characteristics of the low-orbit satellites it is worthwhile to carry out the a posteriori analysis.

For the a posteriori analysis the algorithms,based on the above-mentioned methods of statistical estimation, are applied.The estimates of the motion parameters,obtained before,can be used as measurements. One gets the possibility to study the effect of various factors on the precision of the problem solution. Such algorithms are called the algorithms of the secondary processing.

The applying of various algorithms of the motion parameter specification enables to define the low-orbit satellite lifetime more correctly. In the table 1 below the results of the determination of the space complex "Salute -7-Cosmos-1686" re-entry time, which were obtained by means of various algorithms on the last stage of its lifetime are presented.

These data indicate that the results of various algorithms corresponding to a given moment of specification differ no more than by 5 % from the rest of the lifetime.It gives the confidence in their correctness. One can see the tendency to the approaching of re-entry time at the expense of the increasing of the ballistic coefficient at the last revolutions of the satellite motion. This tendency is especially evident in the recurrence algorithm.

Table 1 Calculation of the re-entry time by different methods 07.02.1991.

The method of specification	Least-square method	Recursive filter	Secondary processing
N=50198	04h 25m	04h 29m	04h 07m
N=50199	03h 58m	04h 05m	04h 09m
N=50200	04h 04m	04h 03m	03h 57m
N=50201	03h 57m	03h 52m	04h 00m
N=50202	03h 47m	03h 48m	03h 49m

Here N - number of revolution.

6. PREDICTION OFTHE INDICES OF THE GEOMAGNETIC AND SOLAR ACTIVITY

Two approaches(Ref.19,20,23) are used for predicting indices of geomagnetic and solar activity. The first of them is based on studying processes, which occur

on the Sun, the second-on the analysis of time series. In operative calculations of ballistic problems the methods of prediction, based on the second approach are used.
For the prediction of the values of 3 hour k_p-indices the autoregression model is recommended:

$$k_{p,t} = \bar{k}_p + \sum_{j=1}^{n} (k_{p,t-j} - \bar{k}_p) \cdot \Phi_j + a_t$$

Here $\bar{k}$ -mean value of an index, a_t - white noise, Φ_j -unknown parameters. The optimal value of order n of autoregression and the dimension L of the sliding window: n=1, L=280-380 counts (35-40 days) were defined as a result of the studying.
The comparison of statistical characteristics of prediction errors obtained as a result of processing real data, showed that the best method of prediction was the method, based on the autoregression model. The form of this model is specified:

$$(1 - \sum_{j=1}^{3} \Phi_j \cdot B^j) \cdot (1-0.25 \cdot B^{27}) y_t = a_t ,$$

$$y_t = F_{10,7}(t) - \bar{F}.$$

Here B - shift operator, Φ_j, j=1,2,3 -required parameters, a_t - white noise.
The application of the given methodology is connected with the necessity of processing initial information on the particular measured interval (with the fixed sliding window). The dimension L of the sliding window was also determined on the basis of minimizing the root mean-square error of prediction. As a result the following optimal estimate of the sliding window dimension was obtained: L = 150 days. The parameters Φ_j are defined by means of the lest-squares methods or on the basis of the solution of Yule - Walker equations (Ref.20). The examples of using the technique of index prediction are presented below in section 10.

7. CONTROL OF THE BALLISTIC COEFFICIENT VARIABILITY AND ATMOSPHERE MODEL ERRORS

The most simple and operative way of controlling is based on the analysis of time corrections which are inserted into the results of the specification of orbit elements during the satellite tracking. For example, let the correction be inserted in the process of specification of orbit elements at the N^{th} revolution at the defined time of crossing the equator δt. This correction reflects the influence of the main sources of errors the ballistic coefficient variability and the unpredicted variations of atmospheric density, which are well revealed on the prediction interval of order 1 day. It is useful to compare this correction with the value of the disturbing effect of the atmosphere on the interval of prediction, which is easely calculated according to the approximate formula

$$\delta t_a = 0.5 \cdot |\Delta T| \cdot \Delta N^2,$$

where ΔT - variation of the period under the influence of the atmosphere during one revolution, ΔN - interval of prediction in revolutions. The ratio

$$\eta = \delta t / \delta t_a$$

is rather informative characteristic of the prediction quality.
Table 2 The relative prediction errors

Time	31.01	01.02	02.02	03.02	04.02	05.02	06.02
η	-.150	-.002	+.010	-.041	-.053	-.041	-.012

In the table below the values of η obtained during the space complex tracking, are presented. It is evident from these data that on the interval 1.02 - 6.02 the prediction errors didn't exceed 5 % from the value of the atmospheric drag. Hen c e, one can conclude with sufficient certainty that the variations of the ballistic coefficient on the prediction interval didn't also exceed 5 %. The negative values η on the time interval 3.02-6.02 indicate that the real drag of the space complex was slightly stronger than the calculation ones, that is on the interval of prediction not decrease but some increase of the ballistic coefficient was observed.
As an example some estimates of the ballistic coefficient of the space comlex "Salute -7" - "Cosmos-1686" obtained during tracking on the interval 1.01.-27.01.1991,are given in figure 3. The analysis of these data enables to conclude the following:

1) The estimates of the ballistic coefficient are in the range (0.255-0.310)·0.01m²/kg with the mean value 0.0028. The daviations of the separate values from the mean does not exceed 10 %.

2) In the evolution of the ballistic coefficient estimates one can distinguish three components: long period one with the deviation up to 5%, periodic one with the period 5-6 days and the amplitude up to 8%, and the random one, the value of which reaches 2-3%.

3) The most probable causes of appearance of above-mentioned components are :
- for a random component - measurement errors;
- for a periodic component - variations of a real ballistic coefficient;
- for a long period component - errors of atmospheric density calculations.

Here it is nesessary to note, that on the interval 27.01 - 02.02 the very high level of solar activity was observed - the index values $F_{10,7}$ exceeded 300 and on the 30. 01 reached the record level (373). Earlier it was found(Ref.21), that at the high level $F_{10,7}$ the model (Ref.10) gives the overstated values of density. It manifested itself in that fact, that

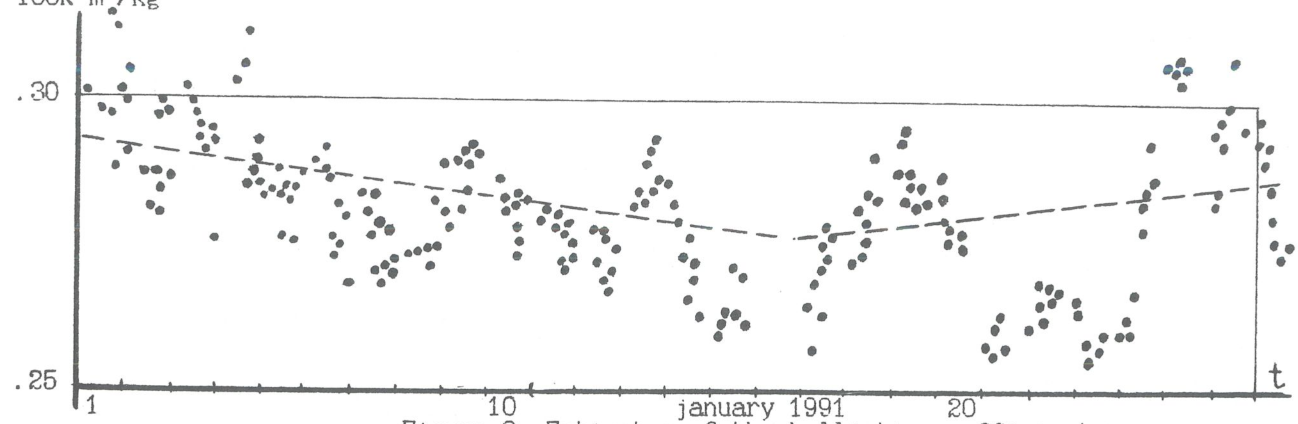

Figure 3. Estimates of the ballistic coefficient

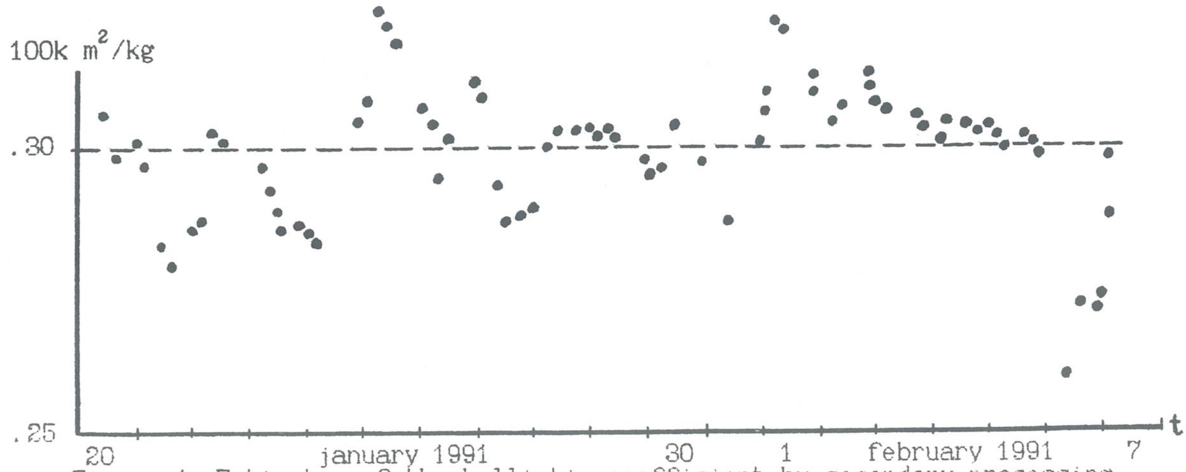

Figure 4. Estimates of the ballistic coefficient by secondary processing

the decrease of obtained ballistic coefficient estimates of many satellites was observed. Therefore the correction was inserted into the calculations in determining density on the 31-th of January.

$$\rho(t) = \rho_m \left\{ 1 - 0.001 \cdot \left[F_{10,7}(t) - 200 \right] \right\}$$

To verify appropriateness of inserting the mentioned correction the secondary processing of the obtained series of orbit elements was carried out according to the technique, presented in a brief form in chapter 5. The obtained ballistic coefficient estimates are given in figure 4. The analysis of the data, presented in this figure, and the comparison of them with figure 3 shows the following.

1. After inserting the correction the systematic (slow variable) component practically vanished which indicates the sufficient good compensation of the atmosphere model errors and the constancy of the ballistic coefficient mean value, equal in this case to $0.00298 m^2/kg$.

2. The level of a random component of estimates k, obtained as a result of the secondary processing doesn't exceeb 1%.

3. The character of a periodic component of variation k on the time interval 20.01-27.01 in both figures is identical. In the last figure the amplitude of a periodic component is slightly less-it equals 7% .

4. On the time interval 28.01-3.02 the character of variation of the ballistic coefficient was preserved.

5. On the interval 3th-5th of February the devitations of the current estimates from the mean value decreased and made up only 2%. It seemed to be connected with the fact, that with descending the rotational motion of the space complex relative to the center of mass became more and more regular.

6. At night from the 5th to the 6th of Febrary the control of the space complex orientation was performed, that is, the space complex was turned so that its longitudinal axis was directed along the velocity vector. In figure 4 it is clearly seen from the decreasing of the ballistic coefficient estimates up to 0.0027, that is by 10% from the mean value .The active control of the space complex was short-term for the lack of propellant. Approximately at 3 o'clock on the 6th February the flight of the space comlex became again uncontrolled.

7. On the last stage of the space complex lifetime , that is beginning with 20 o'clock on the sixth of February the increasing of the ballistic coefficient, the value of which approached the mean and was equal to $0.00297 m^2/kg$ at the end, was fixed. Thus the data of the figures 3 and 4 demonstrate the possibility of regular and operative control of variation of the satellite ballistic coefficients on the last stage of the satellite lifetime.

Together with the above mentioned technique of operative control the method of the detailed analysis of ballistic coefficient variability and variations of atmospheric density was developed (Refs.20,21). This method is based on the "post flight" combined processing of the orbital data on a large number of low-orbit satellites, which cease to exist on the considered time interval. The method was widely used in 1989 during the experimental investigations of the upper atmosphere density by means of passive satellites ПИОН. Then 25000 measurements of density, concerning the satellite drag, were collected and processed.

Besides this during the work, connected with the space complex in January-February of 1991, the bank of data on 40 low-orbit satellites was also collected. In general 2500 sets of orbital elements, including data on satellite drag in the atmosphere, were collected. The detailed processing of these data is expected to be carried out in the years 1991-92.

8. DETERMINATION OF THE RE-ENTRY TIME AND THE AREA OF FALLING OF THE SATELLITE.

The particular calculation of the re-entry time of the satellite and the coordinates of its probable falling point is connected with the necessity of applying numerical integration of motion equations. Here it is necessary to note that notion of the re-entry is conditional.

For the space complex the altitude of the re-entry point can be admitted in the range 75-105 km .It is convenient also because in the indicated range of altitudes the destruction of the space complex construction into separate elements was assumed and confirmed by observation. Subsequently debris followed different trajectories depending on the ballistic coefficient value. The debris with large values of k fell nearer, with small values- farther. One can get an idea of the extent of the probable range of debris falling by admitting the altitude of their formation and the range of the ballistic coefficient variation. In figure 5 the trajectory of the space complex debris are presented for two variants of its destruction altitude (100 and 75 km) and two variants of values of the ballistic coefficients of debris (0.029 and $0.00029 m^2/kg$) It is obvious that the extent of the probable range of debris falling equals 900 and 3900 km. for the destruction altitudes 75 and 100 km.,correspondingly.

The questions of the proper definition of the satellite destruction altitudes are beyond the scope of the given work. Taking into account the fact that the essential velocity loss of the space complex occurs at the altitude less than 80 km, one can consider the most probable extent of the range of its debris falling to be the value of the order of 1000 km.

Determining the coordinates of debris falling it is necessary to take into consideration possible errors of the determination of the satellite re-entry time.

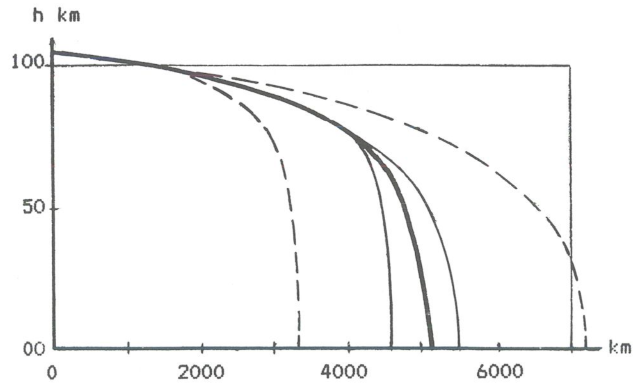

Figure 5. Trajectory of the falling for two variants of the distructions altitude

This problem is solved usually by method of variation of initial conditions within the range of their possible errors with the subsequent integration of motion equations up to the moment of the satellite re-entry. In many cases it turns out to be sufficient to vary only the ballistic coefficient. The sensitivity of the re-entry time to slight deviations of the ballistic coefficient can be determined according to the approximate formula

$$\delta t_{re\text{-}entry} = - t_l \cdot \frac{\delta k}{k} , \qquad (8)$$

where t_l - the interval of the motion prediction to the moment of the re-entry (lifetime), which was equal to 40 minutes for works, concerning the space complex. The variation of the ballistic coefficient by 1 % is reflected in this case in the variation of re-entry time, equal to 24 sec. This leads to the shift of the re-entry point by 180 km. along the trajectory.
The reliable prediction of the possible range of the ballistic coefficient variation seems to be impossible. Nevertheless taking into account the facts, meintioned above in chapter 7 one can assume that its variability is on the level, on which it was before the space complex orientation control, that is ±(2-3)%. This is reflected in the shift of the re-entry point by approximately ±500 km along the trajectory.

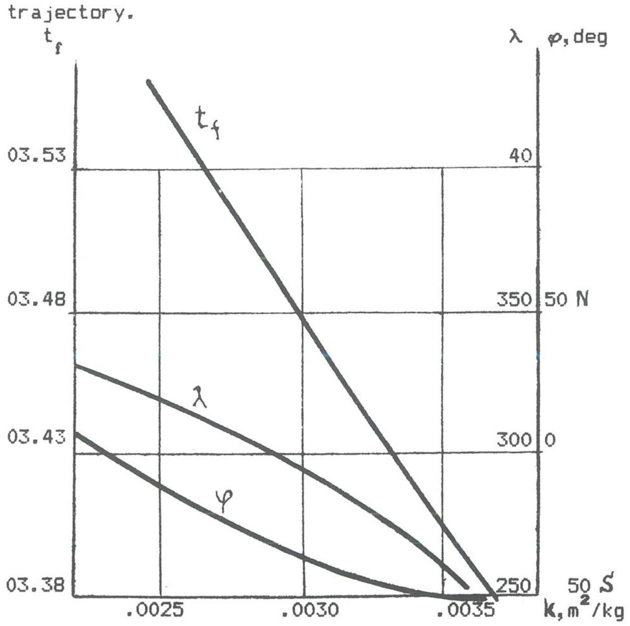

Figure 6. Depedence of time, latitude and longitude estimates of the falling point on the ballistic coefficient at the time moment 02h.31m.07.02.1991

The results of the the numerical experiment, presented in figure 6, give the more complete idea of the influence of k on the determination of the falling point. The variations of k in the limits ±2.5 % lead to deviations of time, latitude and longitude of the falling point by ±75 sec, ±3,5 deg, ±4 deg , correspondingly.
Thus, the approximate estimate of the probable debris falling range with taking into account the errors of the definition of the re-entry point and the space complex destruction at the altitude 75 km is equal to ±1000 km relative to the calculated point of the space complex falling. This corresponde to the sub-satellite band, crossing Chili and Argentine from the South-West to the North-East from the Coast in the area of Puerto-Mont to the border with Brazil in the area of Parana.

9. A POSTERIORI ESTIMATE OF THE ERRORS OF THE PREDICTION OF THE SATELLITE MOTION IN THE ATMOSPHERE

The techniques, considered in the previous chapters, were tested in the large number of experiments, connected with estimating the accuracy of the prediction of the low-orbit satellite motion. The data array of orbit elements, mentioned in chapter 7, was used as initial and standard data.
The initial conditions for the prediction (including the ballistic coefficient) were determined on the daily dimensional interval, preceding the moment of their definition. The differences δt in time of crossing the equator between the calculated and the stanard values were defined on the basis of the results of predicting the motion for 1-6 days. As the absolute values of errors δt vary over the wide range (by 2-3 orders) depending on prediction interval, the values of the ballistic coefficient, altitude and other factors, with the aim of getting the more uniform array of accuracy indices the normalization of errors was carried out:

$$\xi = \frac{\delta t}{\sqrt{2 \cdot \sigma_0^2 + (0.1 \cdot \delta t_a)^2}}$$

Here σ_0^2-dispersion of the determination of time at the initial conditions, δt_a - perturbation influence of the atmosphere in the present realization of prediction. With the prediction intervals being more than a day, the second item in the expression under the radical sign is usually much larger than the first one. Therefore in this case the index ξ has the meaning of ratio of time error δt to 10 % - part of atmospheric perturbation.
All prediction were carried out with three variants of atmospheric density calculations.
In the first variant the atmospheric density was calculated according to the model (Ref. 10) with the constant (mean) values of indices k_p=const. and $F_{10,7}$=const.
In the second variant the indices were considered to be known.
The third variant differed from the first one by inserting the correction $\frac{\delta \rho}{\rho}(t)$ into the density, calculated according to the model (5). The corrections were determined according to the technique described in (Ref.22).
For each of the variants of the density calculation 23467 predictions were carried out. The obtained characteristics of accuracy are presented in the table. Thus taking into account the variability of atmospheric density in accordance with the second and the third computation variants provides the increase of prediction accuracy in the mean by a factor of 1.5 and 1.75 as compared with the first variant.
The obtained results have the averaged character and reflect the particular conditions of the conducted

experiment: the geophysical environment, the distribution of satellites in heights, the values of their aerodynamic characteristics. The using of these data in other conditions is difficult. The knowledge of the main factors, effecting the accuracy of the prediction and the determination of the quantitative characteristics of this effect are of great interest.

Table 3
Values of characteristics of normallized errors ξ of prediction with the differents ways of the density calculation.

characteristics of errors	1	2	3
m_ξ	0.06	0.08	0.06
σ_ξ	0.97	0.64	0.55

The regression analysis of the obtained values of the index ξ enabled to find the main arguments, on which the accuracy of prediction of the low - orbit satellite motion depends: the altitude (h), the variability of indices of the geomagnetic and solar activity (Δk_p, ΔF), and of the ballistic coefficient (δk). The dependence of the mean-square value of the index σ_ξ on the enumerated arguments can be expressed as:

$$\sigma_\xi = \bar{\sigma}_\xi \cdot f(\Delta F) \cdot f(\Delta k_p) \cdot f(h) \cdot f(\sigma_k) \qquad (10)$$

In the table 4 below the data on the influence of the arguments Δk_p and ΔF on the index σ_ξ, which were obtained over 8000 realizations of the prediction of the satellite motion with altitudes up to 260 km (m_ξ = 0.12, σ_ξ = 0.53) are presented for the second variant of the density calculation.

Table 4 Dependence of the index σ_ξ on the variability of the solar and geomagnetic activity

Δk_p	ΔF		
	< -20	-20 ,+20	> +20
< -1.25	0.36	0.59	0.52
-1,25 , +1,25	0.53	0.51	0.49
> +1.25	0.83	0.66	0.69

One can see from these data that the variation of geomagnetic activity has stronger effect than the variation of the index $F_{10,7}$. With slight variability of the index k_p (the middle row) the values of the index σ_ξ are equal to approximately 0.5. This means that the root mean-square value of the prediction error is 5 % of the value of the atmospheric disturbance. For relations f(h) and $f(\sigma_k)$ the following formulas are developed

$$f(h) = 0.71 + 0.50 \cdot \left(\frac{h - 200}{200}\right) , \qquad (11)$$

$$f(\sigma) = 0.75 + 2.5 \cdot \frac{\sigma_k}{k} . \qquad (12)$$

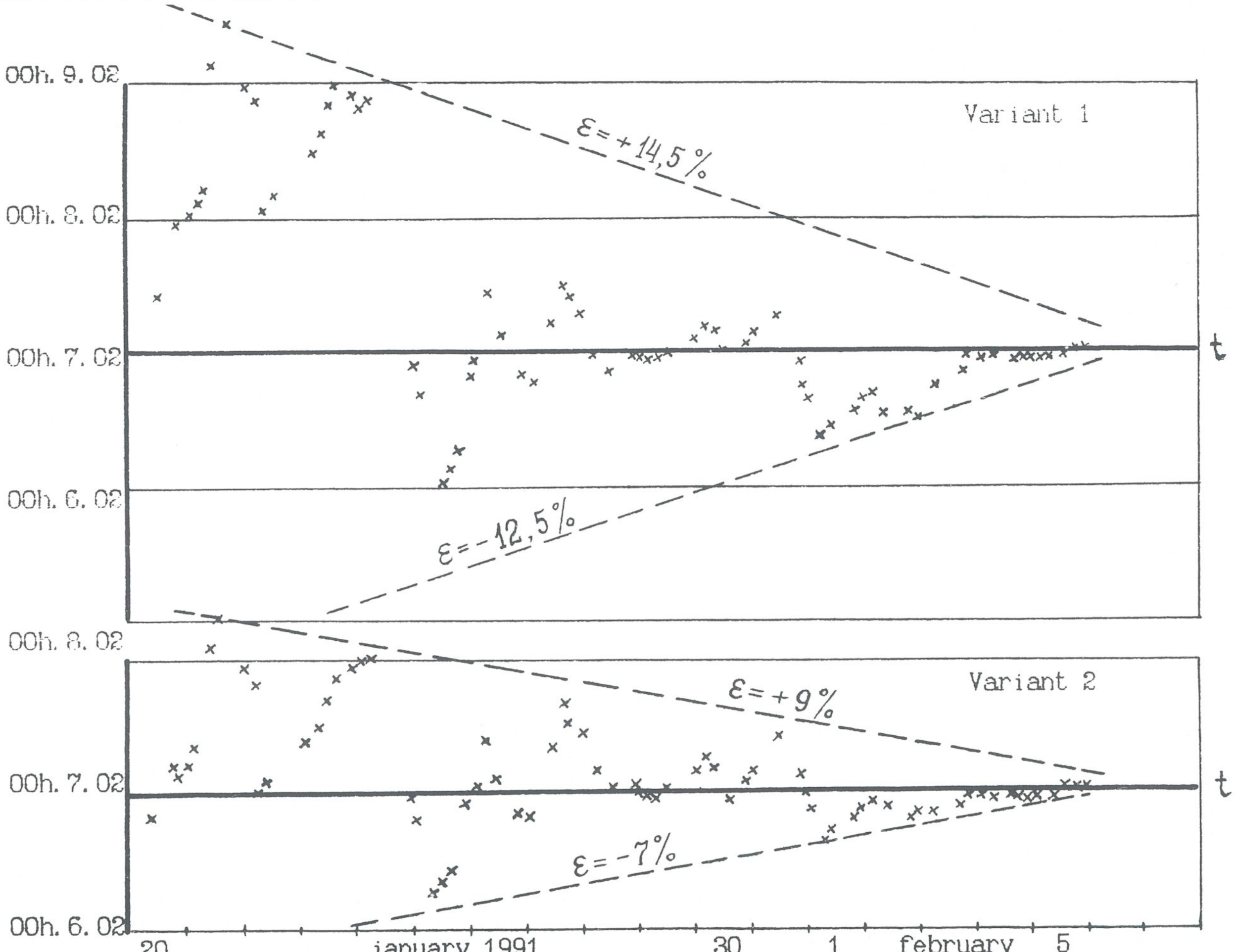

Figure 7. Values of the time of the spase complex falling with the various ways of the density calculation

The application of these relations and the table 4 data to the conditions of the work, connected with the space complex, enables to conclude that the expected mean-square value of the relative prediction error σ_{ξ} on the last stage of the space complex flight is equal to 0.24, that is 2.4 % of the value of the atmospheric disturbance. The data on the prediction accuracy, presented in chapters 7 and 10, prove the correctness of this estimate.

10. A POSTERIORI ANALYSIS OF DETERMINING THE TIME AND THE RANGE OF THE SPACE COMPLEX FALLING

The a posteriori analysis was carried out on the basis of the secondary processing of obtained orbit elements, concerning the space complex, by the using of the real data on indices of solar and geomagnetic activity.
The relative error of determing the falling time was used as the index of accuracy

$$\varepsilon = \frac{\delta t}{t_l}, \qquad (13)$$

where δt - absolute error,equal to the difference between calculated and standard falling time, t_l- satellite lifetime,equal to the interval between the moment of the orbit determination and falling time.
It should be noted that the mentioned relative error ε, and the normalized prediction error, introduced into the chapter 8, ξ, are related by the approximate formula ε=0.1ξ.
The secondary processing of orbital data was conducted on the interval 19.01 - 05.02 1991. The atmospheric drag was calculated on the basis of five different approaches. The first two variants coinside with the variants, presented in chapter 9.
In the third variant the predicted values of the indices of solar and geomagnetic activity were used in calculating the density. The prediction technique is described in chapter 6. This last variant is suitable for the conditions of real work,connected with the control of the space complex motion.The distinction of the fourth and the fifth variants from the first and the second ones is that the ballistic coefficient of the space complex on the prediction interval was assumed to be variable: like exponent it moved from the initial current value to the mean value.
All calculations were carried out with the help of the semi-analytical algorithm of prediction (Ref.11). The initial date referred to the time interval,preceding the control of the space complex orientation. For variants 1 and 2 the obtained estimates of the decay time are presented in figure 7. For each of the variants 72 definitions were obtained. Earlier it was noted that the control of the space complex orientation had leaded to the delay of the falling time by 3 hours. Therefore the falling time 01h.02.07 (01 o'clock on the 7-th of February) was assumed to be standard. It is obvious that it did not influence on the calculated time of falling. As one should expect, with the approaching of the falling time , the absolute errors decrease but their relative values for each variant are approximately on the same level. In the process of the prediction with the constant values of the ballistic coefficient the most significant errors appear in the work with constant indices (variant1), the most insignificant ones - in the work with the a posteriori application of the known index values (variant 2).
The data, presented in the table below , give more complete idea of comparative characferistics of various variants of drag calculation. The mean (m_{ε}) and mean-square (σ_{ε}) values of a relative error ε and also quantiles of module of this error over propability levels 0.8, 0.9 and 0.95 were obtained for each variant. Variants 4 and 5 demonstrate the possibility of decreasing errors (in this case at least by a factor of 1.5) at the expense of using the assumption of stationarity of the ballistic coefficient variability , and the data on the deviation of the current value k of the space complex from the mean value.

Table 5 Statistical characteristics of the falling time determination error

the ways	m_{ε}	σ_{ε}	quantiles $\lvert\varepsilon\rvert$, probab.P		
			0.80	0.90	0.95
1	-0.004	0,073	0.113	0.128	0.132
2	-0.004	0.037	0.051	0.061	0.066
3	-0.002	0.036	0.051	0.054	0.056
4	-0.012	0.043	0.051	0.070	0.080
5	-0.008	0.025	0.035	0.040	0.044

CONCLUSIONS

1. In the process of control of the motion of the space complex "Salute-7"-"Cosmos-1689" on the last stage of its life time it became evident that the effectiveness of the theoretical and the experimental studies, conducted in the USSR in the above mentioned directions , was rather high. The obtained accuracy of determining the time and the area of falling turned out to be sufficiently high.

2.In the process of work , connected with the space complex , the informative cooperation of our Space Control System with ESA and NASA took place for the first time. As in the process of this cooperation both sides showed their interest in the developing a set of scientific areas, it is desirable that the scientific and business-like cooperation in such directions as information support of space programs, ecological monitoring of space, studying of the upper atmosphere, control of international treaties on utilization of outer space should be continued and extended.

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