METHODS FOR ORBIT DECAY PREDICTIONS USED AT THE FGAN-FHP

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ABSTRACT

This article deals with two methods for satellite lifetime prediction used at the FHP. One of these methods is the well-known procedure due to King-Hele. It is described how the second method originated as by-product of the effort to overcome certain difficulties in the supply of the input parameters for King-Hele's method.

Keywords: Satellite Lifetime, Derivative of the Mean Motion, Lagrange Planetary Equation.

1. INTRODUCTION

On the occasion of the decaying satellite COSMOS 954 the FHP, an institute of the Forschungsgesellschaft für Angewandte Naturwissenschaften (FGAN) operating a high power radar facility at Werthhoven FRG, was asked to predict the lifetime of this object. Up to this time, no pertinent activities had been carried out at the FHP. Since there was not much time left to prepare extensive computer software, it was decided to use an analytical method for this purpose.
The graphical version of King-Hele's method (Refs. 1,2) which was chosen is very easy to use and needs no computer assistance. The results obtained with this method proved to be very good. For the short-time predictions in question which were in the order of a few days the accuracy was better than 10% of the remaining lifetime, a limit given by King-Hele. However, since a graphical procedure can cause uncertainties and reading a graph is somewhat trying to the eyes, the equations on which this graphical method is based were used to write a computer program for the future use of King-Hele's method.

2. PROVISION OF THE INPUT PARAMETERS

The application of King-Hele's method requires the knowledge of some of the orbital elements, especially $\dot{n}$, the time derivative of the mean motion n. These parameters were taken from the two-line elements of the NASA prediction bulletins. In the cases of the decaying satellites COSMOS 954 and COSMOS 1402 the values of $\dot{n}$ appearing in the NASA bulletins were very accurate. But this is not always so. Sometimes, these values of $\dot{n}$ are obviously bad. In these cases one has to determine the decay rate $\dot{n}$ from several values of the mean motion. Instead of fitting a rather arbitrary polynomial to the given values of n, it was thought that it would be better to use a function which mirrors at least roughly the actual time dependence of the mean motion or - equivalently - the semi-major axis a. For this purpose a suitable function was derived by solving the Lagrange planetary equation for the semi-major axis. On the supposition that drag is the only perturbing force this Lagrange planetary equation reads

$$\frac{da}{dt} = \dot{a} = \frac{2a^2}{\mu} \underline{v}\,\underline{d} \tag{1}$$

(with μ = geocentric gravitational constant, $\underline{v}$ = velocity of the satellite, $\underline{d}$ = drag).
It is assumed that velocity and drag are parallel, therefore the scalar product $\underline{v}\,\underline{d}$ may be written as the product v d of the magnitudes of the respective vectors. Since d is proportional to the product of air density ρ and v^2, we have

$$\dot{a} = \dot{a}_0 \frac{a^2}{a_0^2} \frac{v^3}{v_0^3} \frac{\rho}{\rho_0} \quad , \tag{2}$$

where a_0, v_0, ρ_0 are the initial values of a, v, ρ.

As a first step, only circular orbits are considered. In this case v^2 is inversely proportional to a, and Eq.2 can be simplified to

$$\dot{a} = \dot{a}_0 \sqrt{\frac{a}{a_0}} \frac{\rho}{\rho_0} \tag{3}$$

Introducing the notion of scale height

$$H = -\left(\frac{d\ln\rho}{dh}\right)^{-1} = -\left(\frac{d\ln\rho}{da}\right)^{-1}$$

the quotient ρ/ρ_0 can be written as

$$\frac{\rho}{\rho_0} = \exp\left(- \int_{a_0}^{a} \frac{da}{H} \right) \tag{4}$$

It is assumed that H depends solely on the height h and that it varies linearly with h:

$$H = H_0 + \lambda (a-a_0) \tag{5}$$

with $\lambda = \frac{dH}{dh} = \frac{dH}{da}$ = const..

Substituting Eq.5 into Eq.4 and using the resulting expression to eliminate ϱ/ϱ_0 in Eq.3 one finds

$$\dot{a} = \dot{a}_0 \sqrt{\frac{a}{a_0}} \left[1 + \lambda\, \frac{a-a_0}{H_0}\right]^{-1/\lambda} \qquad (6)$$

Finally, if $a-a_0$ is small with respect to a_0, Eq.6 reduces to

$$\dot{a} = \dot{a}_0 \left[1 + \lambda\, \frac{a-a_0}{H_0}\right]^{-1/\lambda}, \qquad (7)$$

which has the solution

$$\frac{H_0}{\dot{a}_0\,(1+\lambda)} \left(\left[1 + \lambda\, \frac{a-a_0}{H_0}\right]^{\frac{1+\lambda}{\lambda}} - 1 \right) \qquad (8)$$

On setting $t_0 = 0$ and $-\frac{H_0}{\dot{a}_0\,(1+\lambda)} = L$, Eq.8 gives

$$\frac{a-a_0}{H_0} = \frac{1}{\lambda}\left[\left(1 - \frac{t}{L}\right)^{\frac{\lambda}{1+\lambda}} - 1\right], \qquad (9)$$

which is the desired functional relation between semi-major axis and time. By force of Kepler's third law an equivalent relation between mean motion and time can be obtained.
Given a sequence a_0, a_1, ...a_m of values of the semi-major axis, the unknown parameter L can be determined by the method of least squares.

3. AN INDEPENDENT METHOD OF LIFETIME PREDICTION

Obviously, the fore-mentioned quantity L has the dimension of time. The physical meaning of L emerges, when Eq.9 is utilized to substitute the term $1 + \lambda\,(a-a_0)/H_0$ in Eq. 7:

$$\dot{a} = \dot{a}_0 \left(1 - \frac{t}{L}\right)^{-\frac{1}{1+\lambda}} \qquad (10)$$

Since $1 + \lambda = 1 + \frac{dH}{dh}$ is greater than zero, the following consideration can be made:
As the time t approaches L, the decay rate $\dot{a}$ approaches $-\infty$ which suggests, that L is an approximation of the lifetime of the satellite. So, what was originally planned as a procedure to determine the decay rate $\dot{n}$ as input for King-Hele's method, is found to be a method for estimating the lifetime of a satellite in its own right.
But, so far, this has been proven only for circular orbits. The extension to elliptic orbits requires some modification. Let the satellite S have an elliptic orbit with semi-major axis a and eccentricity e then two imaginary satellites S_- and S_+, identical to S, but with circular orbits of radius a (1-e) and a (1+e) respectively are introduced. At every moment of a single revolution the drag exerted upon S will be less or equal than the drag exerted on S_- and greater or equal than the drag exerted upon S_+. Therefore the lifetime of S_+ will be greater than the lifetime of S, which for its part will be greater than the lifetime of S_-. Consequently, there must exist an intermediate value $\bar{a} = a(1-qe)$, say, with

$$a(1-e) < \bar{a} < a(1+e) \quad \text{or} \quad -1 < q < +1 ,$$

so that a satellite $S_{\bar{a}}$ identical to S, but with circular orbit of radius $\bar{a}$ has the same lifetime as S.
If to given values of a the corresponding intermediate values $\bar{a}$ were known, the elliptic orbit of S would be reduced to the equivalent circular orbit of $S_{\bar{a}}$. Certainly, it will be difficult to establish a theoretical relation for q, which is mainly a function of the eccentricity. But by the study of - admittedly only few - decaying orbits some values of q were obtained empirically for eccentricities in the interval between 0 and .02.
A rational function of the form $\frac{e}{A+Be}$ with constants A, B has been tentatively fitted to these values of q.
Thus, if there are reasons to doubt the accuracy of the $\dot{n}$-values given by the NASA-bulletins and the above described procedure is used, the additional application of King-Hele's method seems to be hardly necessary for orbits with eccentricities within the stated range.

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