ANOMALOUS PERIGEE SHIFT AND ECCENTRICITY VARIATION DUE TO AIR DRAG IN THE REENTRY PHASE

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ABSTRACT

In order to obtain new insight into the detailed structure of the lower thermosphere the long-established method of drag analysis again proves to be a powertool. For near-circular satellite orbits, in addition to the semi-major axis, the eccentricity and the argument of perigee are strongly influenced by atmospheric drag. With the help of a new computational scheme, which is based on fundamental equations of satellite drag analysis, the amplitudes and phases of global density variations are derived.

Keywords: Drag Analysis, Density Variations, Lower Thermosphere, Reentry Phase, Lifetime Prediction, Perigee Shift, Eccentricity Variation

1. METHODOLOGY

In this paper a new method of analysis is demonstrated which is able to supply large amounts of lower thermospheric total density information with very little technical effort. This is acchieved by an extension of drag analysis to the orbital parameters 'eccentricity' and 'argument of perigee'. Drag effects on the eccentricity and the argument of perigee have already been discussed in the literature (Refs. 1-4). But the mathematical expressions underlying these analytical approaches are complicated even with simplifying assumptions, and the authors therefore restrict themselves to a basic and to a large extent phenomenological considerations. The numerical approach presented in this paper is mathematically much simpler and can easily be expanded to process density structures of great complexity.

In the terrestrial thermosphere there are two major causes which lead to secular perturbations of satellite orbits:

(a) Gravitational perturbations due to the second zonal harmonic of the earth's gravitational potential

(b) Atmospheric drag Due to gravitational perturbations, the right ascension of the ascending node Ω and the argument of perigee ω are changed secularly, whereas the orbital eccentricity e performs only periodic perturbations with small amplitudes (Ref. 5). Due to the orbital drag the semi-major axis a decreases according to

$$\Delta a = -2 \pi a^2 \rho CA/M$$
 (per orbit) (1)

where Q is the mean atmospheric density along the orbit, C the drag coefficient, A the cross section and M the mass of the satellite. ${\boldsymbol{\Omega}}$ is practically not influenced by drag because it is affected only by forces normal to the orbital plane. e and ω are always changed by atmospheric drag. In the case of e, the drag leads to a steady decrease of the eccentricity. In the case of $\boldsymbol{\omega},$ drag effects are negligible in the upper thermosphere compared to the gravitational influence.

A drastic transition in the behaviour of e and ω occurs when the density increases and the eccentricity decreases. These conditions are usually met when a satellite drops below about 250 km. From a straightforward evaluation of the perturbation equations of e and ω we obtain for nearcircular orbits the drag-induced changes of e and ω according to

$$\Delta e = - a K \int_{0}^{2\pi} \cos \nu \exp(L(\nu)) d\nu$$

$$\Delta \omega = - \frac{a}{e} K \int_{0}^{2\pi} \sin \nu \exp(L(\nu)) d\nu$$
(2)

$$\Delta \omega = -\frac{a}{e} K \int_{0}^{2\pi} \sin \nu \exp(L(\nu)) d\nu$$
 (3)

per orbit, where ${m v}$ is the true anomaly and K = ${m Q}$ CA/M. The function L is an expression for the variations of density along the orbit. In a spherically symmetrical atmosphere there is L = 0 along a circular orbit, and no changes of e and w occur. Otherwise, e is predominantly influenced

by the density difference between perigee and apogee, whereas $\boldsymbol{\omega}$ is predominantly influenced by the density difference between locations near the semiminor axis. Therefore the amplitude and phase of the fundamental density variation along the orbit can be derived from Eq. 2 and Eq. 3 simultaneously. The determination of the factor CA/M can be avoided if only the relative density variation L is examined. In this case the value K is derived from observed changes of the semi-major axis according to Eq. 1, but no conclusions can be made then on the absolute density.

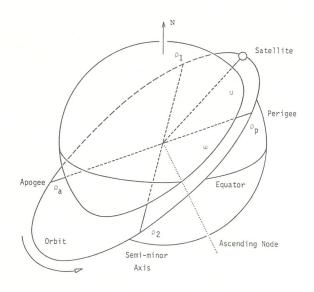


Figure 1. Geometry of the satellite orbit

2. RESULTS AND DISCUSSION

Only the careful tracking of satellite decay can provide us with the orbital parameters necessary for an application of the proposed drag analysis. Such information became available from the decay of the fragments Cosmos 1402 A and Cosmos 1402 C. These two sets of observations are supplemented by orbital elements from Skylab 1 (Ref. 4). For the analysis, the orbital elements were processed by a smoothing cubic spline fit in order to obtain equally spaced data at 1-day intervals.

The density variation function L occuring in Eqs. 2 and 3 is made up of two different contributions L1 and L2. Variations due to the eccentricity of the orbit and to the oblateness of the earth are given by

L1 = (a e
$$\cos \mathbf{v}$$
 - a + R
- 21.4 $\sin^2 \mathbf{p}$) / H

where R is the equatorial radius of the earth, H the mean atmospheric scale height, and \emptyset the geographical latitude.

Variations due to density changes with latitude, local time, day of the year, solar flux and geomagnetic activity are represented by a term L2. In semi-empirical models L2 is generally given by spherical harmonic functions.

In Figs. 2 and 3 the arguments of perigee and the eccentricities of Skylab 1, Cosmos 1402 A and Cosmos 1402 C are shown as functions of time during the late decay phases of the satellites. The short dashed lines show the variations of e and ω including only gravitational perturbations and the contribution L1. According to L2 = 0, no density variations are assumed at ellipsoidal surfaces of equal height above the earth's surface. The disagreement between observations and computations shows that density variations represented by L2 play a major role in the satellite dynamics at lower thermospheric altitudes.

In a second approach a more sophisticated density distribution is introduced by calculating L2 from a semi-empirical model. It is well known that all available thermospheric models are predominantly based on upper thermospheric density data, and none of the models claim to give an accurate description of the lower thermosphere. But all other data available give only momentary information on the atmospheric density and are therefore not suitable for examining global aspects; the models are therefore representative of our present standard of lower thermospheric modeling.

The long-dashed lines in Figs. 2 and 3 show the calculated variations of e and $\boldsymbol{\omega}$ where L2 is derived from the DTM model (Ref. 6). Still, there is striking disagreement between the computational results and the observations. This is not a feature characteristic for the DTM model; the application of other semi-empirical models results in comparable discrepancies. Moreover, our results are confirmed by a much more detailed approach by Klinkrad (Ref. 7). This leads to the conclusion that phases and amplitudes of density in these models do not correctly represent the actual state of the lower thermosphere.

In a third and final approach the application of the scheme is reversed: instead of calculating the orbits on the basis of given density structures, the density structure is derived from the observed orbits. As only three sets of observations are analyzed, theanalytical approach of the density variation L2 can only be relatively simple, and only fundamental diurnal and seasonal variations are considered. L2 is then given by

$$L2 = D \cos(t - T) + S \sin \emptyset$$
 (5)

where S is the seasonal amplitude, D and T are the diurnal amplitude and phase and t is the local solar time. The simplicity of the density model is physically justified by the fact that e and ω are influenced by higher harmonics to a less extent.

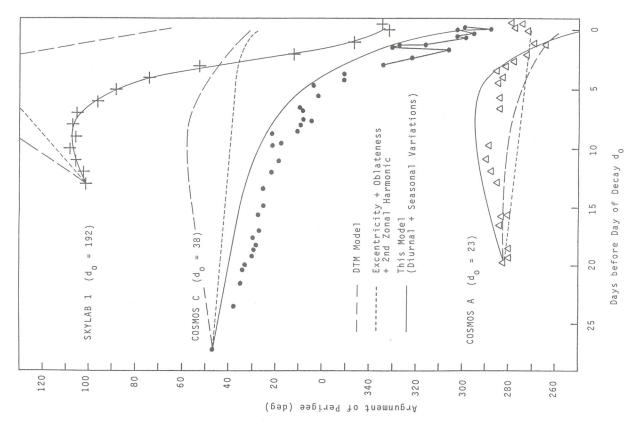


Figure 2. Argument of perigee

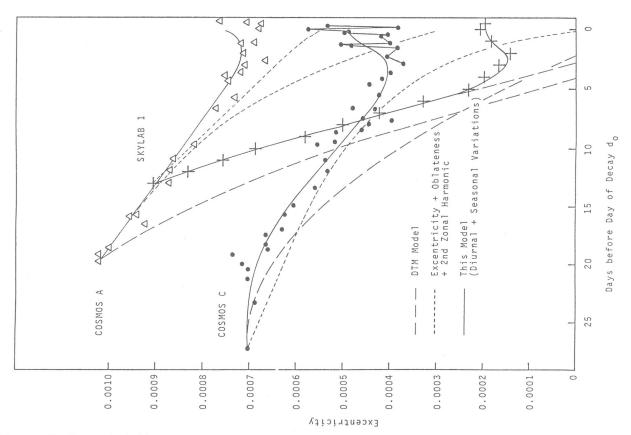


Figure 3. Excentricity

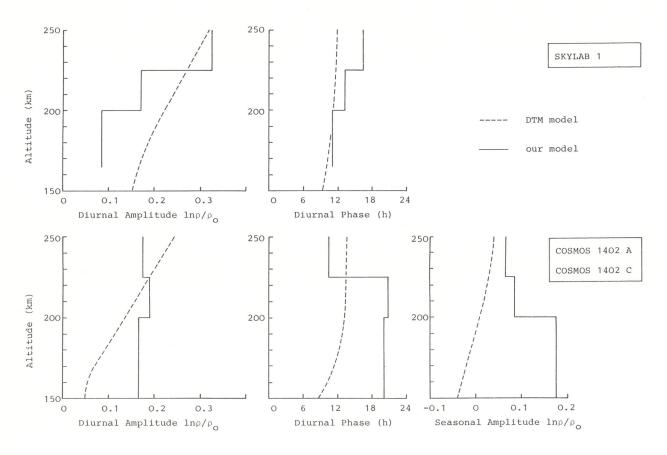


Figure 4. Density amplitudes and phases from the DTM model and from drag analysis

S, D and T are calculated by a least-squares-procedure adjusting the computed variations of the orbital elements e and ω to the observations. In order to account for possible variations of the parameters S, D and T with height, three different altitude regions above 225 km, between 225 and 200 km and below 200 km are considered, where the parameters can attain different values. Accordingly, each fit determines nine parameters.

Because the Cosmos 1402 fragments decayed in early 1983 and the Skylab 1 rocket in the middle of 1979, different seasons and different solar and geomagnetic conditions are covered by the data, so that different density structures are to be expected. For this reason two calculations were carried out, the first based on the data of Skylab 1 and referring to northern summer conditions, the second based on the data of both Cosmos 1402 A and C simultaneously and referring to northern winter conditions. The changes of e and ω which result from using the simple atmospheric model (Eq. 5) are shown in Figs. 2 and 3 by solid lines. Observations and calculations show good to excellent agreement.

A comparison between the parameters S, D and T as derived from our calculations and as given by the DTM model is shown in Fig. 4. The DTM model values are derived

from a Fourier analysis of the total density for Kp = 0 and solar fluxes of 187 at day 180 (Skylab 1) and 143 at day 30 (Cosmos 1402 fragments). The explanation of the Skylab orbit requires a diurnal variation only, and the use of a seasonal component does not improve the fit. Only slight deviations from the DTM model are responsible for the direction of motion of the apsidal line. This indicates the high sensitivity of the analyzing scheme. The explanation of the orbits of the Cosmos fragments requires both a diurnal and a seasonal component. The most remarkable feature exhibited in Fig. 4 are the strong differences between our model and the DTM model at altitudes below 225 km, especially the phase shift in the diurnal variation of more than eight hours. The different density structures resulting from the Cosmos fragments and the Skylab rocket are probably due to density variations correlated with geomagnetic activity, solar activity and season. But an expansion of the density model (Eq. 5) is far beyond the data base and physically not justified.

The parameters S, D and T are not restricted by any conditions except that they must yield the observed orbits. In particular, they are not coupled to a thermospheric model at the upper or lower boundary. Nevertheless, we obtain

amplitudes and phases at 250 km which are fairly consistent with the DTM model in a region where this model should still be applicable. This is a good indication for the reliability of our method of analysis.

In the case of Cosmos 1402 C a strong geomagnetic storm with a daily Ap value of 143 occurred two days before the final decay. This is clearly reflected in the orbital parameters e and $\pmb{\omega}$. With more satellite decay data, it will be possible to deduce important details of the magnetic storm behaviour of the lower thermosphere from changes of e and $\pmb{\omega}_{\bullet}$ The application of the scheme to an expanded data base will thus lead to a considerable step forward in our present knowledge of the lower thermospheric global density structure and, from a more practical point of view, the accuracy of orbit and lifetime predictions for satellites in their late decay phase will be improved.

3. REFERENCES

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