LIFETIME PREDICTIONS FOR THE RAE TABLE OF SATELLITES

D.M.C. Walker D.G. King-Hele

Royal Aircraft Establishment, Farnborough, Hants, UK

ABSTRACT

After studies of Earth satellites in 1955-56, we operated the UK satellite prediction service in 1958 and began a Table of satellites in response to requests. This has now grown to the 770-page *RAE Table of Earth Satellites 1957-1982*, which includes lifetime estimates for all satellites still in orbit, excluding fragments. We have made about 10,000 such lifetime predictions and in this paper we describe the methods of calculation. For most low-eccentricity satellites decaying under the influence of air drag, we use a simple graphical method, and an accuracy of ±10% is about the best that can be achieved because of the unpredictable variations in air density. For high-eccentricity satellites we have a graphical method for transfer orbits at inclinations between 0 and 30°, and otherwise we rely on two computer programs, PROD and PTDEC, where the limits are on computer time and cost.

Keywords: Satellite lifetimes, orbits, orbit theory, orbit computation.

1. INTRODUCTION

It is appropriate to begin with a few words of past history, to explain how we came to start this curious and exacting task of predicting the lifetimes of all satellites launched, as part of the information given in the *RAE Table of Earth Satellites* (Ref. 1).

In the autumn of 1955 we were working on a design study of an expendable Earth satellite for reconnaissance. We had a satellite of mass 1 ton launched by a two-stage rocket having liquid-propellant motors and a weight at launch of 60 tons. The climb trajectory was very flat, with a coasting phase of several thousand miles. We chose a circular orbit at a height of 200 nautical miles (370 km), quite near what has subsequently been used by many of the reconnaissance satellites launched by the USSR. We concluded that the skin temperatures of the satellite would be acceptable and meteor hazards negligible: Fig 1 shows the title page of the report (Ref. 2), issued in January 1956. Maps of the satellite track over the Earth were also calculated, and Fig 2 shows one of these diagrams, the appearance of which has subsequently become familiar.

FIG. 1

Technical Note No. G.W.393

January, 1956

ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

Preliminary assessment of an earth-satellite reconnaissance vehicle

(excluding problems of photography, etc.)

by

D. G. King-Hele
and
Miss D. M. C. Gilmore

SUMMARY

The problems of placing an expendable satellite in a circular orbit about the earth are investigated briefly, and preliminary estimates are made of the likely weight at launch, flight path, etc. An orbital altitude of about 200 n.miles should be low enough for reconnaissance purposes and high enough to avoid serious aerodynamic retardation. At this altitude the satellite would have an orbital speed of 25,200 ft/sec and would make nearly 16 revolutions per day. Possible methods of reconnaissance are discussed in ref.1.

Using a conventional liquid-oxygen rocket motor and a lightweight pressurized-tank structure, a 2000-lb satellite could be placed in an orbit at 200 n.miles altitude for a missile weight at launch of about 125,000 lb, using two stages of propulsion and assuming some advantage can be gained from the earth's rotation. See Figs.9 and 23. With no such advantage, e.g. for an orbit over the poles, weight would go up by about 10%. Fig.17. Missile weight at launch increases by about 4% for every 100 n.mile increase in orbital altitude (up to 1000 n.miles).

Other conclusions are as follows. The climb path should be very flat (see Fig.13) including a coasting phase of several thousand miles. With a photographic strip 40-50 miles wide, and a factor of 4 for adverse weather, reconnaissance should be complete in about 100 days: Figs.24-26 give examples of the satellite's track over the earth's surface. Skin temperatures should be acceptable, between 200 and 400°K, and meteor hazards negligible. Guidance, control and power supply set difficult problems, but at first glance none appear insoluble.

Later in 1956 we made a detailed mathematical study of the decay phase of a satellite initially in a circular orbit, including the effects of aerodynamic heating, and going right down to sea level, where the angle of descent was calculated as 73°, if the satellite was sufficiently robust to survive the intense aerodynamic heating. Fig 3 shows the title page of this report (Ref. 3), and Fig 4 shows the decrease in altitude during the 100 days of the descent phase.

We have shown these little glimpses of work from the past because they have not previously been published and they may explain why we were so

From "Preliminary assessment of an Earth Satellite Reconnaissance Vehicle" by D G King-Hele and Miss D M C Gilmore. RAE Technical Note GW393 (January 1956)

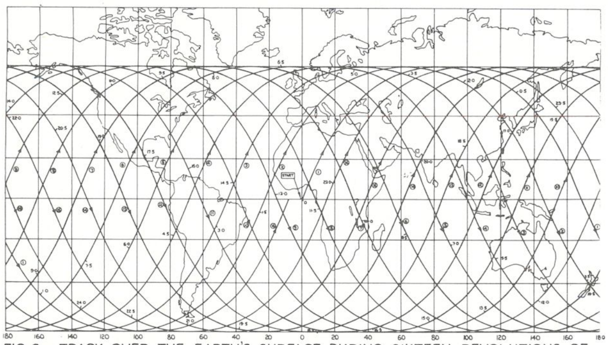

FIG. 2 TRACK OVER THE EARTH'S SURFACE DURING SIXTEEN REVOLUTIONS OF A SATELLITE WITH MAX. LATITUDE 60°.
THE TRACK STARTS WHEN THE SATELLITE PASSES OVER THE POINT WHERE THE GREENWICH MERIDIAN CUTS THE EQUATOR.
UN-RINGED NUMBERS ON THE CURVES INDICATE THE TIME IN HOURS AFTER THE START.
A RINGED NUMBER, ⓝ, INDICATES THAT THE SATELLITE IS THEN ON ITS n-TH REVOLUTION.

immediately involved in the work of tracking and orbit determination when Sputnik 1 was launched in October 1957. We found values of air density from Sputnik 1 within two weeks (Ref. 4), and these values have stood the test of time (Ref. 5). We also developed methods for predicting the decay of that satellite and its rocket at the end of 1957. Early in 1958 we became responsible for the prediction service for the UK and in response to requests we produced a list of satellites in orbit. That single sheet was the beginning of the RAE Table of satellites, for which we make estimates of the lifetimes of all satellites, final-stage rockets and other main components, but not fragments. We have continuously updated the estimates as necessary in successive issues of the Table, which now runs to more than 800 pages. We have made lifetime predictions for about 7000 satellites over the years, some of them only once and a few of them 10 or 20 times - probably about 10,000 predictions in all.

Before going on to describe our methods of prediction, we should point out that predicting satellite lifetimes is a foolish thing to do. If you are right, people wrongly think it is easy. If you are wrong, they condemn you for being incompetent.

When we were first enticed into making lifetime estimates regularly in the summer of 1958, we suspected that solar activity might control the air density, but we did not become certain about the effects of solar activity until November 1958, as a result of the clear 28-day variation in the orbital decay rate of the rocket of Sputnik 3. We were predicting the transits of this satellite and keeping an up-to-date record of the decay rate by analysing the observations each day as they came in, and Fig 5 shows the variation in density that was obtained.

Today it is well known that air density is greatly dependent on solar activity; but no one knows how the solar activity will vary in the next 50 or 100 years, or even in the next 5 or 10 years; so accurate long-term prediction is impossible. Nevertheless we must do the best we can, and we now describe our methods.

2. GRAPHICAL METHODS FOR LOW-ECCENTRICITY ORBITS

With most of the satellites launched we use simple graphical methods for estimating lifetime, based on orbital theory. For satellites that are continuously subject to air drag, elaborate computational

FIG. 3

Technical Note No. GW 430

September, 1956

ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

The descent of an earth-satellite through the atmosphere

by

D. G. King-Hele

and

Miss D. M. C. Gilmore

SUMMARY

A theoretical study is made of the flight path of an uncontrolled but aerodynamically stable satellite when, under the action of air drag, it spirals down through the earth's atmosphere from an initially circular orbit. The earth and its atmosphere are taken as spherically symmetrical and the assumed variation of air density with altitude is shown in Fig.1.

Under these assumptions the speed of the satellite during its descent is, for altitudes above about 110 n. miles, independent of its shape, size, weight and initial altitude, and equal to the orbital speed appropriate to its current altitude (see Fig.8). The angle of descent, in radians, is simply twice the drag/weight ratio, again for altitudes above about 110 n.m. Below this the speed and angle of descent at a given altitude are increasingly dependent on SC_D/m, where C_D is the drag coefficient based on area S, and m is the mass. For the satellite taken here as standard (base diameter 5 ft, mass 2000 lb) speed remains between 26,000 and 22,000 ft/sec down to 20 n.m. altitude, then drops quickly towards the terminal value of 700 ft/sec (see Fig.13), while the angle of descent increases from 0.01° at 100 n.m. to 1° at 45 n.m. and 73° at sea level (see Fig.17). The 'lifetime' of a satellite is inversely proportional to SC_D/m and varies with initial altitude as shown in Fig.11: the standard satellite, initially at 200 n.m. altitude, has a life of 104 days.. A satellite with drag/weight similar to that of the 'standard satellite' is unlikely to survive the aerodynamic heating on its descent without an exceptionally tough heat shield: a steel skin ½" thick would not give adequate protection.

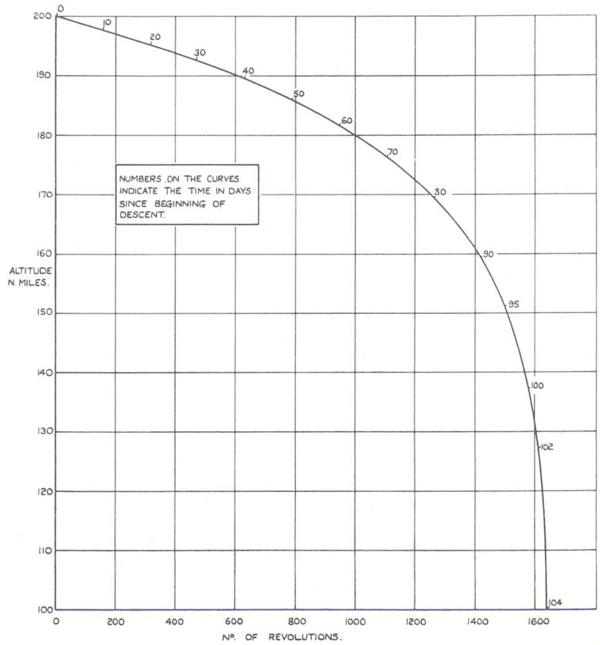

FIG.4 NUMBER OF REVOLUTIONS PERFORMED BY THE STANDARD SATELLITE DURING ITS DESCENT FROM 200 N.MILES TO 100 N.MILES ALTITUDE

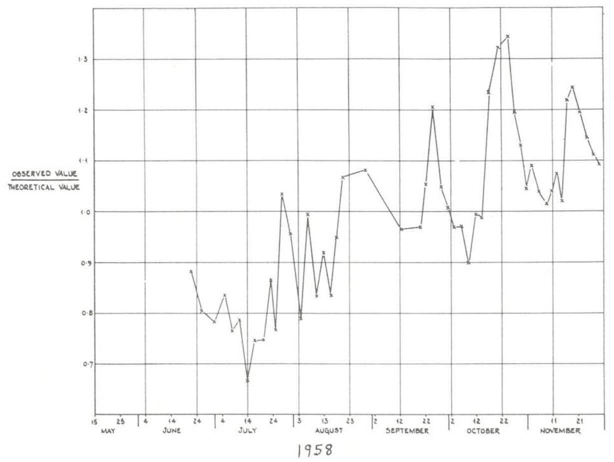

FIG. 5 RATE OF CHANGE OF PERIOD OF SPUTNIK 3 ROCKET (1958 δ1) RATIO OF OBSERVED VALUES TO THEORETICAL

methods are inappropriate because the air density suffers unpredictable day-to-day variations of up to ±10%. Thus even the most sophisticated computations, with full gravity fields and detailed model atmospheres, do not in general achieve an accuracy in lifetime estimation better than about 10%. Errors of up to about 3% in reading off graphs are therefore quite acceptable.

Our graphical methods, which will now be summarized, are described more fully in Ref 6. We express the lifetime L in terms of the decay rate $\dot{n}$, where n is the mean motion (the number of revolutions per day) and $\dot{n} = dn/dt$, by the equation

$$L = \frac{Q}{\dot{n}} ,$$

where the quantity Q is read from graphs. If $\dot{n}$ is in rev/day^2, the lifetime is obtained in days.

Fig 6 shows the values of Q for near-circular orbits, $0 \leqslant e \leqslant 0.01$. For this and for subsequent graphs there are two diagrams, one for low solar activity and one for high solar activity.

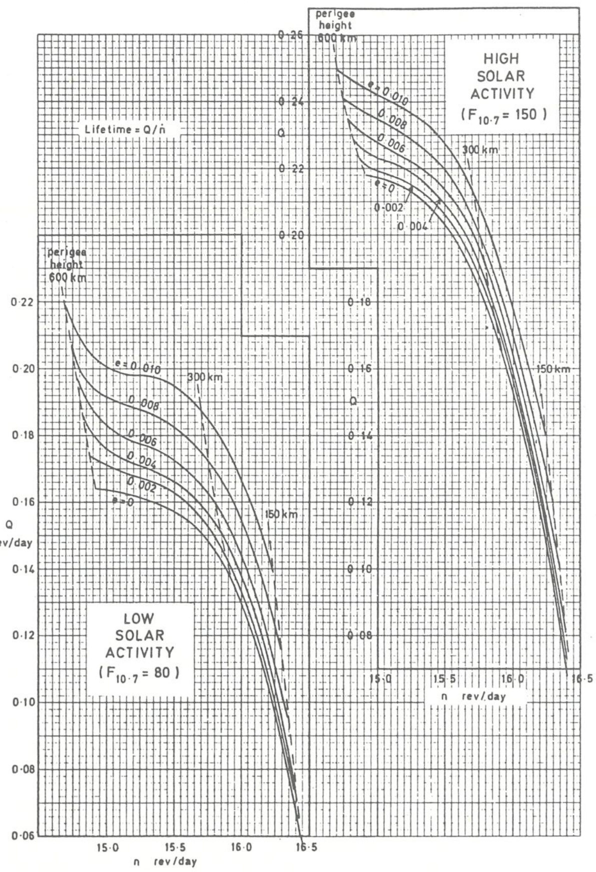

Fig 6 Variation of Q with n for $e \leqslant 0.01$

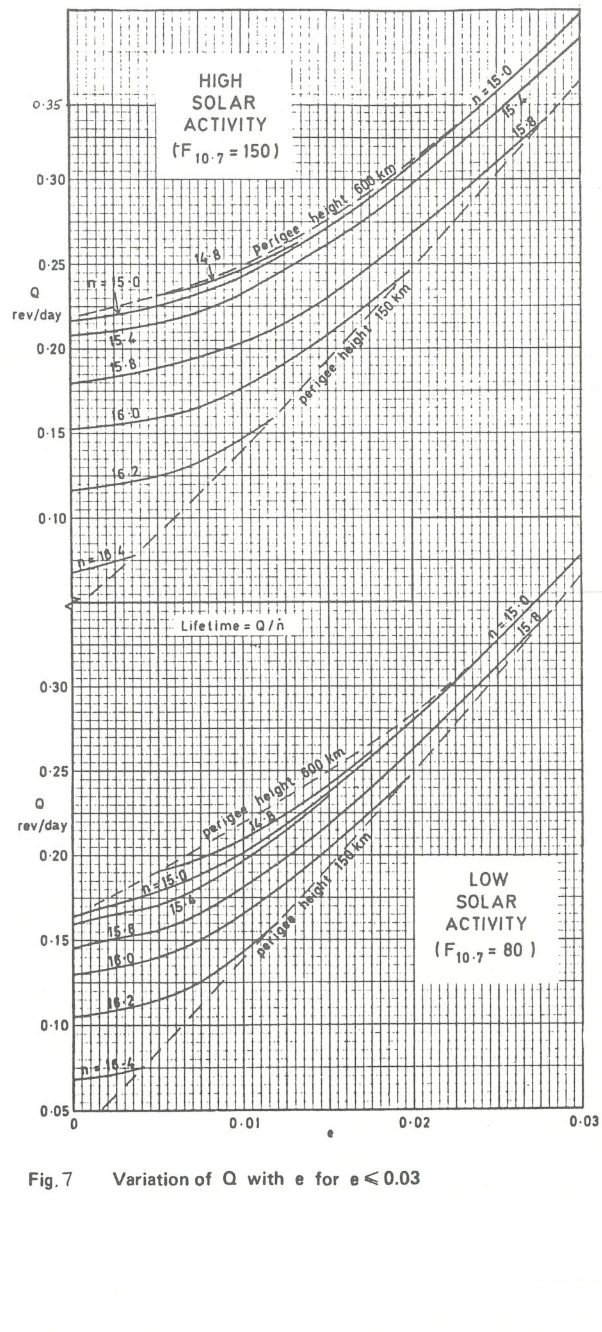

Fig. 7 Variation of Q with e for $e \leqslant 0.03$

Interpolation between these diagrams is necessary: the greatest errors due to interpolation occur for circular orbits, and may sometimes be as great as 5%.

Fig 7 shows the graphs used for $0.01 \leqslant e \leqslant 0.03$. Here the interpolation for solar activity should not cause significant error. In calculating $\dot{n}$, we usually take values of n from two recent NORAD 2-line elements at dates sufficiently far apart to give a reliable value for the difference in n, and then divide that difference by the time interval. If there is any suspicion of error, we try other sets of elements, and for decays of special interest we plot all the values of n against time to identify any 'rogue' values. We try never to use the values of $\dot{n}$ given in the 2-line elements, as these are essentially instantaneous values and are affected by transient variations in air density and sometimes also by poor observations.

Fig 8 shows the diagrams for $0.03 \leqslant e \leqslant 0.2$. By plotting Q/e instead of Q, the values can be read very accurately; and solar activity has little effect.

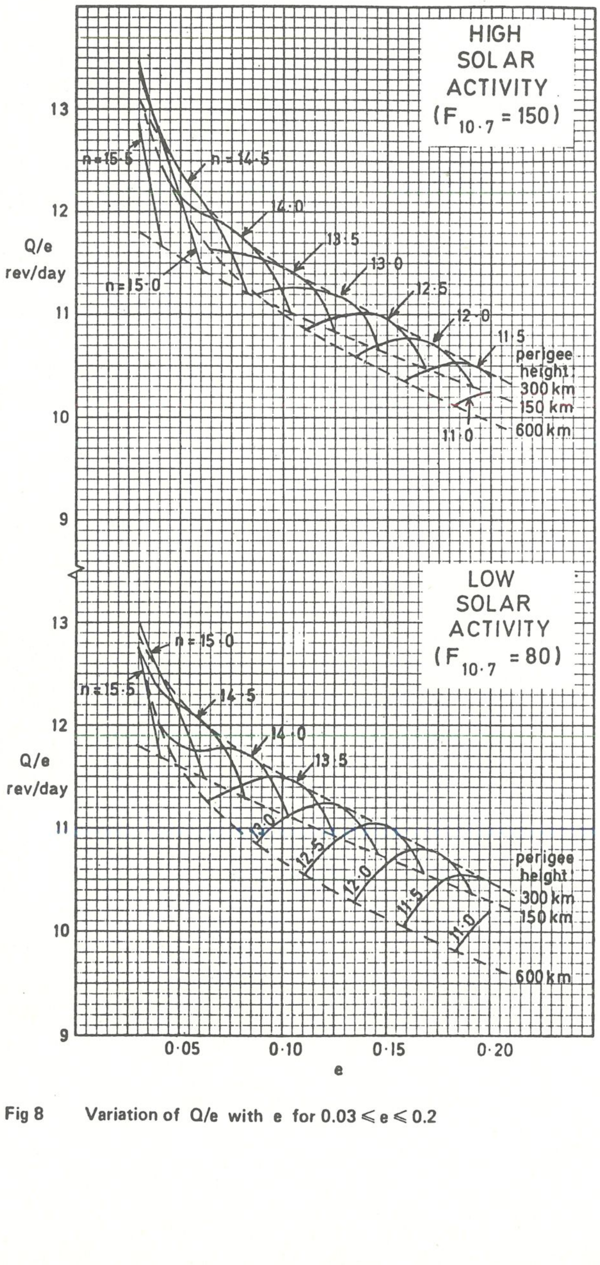

Fig 8 Variation of Q/e with e for $0.03 \leqslant e \leqslant 0.2$

Finally, Fig 9 shows the graphs for $0.2 < e < 0.8$. This is not very useful for eccentricity greater than 0.4 because lunisolar perturbations usually have an important effect on the lifetime and have to be taken into account, but the diagram is quite satisfactory for $0.2 < e < 0.4$ and serves as an approximate guide for higher eccentricities with satellites having orbital inclinations less than 30°.

These graphs are all calculated for a spherical Earth having a spherically symmetrical atmosphere which does not vary with time. The real situation is unfortunately much more complex. The odd zonal harmonics in the Earth's gravity field make the perigee height oscillate as perigee travels from the northern to the southern hemisphere, and the oblateness of the Earth produces a further oscillation in perigee height. Together these effects produce quite a complex variation in perigee height (shown in Fig 11 of Ref. 6), and corrections can be made for this effect; but we have to confess that we do not often make these corrections fully, and this is probably the greatest source of error in our routine lifetime predictions over time intervals of a few months.

The other simplification which is too inaccurate to accept is the idea that density is independent of time. Although the variations due to solar activity are the most important in the long term, and sometimes for short-term predictions of a few months, there is one continuous variation which occurs even when solar activity is constant. This is the semi-annual variation, which has a strong effect on the estimation of lifetimes between

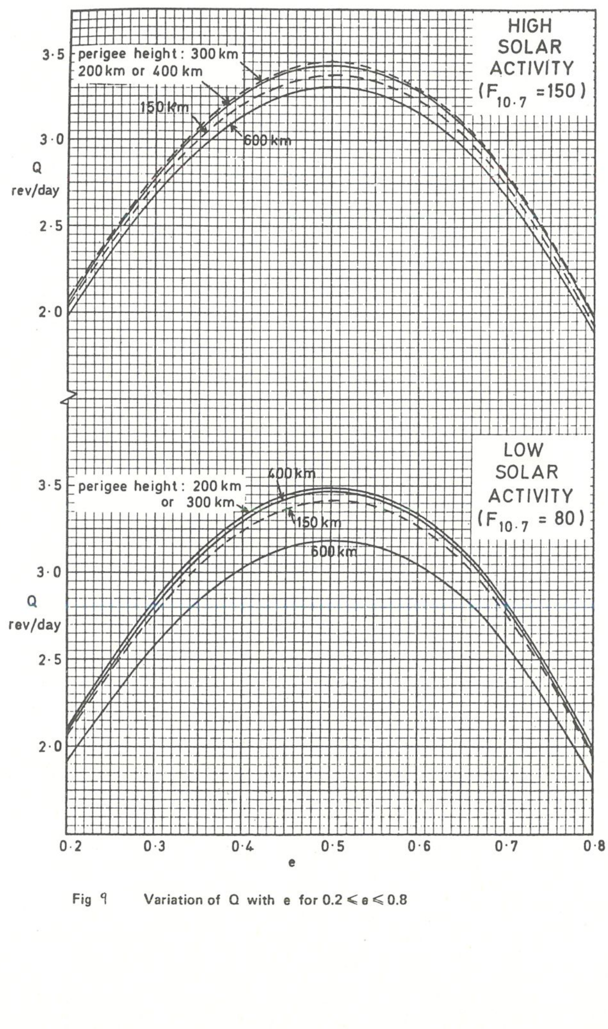

Fig 9 Variation of Q with e for $0.2 \leqslant e \leqslant 0.8$

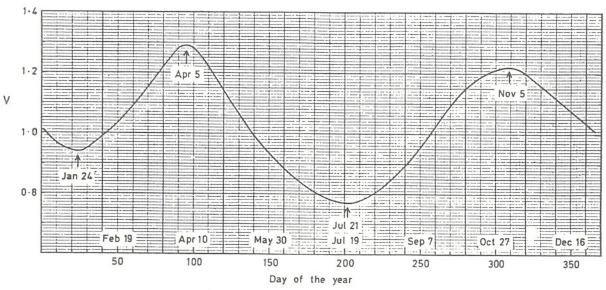

Fig 10 Semi-annual variation V recommended in Ref 7 for heights of 200-250 km

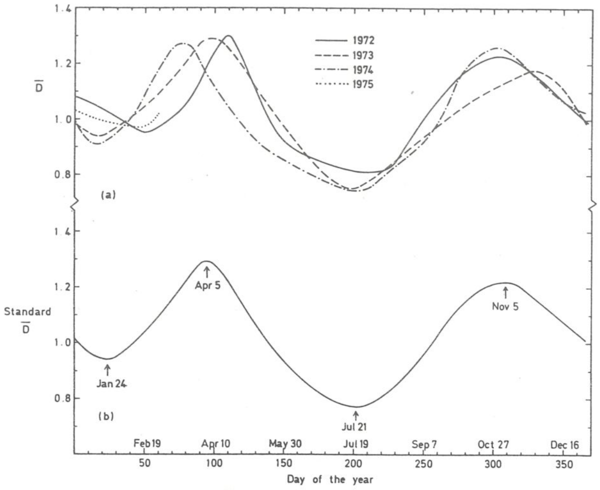

Fig 11 Variation of D̄ and standard D̄ during the year (Ref. 7)

2 months and 18 months. Fig 10 shows the standard curve we use, but it should be emphasized that the form of the variation is somewhat different each year, as shown in Fig 11 (from Ref. 7). The effect of the semi-annual variation on lifetime is shown

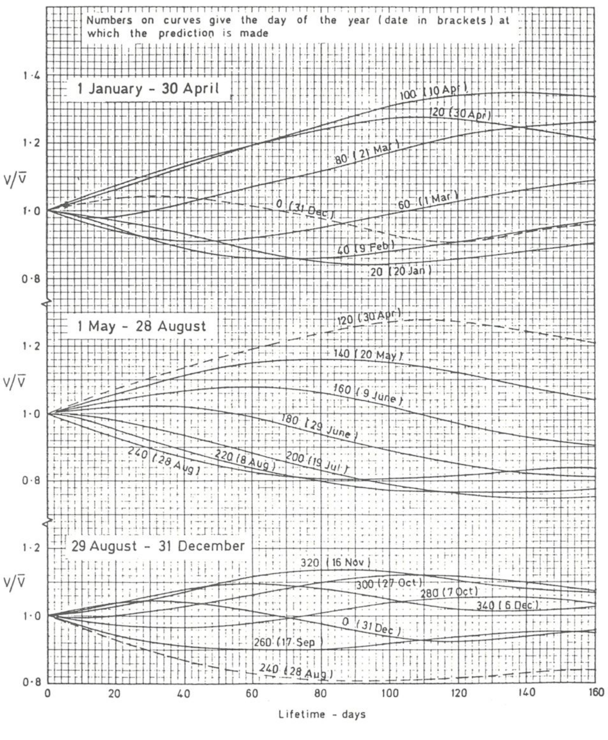

Fig 12 Variation of semi-annual correction factor V/V̄ with lifetime

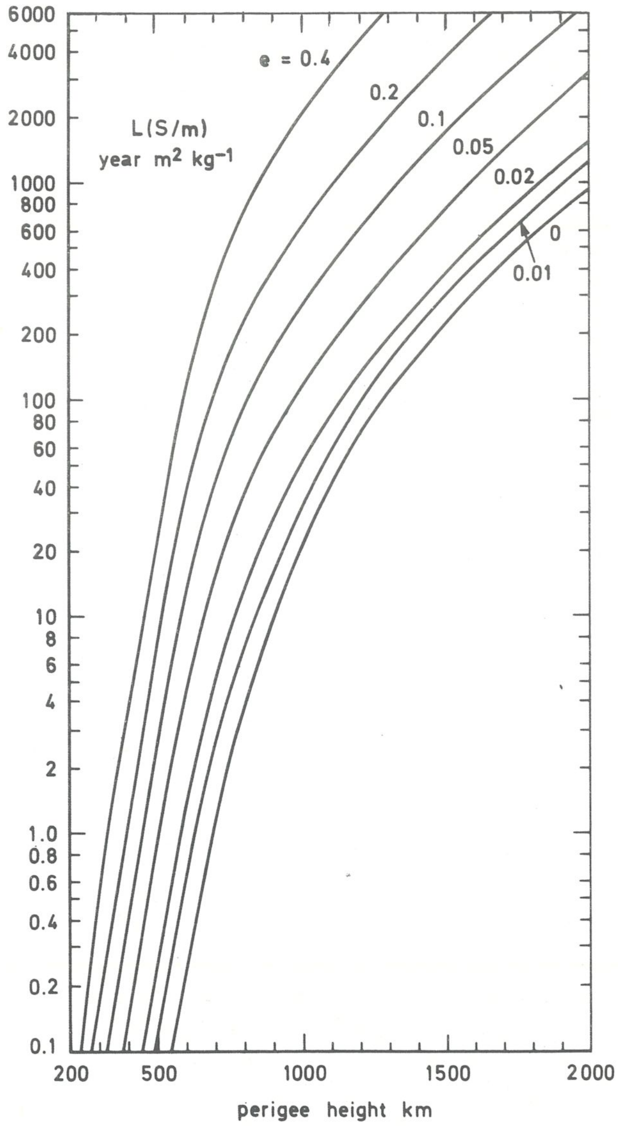

Fig 13 Lifetimes of long-lived satellites, based on mean density over an average solar cycle

in Fig 12, and this is the graph we regularly use: it shows that variations in lifetime by a factor of up to 1.3 may be produced at certain times of year. For example, a lifetime of 100 days calculated on 10 April would be quoted as 130 days because the factor is 1.30: the lifetime is lengthened because the satellite spends its last month or two in the trough of the semi-annual minimum in July-August.

These are the basic methods for short-lifetime satellites. For long lifetimes, we can use graphs such as Fig 13, giving the lifetime in terms of the mass-to-area ratio, which are based on the assumption that the average solar activity in future solar cycles will be the same as in the past few cycles: as already mentioned, this is an assumption which may be considerably in error.

3. GRAPHICAL METHODS FOR HIGH-ECCENTRICITY ORBITS

The low-eccentricity satellites are relatively easy. Now we come to the difficult ones, those of high eccentricity, which are seriously affected by lunisolar perturbations. Here we often have to make numerical integrations, but there are two

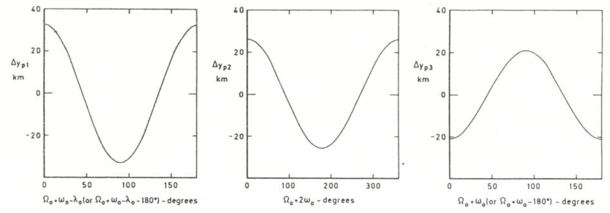

Fig 14 Charts for finding Δy_{p1}, Δy_{p2} and Δy_{p3}

simpler graphical methods that we use whenever possible.

The first method is for satellites in transfer orbits with eccentricity near 0.73 at inclinations between 5^o and about 30^o, and having lifetimes of more than one year. There are very many of these satellites, nearly all of them rockets that have taken geostationary satellites into orbit. At these inclinations the lunisolar forces produce an oscillation in perigee height which has an amplitude of only about 50 km and a period usually somewhat less than 1 year. So, if the lifetime is at least several years, it is possible to use an average perigee height, which can be calculated from the current perigee height. The theory is given in Ref. 8, and Fig 14 shows the corrections which have to be made to the current perigee height in order to determine the future mean perigee height. As can be seen, the magnitudes of the corrections depend on the right ascension of the node Ω, the argument of perigee ω and the solar longitude λ. These results apply for an inclination of 25^o, but Ref. 8 also gives the changes in amplitude for inclinations between 5^o and 30^o. The total amplitude of the variation is close to 50 km for

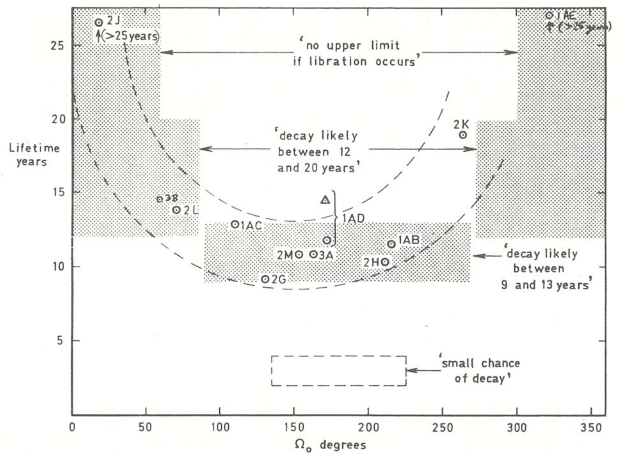

Fig 16 Lifetimes of 63° Molniyas as predicted by PROD, with approximate lifetime domains indicated by theory.

these inclinations, but rapidly increases when inclination exceeds 30^o.

After the average perigee height has been found, it is also necessary to make a small correction to allow for the greater effect of drag in the lower part of the oscillation. When that correction has been made, Fig 15 can be used to estimate the lifetime. This method is usually quite accurate if the initial orbit is known, and it saves us a great deal of time and money in computer running.

Orbits of high eccentricity at inclinations greater than 40^o suffer much larger lunisolar perturbations and are much more difficult to deal with. An approximate theory for Molniya satellites at inclinations of 65^o or 63^o has been developed in Ref. 9, where it is shown that the lifetime depends strongly on the initial longitude of the node.

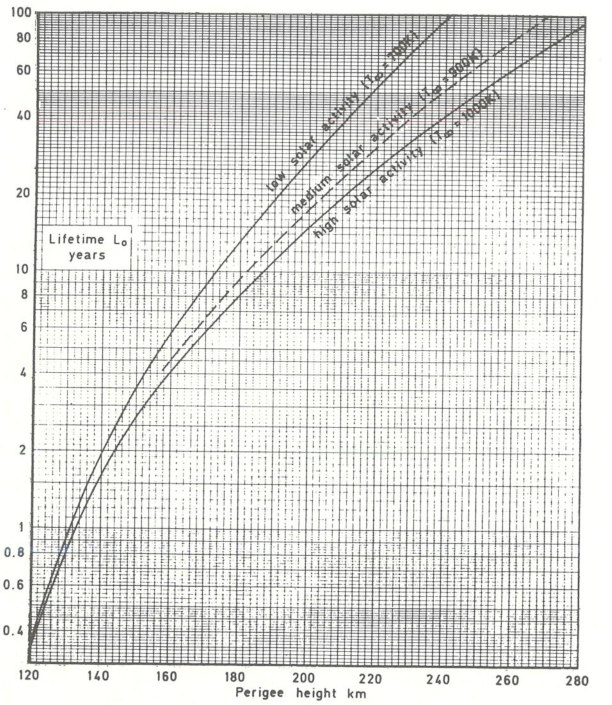

Fig 15 Lifetime L_0 of satellite of mass/area m/S = 100 kg/m^2 in transfer orbit of eccentricity 0.73. If m/S ≠ 100, multiply L_0 by m/100S. Lunisolar perturbations ignored

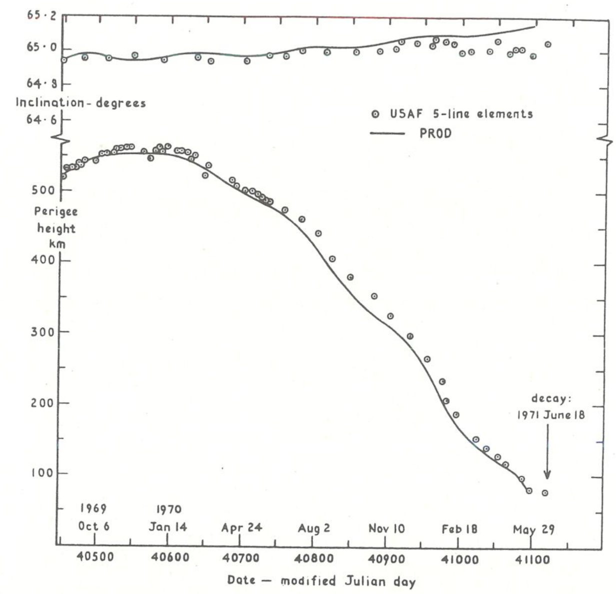

Fig 17 Variation of perigee height and inclination for Molniya 1M, 1969-61A, as given by PROD and USAF elements

Although it is still necessary to confirm the estimates with numerical integrations, we use Fig 16 for making approximate initial estimates of the lifetime: for example we quote a lifetime of '10 years?' if the value of Ω is between 90° and 270°.

4. NUMERICAL INTEGRATIONS

Apart from the graphical method for transfer orbits at low inclinations, numerical integration is needed to determine lifetimes for high-eccentricity orbits. The first computer program that we use for this purpose is PROD (Ref. 10). This has no allowance for air drag, but is an extremely useful program because it is reasonably economical with computer time: we normally integrate ahead for about 20 years with an integration interval of 5 or 10 days. The program includes a full terrestrial gravity field of zonal harmonics and as many harmonics as necessary in the lunar and solar gravity fields. Also the integration interval can be altered as required: we rarely find it necessary to have an interval of less than 5 days for these long-term integrations. Fig 17 shows an example comparing PROD with the actual orbits of a high-eccentricity satellite and the agreement is very good until the perigee dips deeply into the atmosphere. We have found that predictions for satellite lifetimes of 10 or 12 years made with PROD have usually been accurate to within a few months, and this is rather a better percentage accuracy than is achieved with the short-term prediction of satellites subject to continuous drag.

When we have a high-eccentricity orbit which is severely affected by air drag, with a perigee height lower than 180 km, we resort to a more elaborate computer program called PTDEC (Ref. 11). This program numerically integrates the orbit, with full allowance for lunisolar perturbations and air drag, the density model being the *COSPAR International Reference Atmosphere 1972* (Ref. 12). In PTDEC as normally run, there are 96 steps per revolution (Ref. 13) with the step length adjusted according to the distance from the Earth's centre in accordance with the methods developed by Merson (Ref. 14). The number of integration steps can be increased whenever necessary, and this is sometimes needed for eccentricities greater than 0.8. Ideally, PTDEC should give nearly perfect results, and often it is very good indeed, as in Fig 18. There are three problems, the first being the computer time. The program usually takes several hours on the RAE computer, even for an integration of only 200 days. Second, there is the computer cost, which is about \$1 per day of lifetime. So it is not practicable to use the program for satellites of very long life - a lifetime of 10^6 days can easily arise with these satellites. In fact, we confine our use of the program to a maximum time of 200 days and only use it for satellites which cannot be tackled by other methods.

These two problems are limitations on the use of PTDEC, but the really difficult problem is the third one. How do you obtain an accurate initial orbit? For most satellites we have nothing but NORAD orbits, and they can have errors of up to 50 km in perigee height for high-eccentricity orbits of high

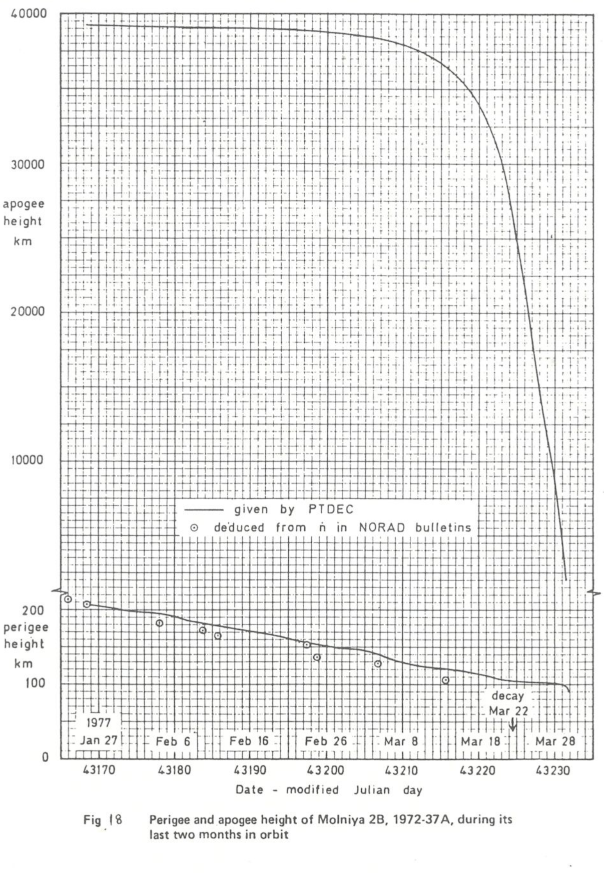

Fig 18 Perigee and apogee height of Molniya 2B, 1972-37A, during its last two months in orbit

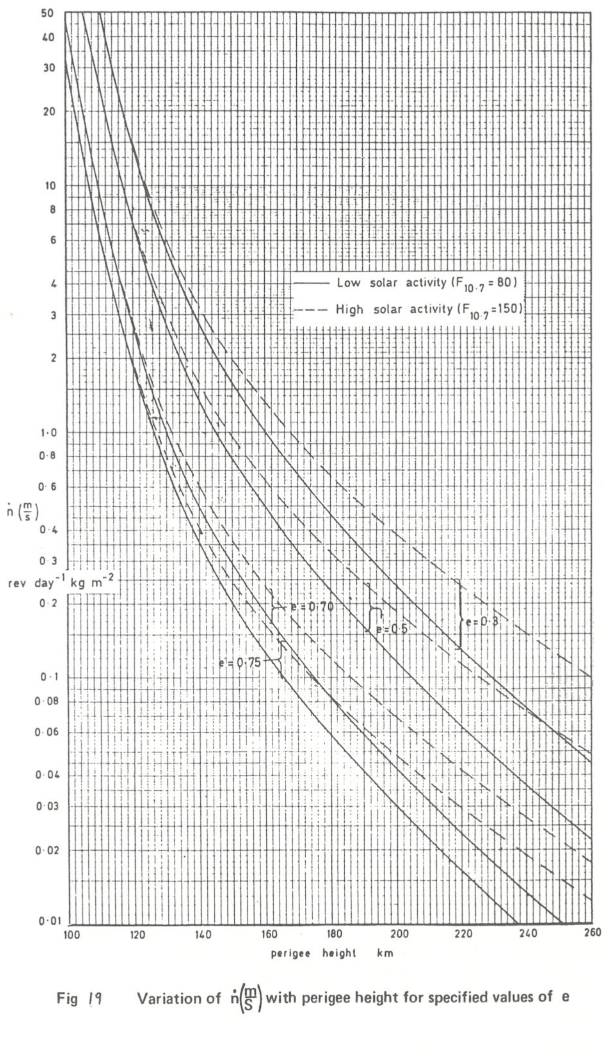

Fig 19 Variation of $\dot{n}(\frac{m}{S})$ with perigee height for specified values of e

```
SDC 2 LINE ELEMENTS

1  2805  67046  A 77284,33830000 0,00001890 *,0000E 00
2  2805  51,8300 212,3400 *,8153200 204,9200  66,0500  1,22798000

ELLIPTIC ORBIT INTEGRATION

DRAG IS     INCLUDED            LUNI-SOLAR PERTURBATIONS ARE INCLUDED          SOLAR RADIATION PRESSURE NOT
ZONAL HARMONICS (J2-J9) ARE INCLUDED BUT TESSERAL HARMONICS ARE NOT INCLUDED                        INCLUDED

AREA/MASS RATIO = 0,001 M2/KG                  SMEAN  =  80,0 X10E-22 W/M2/HZ
DECAY ALTITUDE  =  90.0 KM                     SOLBAR =  80,0 X10E-22 W/M2/HZ
```

TIME (MJD)	SEMI-MAJOR AXIS (KM)	ECCENT-RICITY	INCLINATION (DEG)	RIGHT ASCENSION (DEG)	ARGUMENT OF PERIGEE (DEG)	MEAN ANOMALY (DEG)	MEAN MOTION (DEG/S) (X1000)	TRUE PERIGEE HEIGHT (KM)	TRUE APOGEE HEIGHT (KM)
43427 0,188908	36964,956	0,8157642	51,8384	211,9745	204,9180	0,000002	5,089870	434,589	
43428 0,003306	36964,610	0,8159398	51,8508	211,8834	204,9865	-0,000042	5,089942	428,049	60492,978
43428 0,817726	36964,346	0,8161450	51,8600	211,7907	205,0584	0,000027	5,089996	420,431	60500,598
43429 0,632166	36964,116	0,8163638	51,8629	211,6949	205,1345	-0,000032	5,090044	412,315	60508,934
43430 0,446619	36964,095	0,8165758	51,8575	211,5948	205,2151	0,000025	5,090048	404,489	60517,177
43431 0,261071	36964,081	0,8167592	51,8437	211,4902	205,2991	0,000010	5,090051	397,724	60524,448

Fig 20 Sample PTDEC printout

drag: such an error could alter the lifetime by factors of up to 1000 or more. However, we find that the NORAD orbits are usually quite accurate for the mean motion n; so we obtain values for the rate of change of n by taking the difference in n between two orbits several days apart, or by using more than two orbits if they are available. Having obtained a value of $\dot{n}$ in this way, we use Fig 19 to determine the perigee height. We then alter the eccentricity on the NORAD orbit to give the correct perigee height and run PTDEC with that modified initial orbit. To obtain the perigee height from Fig 19, we need to specify the mass/area ratio of the satellite: this may not always be accurately known, but any errors are cancelled out to the first order because we use the same value of mass/area for the computer run. However, some errors from this source will occur because wrong values of scale height will be used if the perigee height is in error, even though the correct initial decay rate will be obtained.

After completing the computation with PTDEC, we then monitor its accuracy as the weeks pass, by calculating $\dot{n}$ from all subsequent NORAD orbits, finding the perigee height again from Fig 19 and checking this against the perigee height predicted by PTDEC for the appropriate date. Usually the comparisons go quite well but sometimes we have to re-run the program. Fig 20 shows a few lines of a sample PTDEC printout: the orbital elements are given at a time interval of 1 revolution.

There is no allowance in the PTDEC program for the decrease in drag coefficient at heights where the mean free path of the molecules becomes comparable with the dimensions of the satellite (Ref. 15). The mean free path is 3 m at a height of 120 km and 10 m at a height of 130 km, so this effect is usually not of importance until the last day of the satellite's life. On the rare occasions when predictions are being made so late in the life, we assume that the drag coefficient falls to half its normal value at the height where the mean free path is equal to the length of the satellite. This 'step-function bridging' between free-molecule and slip-flow is only a crude approximation, but it is usually adequate because the decrease in drag coefficient does not normally add more than one or two revolutions to the lifetime of the satellite. When PTDEC is being used, we can re-run the orbit from the time when the perigee falls to the crucial height, with the area/mass ratio half the value used on the main computer run.

5. CONCLUSIONS

These are the methods that have evolved gradually over the years, and they make a strange mixture of the simple and the computer-intensive. All the time we have been driven by the need to give lifetimes for all satellites, even those of an unknown size, shape and mass, and even if we only have very poor initial orbits.

For high-eccentricity orbits the poor data available often constitute the chief limitation; but the major source of error in long-term satellite prediction is the lack of knowledge of future solar activity. If the solar activity declines to the levels prevalent in the late 17th century, all the long lifetimes will be much greater than we have predicted. But at least we shall not be there to be blamed for our mistakes!

6. REFERENCES

1. King-Hele D G, Pilkington J A, Walker D M C, Hiller H and Winterbottom A N 1983, *The RAE Table of Earth Satellites 1957-1982* London, Macmillan.

2. King-Hele D G and Gilmore D M C 1956, Preliminary assessment of an earth-satellite reconnaissance vehicle, RAE Technical Note GW 393.

3. King-Hele D G and Gilmore D M C 1956, The descent of an earth satellite through the atmosphere. RAE Technical Note GW 430.

4. Staff of RAE 1957, Observations on the orbit of the first Russian earth satellite, *Nature*, 180, 937-941.

5. King-Hele D G 1983, Geophysical researches with the orbits of the first satellites, *Geophys. J. Roy. Astronom. Soc.*, 74, 7-23.

6. King-Hele D G 1978, Methods for predicting satellite orbital lifetimes, *J. Brit. Interplan. Soc.*, 31, 181-196.

7. Walker D M C 1978, Variations in air density from January 1972 to April 1975 at heights near 200 km, *Planet Space Sci.*, 26, 291-309.

8. King-Hele D G 1982, Lifetime prediction for satellites in low-inclination transfer orbits, *J. Brit. Interplan. Soc.*, 35, 339-344.

9. King-Hele D G 1975, The orbital lifetimes of Molniya satellites, *J. Brit. Interplan. Soc.*, 28, 783-796.

10. Cook G E 1973, Basic theory for PROD, a program for computing the development of satellite orbits, *Celest. Mech.*, 7, 301-314.

11. Palmer M D 1977, The computer program PTDEC, RAE Technical Memorandum, Space 248.

12. *COSPAR International Reference Atmosphere 1972*, 1972, Berlin, Akademie.

13. Cook G E and Palmer M D 1976, The elliptic orbit integration program POINT, RAE Technical Report 76129.

14. Merson R H 1974, Numerical integration of the differential equations of celestial mechanics, RAE Technical Report 74184.

15. Koppenwallner G 1985, The drag of simple shaped bodies in the rarefied hypersonic flow regime. Presented at 20th AIAA Thermophysics Conference.